International Journal of Scientific Research and Engineering Development-– Volume X Issue X, Year

Available at www.ijsred.com

RESEARCH ARTICLE OPEN ACCESS

Static Flexural Analysis of Thick Beam Using Higher Order Shear Deformation Theory

N.G.Patil*, Dr.G.R.Gandhe, D.H.Tupe***, D.B.Gaidhane ******

*N.G.Patil, P.G.Student, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad, Maharashtra, India. Email-patil.nikita13@gmail.com

**Dr. G.R.Gandhe, Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad, Maharashtra, India. Email-gajendragandhe@gmail.com

***D.H.Tupe, Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad, Maharashtra, India. Email-durgeshtupe@gmail.com

****D.B.Gaidhane, Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad, Maharashtra, India. Email-gaidhanedipak23@gmail.com

--*************************---

Abstract –

In the present study, a higher order shear deformation theory is developed for static flexure analysis of thick isotropic beam. Cantilever beam is analyzed for the axial displacement, Transverse displacement, Axial bending stress and transverse shear stress. The governing differential equation and boundary conditions of the theory are obtained by the principle of virtual work. The numerical results have been computed for various length to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, trigonometric and other hyperbolic shear deformation theories and with the available solution in the literature.

***Key Words*: Higher order shear deformation theory, Isotropic beam, virtual work, Shear deformation, thick beam, static flexure, transverse shear stress etc.**

--*************************---

1.INTRODUCTION

Theories of beams and plates are essentially one and two-dimensional approximations of the corresponding two and three-dimensional theories of elastic bodies i.e., beams and plates. These are basically the reduction problems. Since the thickness dimension is much less than the others, it is possible to approximate the distribution of the displacements, strains and stress components in thickness dimension. In the displacement-based theories In-plane and transverse displacements are expanded in the thickness coordinate using Taylor series or power series and truncating the series at the required power of thickness coordinate. This power governs the order of displacement-based theory. In literature such theories are called as higher order shear deformation theories. In general, third order theories are widely used for analysis of thick beams and plates in order to have the quadratic shear stress distribution through the thickness. In TSDT, HPSDT and ESDT approach, series is expressed in terms of trigonometric, hyperbolic and exponential functions with thickness coordinate. In displacement-based theories, generally, principle of virtual work is employed to obtain the variationally consistent governing equations and boundary conditions.

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis that the plane sections which are perpendicular to the neutral layer before bending remain plane and perpendicular to the neutral layer after bending, implying that the transverse shear and transverse normal strains are zero. Thus, the theory disregards the effects of the shear deformation. It is also known as classical beam theory. The theory is applicable to slender beams and should not be applied to thick or deep beams. When elementary theory of beam (ETB) is used for the analysis thick beams, deflections are underestimated and natural frequencies and buckling loads are overestimated. This is the consequence of neglecting transverse shear deformations in ETB.

The various methods of development of refined theories based on the reduction of the three dimensional problem of mechanics of elastic bodies are discussed by Vlasov and leontev [1], Donnell[2], Kil chevskiy [3], Gol denveizer [4].Rankine[5], Bresse[6]were the first to include both rotatory inertia and shear flexibility effects as refined dynamical effects in beam theory. This theory is, referred as Timoshenko beam theory as mentioned in the literature by Rebello,et.al.[7] and based upon kinematics it is known as first order shear deformation theory(FSDT). Stephen and Levinson [8] have introduced a refined theory incorporating shear curvature, transverse direct stress and rotatory inertia effect. The limitations of the elementary theory of bending of beams and first order shear deformation theory (FSDT) for beams forced the development of higher order shear deformation theories.

In this paper, a higher order shear deformation theory is developed for static flexural analysis of thick isotropic beams. The theory is applied to a cantilever isotropic beam to analyzed the axial displacement, Transverse displacement, axial displacement, axial bending stress and transverse shear stress. The numerical results have been computed for various length to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, trigonometric and other hyperbolic shear deformation theories and with the available solution in the literature.

2. Formulation of Problem

Consider a thick isotropic cantilever beam of length L in x direction, Width b in y direction and depth h as shown in fig. Where x,y,z are Cartesian coordinate. The beam is subjected to transverse load intensity sin(pix/l) per unit length beam. Under this condition, the axial displacement, Transverse displacement, axial displacement, axial bending stress and transverse shear stress are to be determined.

