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RP-89: Formulation of Some Classes of Solvable Standard Quadratic Congruence moduloa Prime Integer - Multiple of Three & Ten

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ABSTRACT

In this paper, the authors formulated two special types of standard quadratic congruence modulo a prime integer- multiple of three and ten. The standard quadratic congruence of prime modulus is discussed in the literature of mathematics. But no formulation was found for the congruence of the said type. It was not formulated by earlier mathematicians.So, the authors tried their best to formulate the said congruence. Formulation is the merit of the paper.

Key-words: Composite modulus, Chinese Remainder theorem,Quadratic congruence.

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INTRODUCTION

Congruence of the type:$x^2 \equiv a \ (mod\ p)$, p an odd positive integer, is called a standard quadratic congruence of prime modulus. If $p = m$, a composite integer, then the congruence is called a standard quadratic congruence of composite modulus.

The author[1]had formulated a lot of standard quadratic congruence of composite modulus successfully. Those are:

(1) $x^2 \equiv a \ (mod\ 4p)$[4]
(2) $x^2 \equiv a \ (mod\ 5p)$[5]
(3) $x^2 \equiv a \ (mod\ 6p)$[6]
(4) $x^2 \equiv a \ (mod\ 7p)$[7]
(5) $x^2 \equiv a \ (mod\ 8p)$[8]
(6) $x^2 \equiv a \ (mod\ 9p)$ [9];all these p being an odd prime integer.

Here the authors wish to consider two such standard quadratic congruence yet not formulated.

Such standard quadratic congruence are: $x^2 \equiv b \ (mod\ 3p), with\ (b, 3p) = 1$;

And$x^2 \equiv b \ (mod\ 10p)$, p being a positive prime integer.

It is also found that the quadratic congruence of prime modulus has exactly two incongruent solutions while the congruence of composite modulus has more than two solutions [2].

LITERATURE REVIEW

In the literature of mathematics, no formulation is found for the said congruence. Congruence under consideration can be solved only by the use of Chinese Remainder Theorem(CRT) [3]. No one had attempted to do anything for the students' sake. It is found that Thomas Koshy had attempted to do something on standard quadratic congruence of composite modulus [2]. No other mathematician cared the said congruence.

NEED OF RESEARCH

The use of Chinese Remainder Theorem is not good & affordable for the students. It is time-consuming. Students want to get rid of such method and want to feel comfortable in solving such quadratic congruence. It is only possible if the problem is formulated. The authors have tried their best to formulate the said congruence. Formulation is the need of the research.

PROBLEM-STATEMENT

The problem is "To formulate the standard quadratic congruence of composite modulus of the type:

$(1)x^2 \equiv b \ (mod\ 3p); (2)x^2 \equiv b \ (mod\ 10p),\ p\ an\ odd\ prime\ integer$."

ANALYSIS & RESULT (Formulation)

Case-I: Consider the congruence $x^2 \equiv b\ (mod\ 3p)$ with $(b, 3p) = 1$.

If it is solvable, it can be written as $x^2 \equiv b + k.3p = a^2\ (mod\ 3p)$ for some positive integer k [1].

It is of the type $x^2 \equiv a^2\ (mod\ 3p)$. Such congruence always has four incongruent solutions [3].

The two obvious solutions are $x \equiv \pm a\ (mod\ 3p)\ \equiv a, 3p - a\ (mod\ 3p)$.

Now, consider $x = \pm(2p \pm a)$

Then, $x^2 = (2p \pm a)^2 = 4p^2 \pm 4pa + a^2 = a^2 + 4p(p \pm a) = a^2 + 4p.3m,\ \ if\ p \pm a = 3$

Therefore, if $p \pm a = 3m, then\ x \equiv \pm(2p \pm a)$ are the other two solutions.

But, if $(a, 3p) \neq 1$, then the congruence has only two obvious solutions. Because, then b=$3k\ and\ ultimately, p \pm a \neq 3m$. Hence the second pair of solutions will not exist. And the said congruence must have the one pair of solutions.

