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RP-90: Formulation of Two Special Classes of Standard Cubic Congruence of Composite Modulus- A Power of Three

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Abstract:

In this paper, two special types of standard cubic congruence of composite modulus—a power of three, are considered for study and are formulated. Formulation of the solutions is proved time-saving, simple and quick. It made finding solutions of cubic congruence of composite modulus easy. Solutions can also be obtained orally. Formulation is the merit of the paper.

Key-words: Binomial Theorem, Cubic Congruence, Composite Modulus.

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Introduction

Here, two standard cubic congruence of *composite modulus* are considered for discussion. It is of the type: $x^3 \equiv a^3 (mod\ m)$, m being a positive composite integer.

In different books on Number Theory, no discussion is found for the said congruence. Only the standard quadratic congruence are discussed [3]. Much had been written on standard quadratic congruence of prime modulus. But no discussion and formulation for standard cubic congruence of composite modulus is found. A short discussion is found in the book of Thomas Koshy [2].

Once a student had brought a cubic congruence of composite modulus in the class to have a discussion on the method of finding solutions. It was not a problem of study in the syllabus. So, it was difficult to solve. The author considered the congruence for his research. Many papers on the formulation of standard cubic congruence of composite modulus have been published in different international journals [4], [5], [6]& [7]. Even some remains to formulate. The author considered two of such congruence for formulations.

Problem-Statement

The problem is:

"To formulate the cubic congruence:$x^3 \equiv a^3 (mod 3^n)$&$x^3 \equiv a^3 (mod 3^n.b)$.

Analysis & Results

Consider the congruence $x^3 \equiv a^3 (mod 3^n),\ n \geq 2$.

It can be seen that for$x \equiv 3^{n-1}k + a \quad (mod\ 3^n)$,

$$x^3 \equiv (3^{n-1}k + a)^3$$

$\equiv a^3 + 3^n(t)$, by Binomial Theorem

$$\equiv a^3 \ (mod\ 3^n)$$

Thus, $x \equiv 3^{n-1}k + a \ \ (mod\ 3^n)$ is a solution of the standard cubic congruence:

$$x^3 \equiv a^3 (mod\ 3^n).$$

It can also be seen that for $k = 0, 1, 2$ the said congruence has three incongruent solutions. But for $k = 3,4, 5, \ldots..$ one get the same solutions as for $k = 0, 1, 2$. Thus, the congruence has exactly three incongruent solutions for $k = 0, 1, 2$.

Sometimes the congruence can be of the type: $x^3 \equiv b \ (mod\ m)$.

But it can be written as $x^3 \equiv b + km = a^3 \ (mod\ m) for$ some positive integer k [1].

If $m = 3^n.b, b \neq 2^m,$ then the congruence becomes:

$x^3 \equiv a^3 \ (mod\ 3^n.b)$.

Then, it can be seen that for$x \equiv 3^{n-1}bk + a \quad (mod\ 3^n.b)$,

$$x^3 \equiv (3^{n-1}bk + a\ )^3$$

$\equiv a^3 + 3^n b(t)$, by Binomial Theorem

$$\equiv a^3 \ (mod\ 3^n.b)$$

Thus, $x \equiv 3^{n-1}bk + a \ \ (mod\ 3^n.b)$ is a solution of the cubic congruence:

$$x^3 \equiv a^3 (mod\ 3^n.b).$$

It can also be seen that for $k = 0, 1, 2$ the said congruence has incongruence solutions. But for $k = 3,4, 5, \ldots..$ one get the same solutions as for $k = 0, 1, 2$. Thus, the congruence has exactly three incongruence solutions with $k = 0, 1, 2$.

ILLustrations

Consider the congruence:$x^3 \equiv 512 \ \ (mod\ 729)$.

It can be written as $x^3 \equiv 8^3 \ (mod\ 3^6)$ with $a = 8, \ \ n = 6$.

It is of the type: $x^3 \equiv a^3 (mod\ 3^n)$.

Its solutions are given by $x \equiv 3^{n-1}k + a \ (mod\ 3^n); k = 0,1,2.$

$$\equiv 3^5 k + 8 \ (mod\ 3^6); k = 0, 1, 2.$$

Therefore, required solutions are $x \equiv 0 + 8, 3^5 + 8, 3^5.2 + 8 \ \ (mod\ 3^6)$.

$$\equiv 8, 243 + 8, 486 + 8 \ (mod\ 729)$$

$$\equiv 8, 251, 494 \ (mod\ 729).$$

Consider one more example: $x^3 \equiv 44 \ (mod\ 81)$.

It can be written as $x^3 \equiv 44 + 81 = 125 = 5^3 (mod\ 81)$.