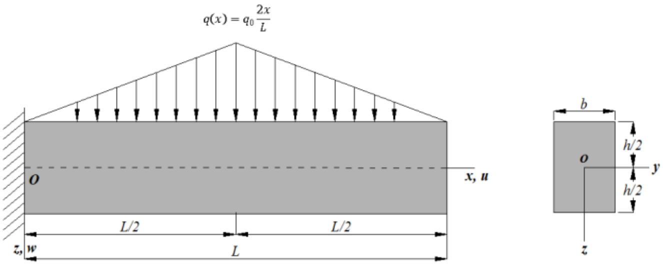

Fig -1: Cantilever beam bending under x-z plane

A. Assumptions made in the theoretical formulation

1. The axial displacement (u) consists of two parts:
   a) Displacement given by elementary theory of bending.
   b) Displacement due to shear deformation, which is assumed to be hyperbolic in nature with respect to thickness coordinate, such that maximum shear stress occurs at neutral axis as predicted by the elementary theory of bending of beam.
2. The axial displacement (u) is such that the resultant of In-plane stress (σx) acting over the cross-section should result in only bending moment and should not in force in x direction.
3. The transverse displacement (w) in z direction is assumed to be function of x coordinate.
4. The displacements are small as compared to beam thickness.
5. The body forces are ignored in the analysis. (The body forces can be effectively taken into account by adding them to the external forces.)
6. One dimensional constitutive laws are used.

7.The beam is subjected to lateral load only.

B. The Displacement field-

Based on the above mentioned assumptions, the displacement field of the present beam theory can be expressed as follows.

$$u(x,z) = -z\frac{\partial w}{\partial x}(x) + \left[\frac{z}{2}\left(\frac{h^2}{4} - \frac{z^2}{3}\right)\right]\phi(x) \quad (1)$$

$$w(x,z) = w(x) \quad (2)$$

Where,

u = Axial displacement in x direction which is function of x and z.

w = Transverse displacement in z direction which is function of x.

ϕ = Rotation of cross section of beam at neutral axis which is function of x.

Normal Strain:

$$\in_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + \left(\frac{zh^2}{8} - \frac{z^3}{6}\right)\frac{\partial \phi}{\partial x} \quad (3)$$

Shear strain

$$\gamma_{xz} = \left[\frac{h^2}{8} - \frac{3z^2}{6}\right]\phi(x) \quad (4)$$

Stresses:

$$\sigma_x = E\in_x = -zE\frac{\partial^2 w}{\partial x^2} + E\left[\frac{zh^2}{8} - \frac{z^3}{6}\right]\frac{\partial \phi}{\partial x} \quad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G\left[\frac{h^2}{8} - \frac{3z^2}{6}\right]\phi \quad (6)$$

Where E and G after elastic constant of the beam material.

C. Governing differential equation

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=h/2}(\sigma_x.\delta\in_x + \tau_{xz}.\delta\gamma_{xz})\,dxdz - \int_{x=0}^{x=L} q\delta w dx = 0 \quad (7)$$

Where δ = variational operator

Employing Green's theorem in equation (3.8) successively we obtain the coupled Euler Lagrange's equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows.

$$EI\left[\frac{\partial^4 w}{\partial x^4} - A_0\frac{\partial^3 \phi}{\partial x^3}\right] = q(x) \quad (8)$$

$$EI\left[A_0\frac{\partial^3 w}{\partial x^3} - B_0\frac{\partial^2 \phi}{\partial x^2}\right] + GAC_0\phi = 0 \quad (9)$$

Where A_0, B_0 and C_0 are the stiffness coefficients in governing equations. The associated consistent natural boundary conditions obtained are of following form along the edges x = 0 and x = L.

$$V_x = EI\left[\frac{\partial^3 w}{\partial x^3} - A_0\frac{\partial^2 \phi}{\partial x^2}\right] = 0 \quad (10)$$

Where w is prescribed

$$M_x = EI\left[\frac{\partial^2 w}{\partial x^2} - A_0\frac{\partial \phi}{\partial x}\right] = 0 \quad (11)$$

Where $\frac{dw}{dx}$ is prescribed.

$$M_x = EI\left[A_0\frac{\partial^2 w}{\partial x^2} - B_0\frac{\partial \phi}{\partial x}\right] = 0 \quad (12)$$

Where ϕ is Prescribed.