Case-II: Considerthe standard quadratic congruence$x^2 \equiv b\ (mod\ 10p), (b, 10p) = 1$,

p being a prime positive integer. If it is always solvable, then it can be written as

$x^2 \equiv b + k.3p = a^2(mod\ 10p)$,for some positive integer k [1].

It is of the type $x^2 \equiv a^2\ (mod\ 10p)$.Such congruence always has four incongruent solutions [3].

Its two obvious solutions are $x \equiv \pm a\ (mod 10\ p)\ \equiv a, 10p - a\ (mod\ 10p)$.

Now, consider $x = (2p \pm a)$

Then, $x^2 = (2p \pm a)^2 = 4p^2 \pm 4pa + a^2 = a^2 + 4p(p \pm a) = a^2 + 4p.5m,\ \ if\ p \pm a = 5m.$

Also, consider $x = (4p \pm a)$

Then, $x^2 = (4p \pm a)^2 = 16p^2 \pm 8pa + a^2 = a^2 + 8p(2p \pm a) = a^2 + 8p.5m,\ \ if 2p \pm a = 5m.$

Thus,if $p \pm a = 5m,\ then\ x \equiv \pm(2p \pm a)$ are the other two solutions; OR

if$2p \pm a = 5m,\ then\ x \equiv \pm(4p \pm a)$ are the other two solutions.

If $(b, 10p) \neq 1$, then the congruence has only two solutions because then other solutions do not exist.

ILLUSTRATIONS

Consider the congruence$x^2 \equiv 1\ (mod\ 33)$

It can be written as $x^2 \equiv 1 = 1^2 (mod\ 3.11) with\ p = 11$

Thus, the congruence is of the type $x^2 \equiv a^2 (mod\ 3p)$

Then $x \equiv \pm a = \pm 1\ (mod\ 33) \equiv 1, 32\ (mod\ 33)$ are the two obvious solutions.

We also see that $p + a = 11 + 1 = 12 = 3.4 = 3m$

Thus, $x \equiv \pm(2p + a) = \pm(2.11 + 1) = \pm 23 = 23, 10\ (mod\ 33)$

are the other two solutions.

Therefore, the said congruence under considerations has four solutions

$$\boldsymbol{x \equiv 1, 32;\ 10, 23\ (mod\ 33)}$$

Consider the congruence $x^2 \equiv 7\ (mod\ 93)$

It can be written as $x^2 \equiv 7 + 93 = 100 = 10^2 (mod\ 3.31) with\ p = 31$ [1]

Thus, the congruence is of the type $x^2 \equiv a^2 (mod\ 3p)$

Then $x \equiv \pm a = \pm 10\ (mod\ 93) \equiv 10, 83\ (mod\ 93)$ are the two obvious solutions

For the other two solutions, we see that $p - a = 31 - 10 = 21 = 3.7 = 3m$

Thus, $x \equiv \pm(2p - a) = \pm(2.31 - 10) = \pm 52 = 52,\ 41\ (mod\ 93)$

are the other two solutions.

Therefore, the said congruence under considerations has four solutions

$$\boldsymbol{x \equiv 10, 83; 41, 52\ (mod\ 93)}$$

Consider congruence: $x^2 \equiv 3\ (mod\ 39)$

It can be written as $x^2 \equiv 3 + 2.39 = 81 = 9^2 (mod\ 3.13)\ with\ p = 13.$

Then two obvious solutions are $x \equiv \pm 9 = 9, 24\ (mod\ 39)$

Also, $(3, 39) = 3 \neq 1\ and\ hence$ the congruence has no other solutions.

Thus, it has only two solutions.

Consider the congruence $x^2 \equiv 144\ (mod\ 310)$

It can be written as $x^2 \equiv 144 = 12^2 (mod\ 10.31) with\ p = 31$

Thus, the congruence is of the type $x^2 \equiv a^2 (mod\ 10p)$

Then, $x \equiv 10p \pm a = 310 \pm 12\ (mod\ 310) \equiv 12,\ 298\ (mod\ 310)$ are the two obvious solutions.