It is of the type: $x^3 \equiv a^3 (mod\ 3^n)\ with\ a = 5, n = 4$.

Its solutions are: $x \equiv 3^{n-1}k + a\ (mod\ 3^n);\ k = 0,1,2.$

$$\equiv 3^3k + 5\ (mod\ 3^4)$$

$$\equiv 27k + 5\ (mod\ 81)$$

$$\equiv 0 + 5, 27 + 5, 54 + 5\ (mod\ 81)$$

$$\equiv 5, 32, 59\ (mod\ 81).$$

Consider the congruence: $x^3 \equiv 343 \ (mod\ 10125)$.

It is seen that $10125 = 81.125 = 3^4.125$ and the congruence can be written as

$x^3 \equiv 7^3\ (mod\ 3^4.125)$with$a = 7,\ n = 4,\ b = 125$.

It is of the type: $x^3 \equiv a^3 (mod\ 3^n.b)$.

Its solutions are given by $x \equiv 3^{n-1}.bk + a\ (mod\ 3^n.b);\ k = 0,1,2.$

$$\equiv 3^3.125k + 7\ (mod\ 3^4.125); k = 0, 1, 2.$$

$$\equiv 3375k + 7\ (mod\ 81.125)$$

Therefore, required solutions are $x \equiv 0 + 7, 3375 + 7, 6750 + 7 \ (mod\ 10125)$.

$$\equiv 7, 3382, 6757\ (mod\ 10125)$$

Consider one more example: $x^3 \equiv 26\ (mod\ 486)$ with $486 = 3^4.6$

It can be written as $x^3 \equiv 26 + 486 = 512 = 8^3 (mod\ 3^4.6)$.

It is of the type: $x^3 \equiv a^3 (mod\ 3^n.b)\ with\ a = 8, n = 4$.

Its solutions are: $x \equiv 3^{n-1}bk + a\ (mod\ 3^n.b);\ k = 0,1,2.$

$$\equiv 3^3.6k + 8\ (mod\ 3^4.6)$$

$$\equiv 162k + 8\ (mod\ 81.6) with\ k = 0, 1, 2.$$

$$\equiv 0 + 8, 162 + 8, 324 + 8\ (mod\ 486)$$

$$\equiv 8, 170, 332\ (mod\ 486).$$

Conclusion:

Thus, it can be concluded that the standard cubic congruence of composite modulus:

$x^3 \equiv a^3 \ (mod\ m)$is formulated.

If $m = 3^n, n \geq 2$, then the congruence $x^3 \equiv a^3 \ (mod\ 3^n)$ has exactly three incongruent solutions: $x \equiv 3^{n-1}k + a \ (mod\ 3^n) \ \ with\ k = 0, 1, 2.$

If $m = 3^n.b, then\ the\ congruence\ x^3 \equiv a^3 \ (mod\ 3^n.b)$ has exactly three solutions

$$x \equiv 3^{n-1}.bk + a \ (mod\ 3^n.b) with\ k = 0, 1, 2.$$

REFERENCE

[1] Roy B M, 2016, Discrete Mathematics & Number Theory, First edition, Das GanuPrakashan, Nagpur (INDIA),ISBN: 978-93-84336-12-7.

[2] Thomas Koshy, 2009, "Elementary Number Theory with Applications", 2/e Indian print, Academic Press, ISBN: 978-81-312-1859-4.

[3] H S Zuckerman at el, 2008, An Introduction to The Theory of Numbers, fifth edition, Wiley student edition, INDIA, ISBN: 978-81-265-1811-1.

[4] Roy B M, 2019,*Formulation of a class of solvable standard cubic congruence of even composite modulus,* International Journal of Advanced Research, Ideas and Innovations in Technology (IJARIIT), ISSN: 2454-132X, Vol-05, Issue-01, Jan-19.

[5] Roy B M, 2019,*Formulation of solutions of a class of standard cubic congruence modulopower of an integer multiple of power of three, International Journal of Recent Innovations in Academic Research (IJRIAR), ISSN: 2659-1561, Vol-03, Issue-01, Jan-19.*

[6] Roy B M, 2019,*Formulation of solutions of a class of standard cubic congruence of even composite modulus- a power of an odd positive integer multiple of power of three,* International Journal for Research, Trends and Innovations(IJRTI), ISSN:2456-3315,Vol-04, Issue-03, Mar-19.

[7] Roy B M, 2019,*Formulation of solutions of a class of standard cubic congruence of even composite modulus- an r^{th}power of an odd positive integer multiple of n^{th} power of three*, Research Journal of Mathematical and Statistical Sciences, (RJMSS), International Science Community Association(ISCA), ISSN:2320-6047,Vol-9, Issue-3, May-19.