D. The General solution of Governing equilibrium equations of beam:

The general solution for transverse displacement w(x) and ϕ(x) can be obtained from Eqn. (8) and (9) by discarding the terms containing time (t) derivatives. Integrating and rearranging the Eqn. (9), we obtained the following equation,

$$\frac{\partial^3 w}{\partial x^3} = A_0\frac{\partial^2 \phi}{\partial x^2} + \frac{Q(x)}{D} \quad (13)$$

where, Q(x) is generalised shear force for beam and it is given by

$$Q(x) = \int_0^x q dx + k1 \quad (14)$$

And by rearranging second governing Eqn. (9) the following equation is obtained.

$$\frac{\partial^3 w}{\partial x^3} = \frac{B_0}{A_0}\frac{\partial^2 \phi}{\partial x^2} - B_0 \quad (15)$$

Now a single equation in terms of ϕ is obtained, by putting the Eqn. (6) in second governing Eqn. (15)

$$\alpha\left(\frac{\partial^2\phi}{\partial x^2}\right)-\beta(\phi)=\frac{Q(x)}{EI} \quad (16)$$

$$\phi=k_2\cosh(\lambda x)+k_3\sinh(\lambda x)-\left(\frac{Q(x)}{\beta EI}\right) \quad (17)$$

The equation of transverse displacement w(x) is obtained by substituting the expression of ϕ(x) in Eqn. (15) and integrating it thrice with respect to x. The general solution for w(x) is obtained as follows:

$$EIw(x)=\int\int\int qdxdxdxdx+\frac{D}{\lambda^3}\left(\frac{B_0}{A_0}\lambda^2-\beta\right)(k_2\sinh\lambda x+k_3\cosh\lambda x)+\frac{k_1x^3}{6}+k_4\frac{x^2}{2}+k_5x+k_6 \quad (18)$$

where k1, k2, k3, k4, k5 and k6 are the constants of integration and can be obtained by imposing natural (forced) and kinematic boundary conditions of beams.

3. Illustrative Example

In order to prove the efficiency of the present theory, the following numerical examples are considered. The following material properties for beam are used. Material properties:

1. Modulus of Elasticity E = 210 GPa
2. Poisson's ratio μ = 0.30
3. Density = 7800 kg/m3

A. **C**antilever beam with sine load $q(x)=q_0\frac{\sin\pi x}{L}$

The beam has its origin on left hand side fixed support at x = 0 and free at x = L. The beam is subjected to varying load, q(x) on surface z = +h/2 acting in downward z direction with minimum intensity of load $\frac{\sin\pi x}{L}$

Boundary conditions associated with this problem are as follows:

At free end: x=L

$$EI\frac{\partial^2w}{\partial x^2}=EI\frac{\partial\phi}{\partial x}=EI\frac{\partial^3w}{\partial x^3}=EI\frac{\partial^2\phi}{\partial^2x^2}=0$$

At fixed end: x=0

$$EI\frac{\partial w}{\partial x}=EI\phi==EIw=0$$

General Expressions obtained for w(x) and Q(x) are as follows.

$$\phi(x)=\frac{A_0}{C_0}\frac{1}{\pi}\frac{q_0L}{Gbh}\left[\sinh\lambda x-\cosh\lambda x+1-\cos\frac{\pi x}{L}\right] \quad (19)$$

$$w(x)=\left\{\begin{array}{l}\frac{120}{\pi}\left[\frac{x^2}{2L^2}-\frac{x^3}{6L^3}+\frac{1}{\pi^3}\frac{\sin\pi x}{L}-\frac{x}{\pi^2L}\right]+\\ 10\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}\frac{1}{\pi}\left(\frac{\cosh\lambda x-\sinh\lambda x-1}{\lambda L}+\frac{x}{L}\right)\\ +\left[10\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\left(\frac{\sin\pi x}{L}\frac{1}{\pi^2}-\frac{x}{\pi L}\right)\right]\end{array}\right\} \quad (20)$$

$$\bar{u}=-\frac{z}{h}\frac{L^3}{h^3}\left\{\begin{array}{l}\left[\begin{array}{l}\frac{12}{\pi}\left[\frac{x}{L}-\frac{x^2}{2L^2}+80\frac{1}{\pi^2}\cos\frac{\pi x}{L}-\frac{1}{\pi^2}\right]+\\ \frac{1}{\pi}\frac{E}{G}\frac{h^2}{L^2}\frac{A_0^2}{C_0}(\sinh\lambda x-\cosh\lambda x+1)\end{array}\right]+\\ \left[20\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\left(\frac{\cos\pi x}{L}\frac{1}{\pi}-\frac{1}{\pi}\right)\right]+\\ \left[\begin{array}{l}\frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\frac{1}{\pi}\left(\frac{zh}{8}-\frac{z^3}{6h^3}\right)\\ \left(\sinh\lambda x-\cosh\lambda x+1+\frac{\cos\pi x}{L}\right)\end{array}\right]\end{array}\right\} \quad (21)$$