We also see that$2p - a = 2.31 - 12 = 62 - 12 = 50 = 5.10 = 5m.$

Thus, $x \equiv \pm(4p - a) = \pm(4.31 - 12) = \pm 112 = 112, 198\ (mod\ 310)$

are the other two solutions.

Therefore, the said congruence under considerations has four solutions

$$\boldsymbol{x \equiv 12, 298;\ 112, 198\ (mod\ 310)}.$$

Consider the congruence $x^2 \equiv 49\ (mod\ 230)$

It can be written as $x^2 \equiv 49 = 7^2 (mod\ 10.23) with\ p = 23$.

Thus, the congruence is of the type: $x^2 \equiv a^2 (mod\ 10p)$.

Then, $x \equiv 10p \pm a = 230 \pm 7 (mod\ 230) \equiv 7, 223\ (mod\ 230)$ are the two obvious solutions.

We see that:$p + a = 23 + 7 = 30 = 5.6 = 5m.$

Thus, $x \equiv \pm(2p + a) = \pm(2.23 + 7) = \pm 53 = 53, 177\ (mod\ 230)$ are the other two solutions.

Therefore, the said congruence under considerations has four solutions

$\boldsymbol{x \equiv 7, 223; 53, 177\ (mod\ 230)}$**.**

Consider the congruence $x^2 \equiv 16\ (mod\ 190)$

It can be written as $x^2 \equiv 16 = 4^2 (mod\ 10.19) with\ p = 19$

Thus, the congruence is of the type $x^2 \equiv a^2 (mod\ 10p)$

Then $x \equiv \pm a = \pm 4\ (mod\ 190) \equiv 4, 186\ (mod\ 190)$ are the two obvious solutions

For the other two solutions, we see that $p - a = 19 - 4 = 15 = 5.3 = 5m.$

Thus, $x \equiv \pm(2p - a) = \pm(2.19 - 4) = \pm 34 = 34, 156\ (mod\ 190)$

are the other two solutions.

Therefore, the said congruence under considerations has four solutions

$\boldsymbol{x \equiv 4, 186; 34, 156 \ (mod\ 190).}$

Consider another congruence as per need: $x^2 \equiv 11\ (mod\ 110)$.

It can be written as $x^2 \equiv 11 + 110 = 121 = 11^2\ (mod\ 110)$.

Two obvious solutions are $x \equiv 10p \pm a = 110 \pm 11 = 11, 99\ (mod\ 110)$.

We see that $(11, 110) \neq 1$; and hence it has only two solutions.

CONCLUSION

Case-I: Therefore, we conclude that the congruence:$x^2 \equiv a^2 (mod\ 3p)$; $(a, 3p) = 1$,

has four incongruent solutions; two are given by:$\boldsymbol{x \equiv \pm a = a, 3p - a\ (mod\ 3p)}$.

And other two solutions are in the followings:

If $\boldsymbol{p \pm a = 3m, then\ x \equiv \pm(2p \pm a)}$ are the other two solutions

But if $\boldsymbol{(b, 3p) \neq 1, then\ the\ congruence\ have\ only\ two\ obvious\ solutions}$.

Case-II:

The congruence:$x^2 \equiv a^2 (mod\ 10p)$ has four incongruent solutions; two are given by:

$$\boldsymbol{x \equiv \pm a = a, 10p - a\ (mod\ 10p)}.$$

And other two solutions are in the followings:

If $\boldsymbol{p \pm a = 5m,\ then\ \ x \equiv \pm(2p \pm a)}$ are the other two solutions. OR

If $\boldsymbol{2p \pm a = 5m,\ then\ \ x \equiv \pm(4p \pm a)}$ are the other two solutions.

MERIT OF THE PAPER

In this paper, some classes of standard quadratic congruence of composite modulus- an odd positive prime integer multiple of three & ten, are formulated. It was not formulated by earlier mathematicians. First time, a formula is established. No need to use Chinese Remainder Theorem. This is the merit of the paper.

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