$$\overline{\sigma_x}=-\frac{z}{h}\frac{L^2}{h^2}\left\{\begin{array}{l}\left[\begin{array}{l}\frac{12}{\pi}\left(1-\frac{x}{L}-\frac{\sin\pi x}{L}\frac{1}{\pi}\right)+\frac{E}{G}\frac{h^2}{L^2}\frac{1}{\pi}\frac{A_0^2}{C_0}\\ (\lambda L\cosh\lambda x-\lambda L\sinh\lambda x)\end{array}\right]+\\ \left[\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\left(-\frac{\sin\pi x}{L}\right)\right]+\\ \left[\begin{array}{l}\frac{A_0}{C_0}\frac{1}{\pi}\frac{E}{G}\left(\frac{zh}{8}-\frac{z^3}{6h}\right)\\ \left(\lambda L\cosh\lambda x-\lambda L\sinh\lambda x-\frac{\pi\sin\pi x}{L}\right)\end{array}\right]\end{array}\right\} \quad (22)$$

$$\overline{\tau_{zx}^{CR}} = \frac{A_0}{C_0}\frac{L}{h}\frac{1}{\pi}\left(\sinh\lambda x - \cosh\lambda x + 1 + \frac{\cos\pi x}{L}\right)\left[\left(\frac{h^2}{8}\right) - \frac{3z^2}{6}\right] \quad (23)$$

$$\overline{\tau_{zx}^{EE}} = \frac{1}{8}\frac{L}{h}\left(4\frac{z^2}{h^2} - 1\right)\left\{\begin{matrix}\left[\begin{matrix}\frac{12}{\pi}\left(-1 - \frac{\cos\pi x}{L}\right) + \frac{E}{G}\frac{h^2}{L^2}\frac{{A_0}^2}{C_0}\frac{1}{\pi}\lambda^2 L^2 \\ (\sinh\lambda x - \cosh\lambda x) \\ -\left(\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\right)\pi\frac{\cos\pi x}{L}\end{matrix}\right] + \\ \left\{\frac{1}{\pi}\frac{A_0}{C_0}\frac{E}{G}\frac{h}{L}\left[\left(\frac{z^2h^2}{16}\right) - \left(\frac{z^4}{24}\right) + \left(\frac{5h^4}{384}\right)\right]\right\} \\ \left(\lambda^2L^2\sinh\lambda x - \lambda^2L^2\cosh\lambda x - \pi^2\frac{\cos\pi x}{L}\right)\end{matrix}\right\} \quad (24)$$

4. Numerical Result

The numerical results for axial displacements, transverse displacements, bending stress and transverse shear stress are presented in following non dimensional form and the values are presented in table 1 and table 2.

$$\bar{w} = \frac{10Ebh^3}{q_0L^4}w \qquad \bar{u} = \frac{Eb}{q_0h}u$$

$$\overline{\sigma_x} = \frac{b}{q_0}\sigma_x \qquad \overline{\tau_{zx}} = \frac{b}{q_0}\tau_{zx}$$

Table 1: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = L, z = 0.0), Axial Stress ($\overline{\sigma_x}$) at (x = 0:0, z = h/2), Maximum Transverse Shear Stresses ($\overline{\tau_{zx}^{CR}}$) and ($\overline{\tau_{zx}^{EE}}$) at (x = 0:01L, z = h/2) of Cantilever Beam Subjected to Varying Load for Aspect Ratio 4.

Source	Model	$\bar{w}$	$\bar{u}$	$\overline{\sigma_x}$	$\overline{\tau_{zx}^{CR}}$	$\overline{\tau_{zx}^{EE}}$
Present	HSDT	8.76	34.31	36.55	0.22	6.49
Ghugal	HPSDT	8.75	34.17	36.85	0.24	7.03
Dahake	TSDT	8.74	34.30	36.24	0.18	5.82
Timoshen	FSDT	11.33	36.34	30.55	0.93	0.29
Bernoulli-Euler	ETB	8.86	36.32	30.55	-	0.29

Table 1: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = L, z = 0.0), Axial Stress ($\overline{\sigma_x}$) at (x = 0:0, z = h/2), Maximum Transverse Shear Stresses ($\overline{\tau_{zx}^{CR}}$) and ($\overline{\tau_{zx}^{EE}}$) at (x = 0:01L, z = h/2) of Cantilever Beam Subjected to Varying Load for Aspect Ratio 10.

Source	Model	$\bar{w}$	$\bar{u}$	$\overline{\sigma_x}$	$\overline{\tau_{zx}^{CR}}$	$\overline{\tau_{zx}^{EE}}$
Present	HSDT	7.485	562.827	205.8616	0.686	5.950
Ghugal	HPSDT	7.481	562.491	206.7878	0.729	6.74
Dahake	TSDT	7.485	562.807	205.2157	0.557	5.782
Timoshenko	FSDT	11.564	567.911	190.9859	14.546	0.74
Bernoulli-Euler	ETB	8.862	567.911	190.9859	14.5462	0.748

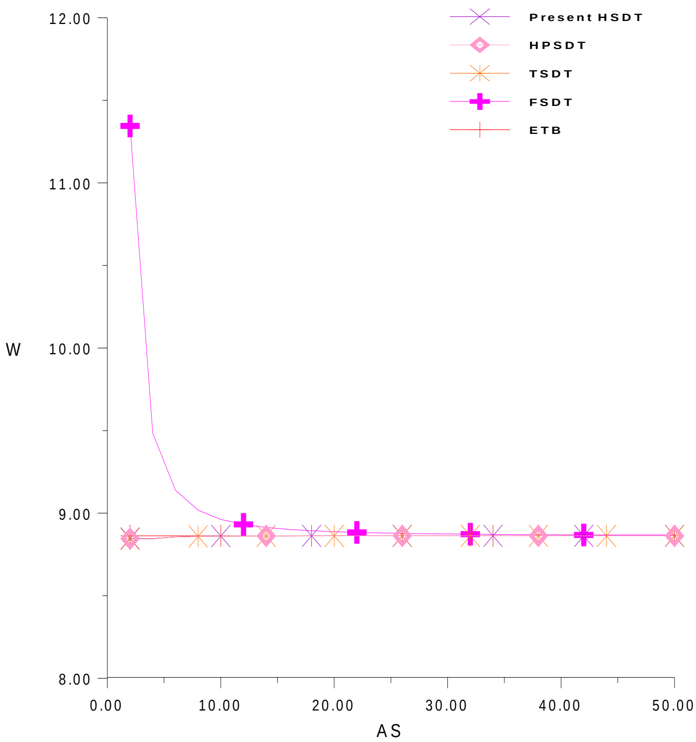

Fig -2: Variation of transverse displacements $\bar{w}$

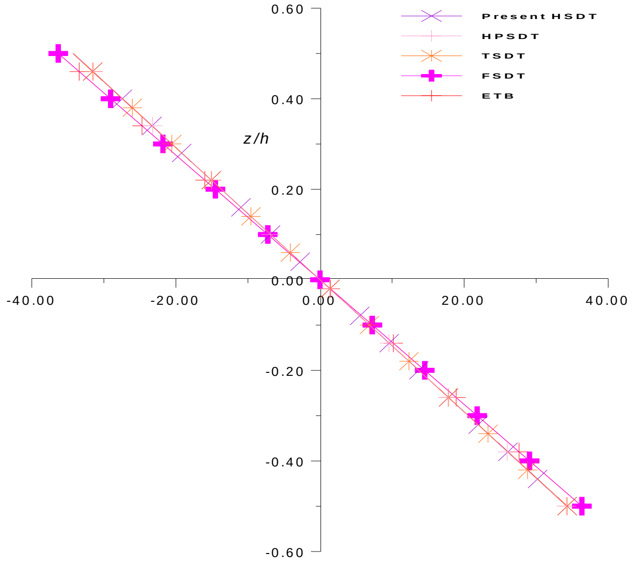

Fig -3: Variation of Maximum Axial displacement $\bar{u}$ for AS 04

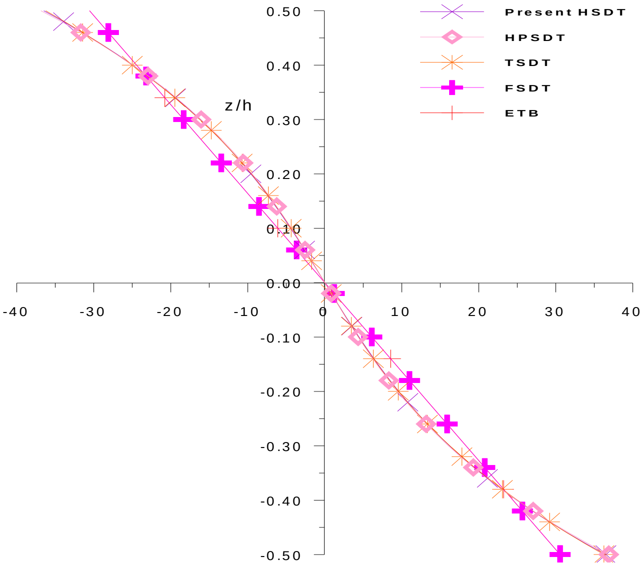

Fig -5: Variation of maximum axial stress $\overline{\sigma_x}$ for AS 04

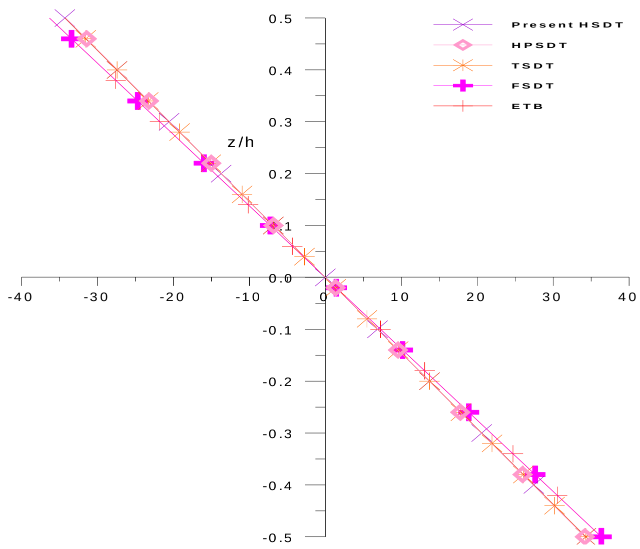

Fig -4: Variation of Maximum Axial displacement $\bar{u}$ for AS 10

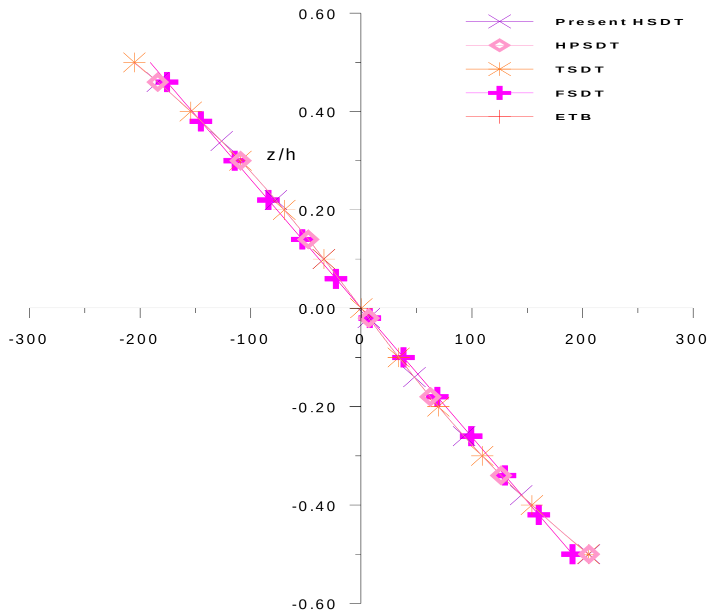

Fig -6: Variation of maximum axial stress $\overline{\sigma_x}$ for AS 10

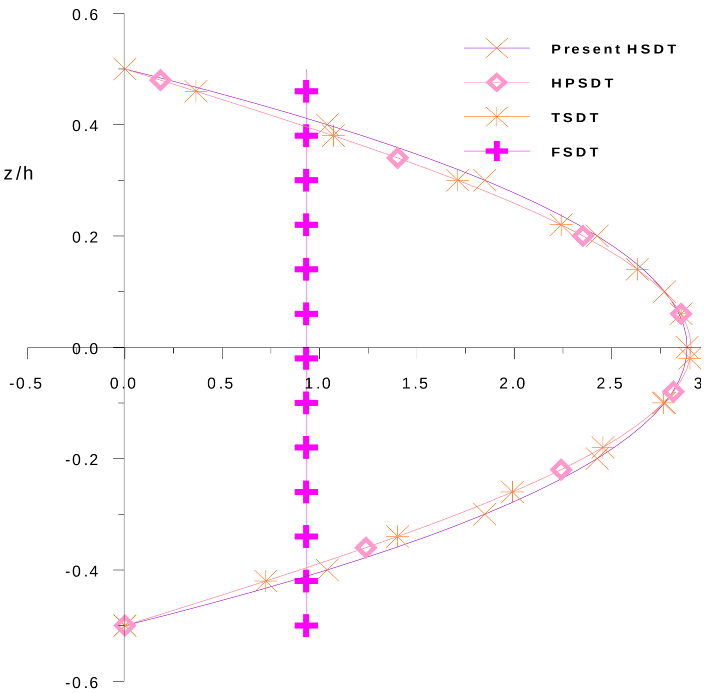

Fig -7: Variation of transverse shear stress $\overline{\tau_{zx}^{CR}}$ for AS 04

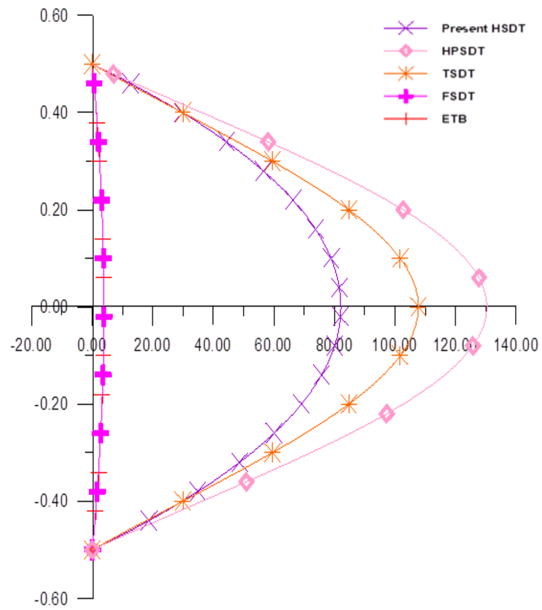

Fig -9: Variation of transverse shear stress $\overline{\tau_{zx}^{EE}}$ for AS 04

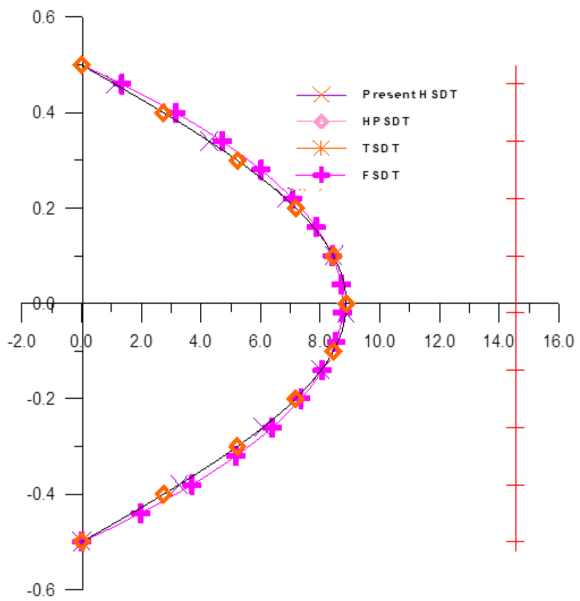

Fig -8: Variation of transverse shear stress $\overline{\tau_{zx}^{CR}}$ for AS 10

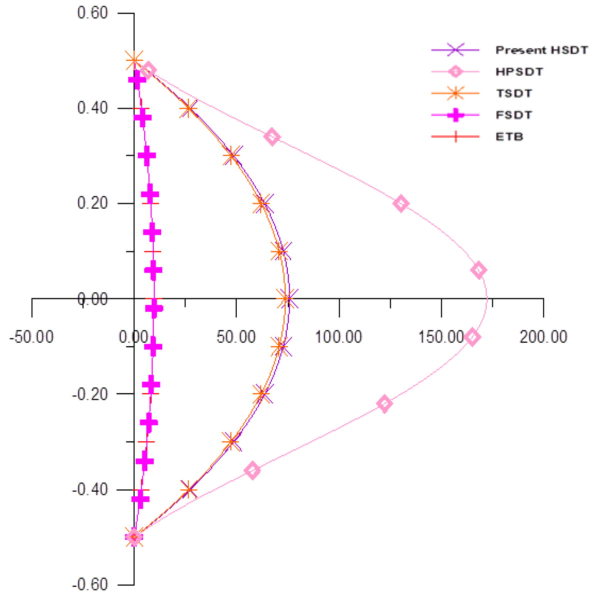

Fig -10: Variation of transverse shear stress $\overline{\tau_{zx}^{EE}}$ for AS 10

3. CONCLUSIONS

From the static flexural analysis of Cantilever beam following conclusion are drawn:

1. The result of maximum transverse displacement $\bar{w}$ obtained by present theory is in excellent agreement with those of other equivalent refined and hyperbolic theories. The variation of AS 4 and AS 10 are present as shown in fig.-2.
2. From Fig. 3 and Fig. 4, it can be observed that, the result of axial displacement $\bar{u}$ for beam subjected to uniform load varies linearly through the thickness of beam for AS 04 and AS 10 respectively.
3. The maximum Non-dimensional axial stresses $\bar{\sigma}_x$ for AS 04 and AS 10 varies linearly through the thickness of beam as shown in Figure 5 and Figure 6 respectively.
4. The transverse shear stresses $\bar{\tau}_{zx}^{EE}$ and $\bar{\tau}_{zx}^{CR}$ are obtained directly by constitutive relation. Fig. 7, 8, 9 and fig.10. shows the through thickness variation of transverse shear stress for thick beam for AS04 and 10. From this fig it can be observed that, the transverse shear stress satisfies the zero condition at top and at bottom surface of the beam.

REFERENCES

[1] Vlasov V. Z., and Leont'ev U. N.,Beams, Plates and Shells on Elastic Foundations, Translated from Russian by Barouch, A., and edited by Pelz, T. Israel program for scienti_c translations Ltd., Jerusalem, Chapter 1, pp. 1-8., 1960.

[2] Donnell L. H. Beams, Plates, and shells, McGraw-Hill Book Company. New york,1976.

[3] Kilchevskiy, N. A. Fundamentals of the Analytical Mechanics of Shell,NASA TT F-292, Washington, D. C., pp. 80-172. 1965.

[4] Goldenviezer, A. L., Methods for justifying and Refining the Theory Shells, Journal of Appied Mathematics and Mechanics, Vol. 32, No. 4, pp.704-718,1968.

[5] Rankine W. J. M., 1858, A Manual of Applied Mechanics, R. Grin and Company Ltd., London, U. K., pp. 342-344.

[6] Bresse J. A. C., 1859, Cours de Mecanique Applique, Paris: Mallet-bachelier, (1866 2nd ed.), Gauthier-Villars, Paris.

[7] Rebello C. A., Bert C. W., and Gordaninejad F., 1983, Vibration of Bi modular Sandwich Beams with Thick Facings: A New Theory and Experimental Results, Journal of Sound and Vibration, Vol. 90, No.3, pp. 381-397.

[8] Rayleigh Lord, 1880, Theory of Sound, Macmillan Publishers, London, U.K.

[9] Timoshenko S. P., 1921, On the Correction for Shear of the Di_erential Equation for Transverse Vibrations of Prismatic Bars, Philosophical Magazine, Series 6, Vol. 41, pp. 742-746.

[10] Ghugal Y. M., and Shimpi R. P., 2000, A Trigonometric Shear Deformation Theory for Flexure and Free Vibration of Isotropic Thick Beams, Structural Engineering Convention, SEC-2000, IIT Bombay, India, pp. 255-263.

[11] Ghugal Y. M. and Waghe U. P., 2011, Flexural Analysis of Deep Beams using Trigonometric Shear Deformation Theory, Journal of Institution of Engineers (India), Civil Engineering Division, Vol. 92, pp. 24-30.

[12] A. S. Sayyad, 2012, Static exure and free vibration analysis of thick isotropic beams using di_erent higher order shear deformation theories, International Journal of Applied mathematics and Mechanics, Vol. 8, No. 14, pp. 71-87.

[13] A. S. Sayyad, 2011, Comparison of various re_ned beam theories for the bending and free vibration analysis of thick beams, Journal of Applied and Computational Mechanics, Vol. 5 pp. 217-230.

[15] Tupe D.H. and Dahake A.G, 2015, Trigonometric Shear Deformation Theory For Thick Simply Supported Beams, International Journal of Research in Engineering and Technology, Vol. 4, No. 13, pp. 476-483.

[16] Vikas N. Nimbalkar and Dr. Ajay G. Dahake, 2015, Displacement and stresses for Thick Beam using New Hyperbolic Shear Deformation Theory, International Journal of pure and applied research in Engineering and technology, Vol. 3, No. 9, pp. 120-130.

[17] Yuwaraj M. Ghugal, Ajay G. Dahake, 2013, Flexure of Simply Supported Thick Beams Using Re_ned Shear Deformation Theory, World Academy of Science, Engineering and Technology International Journal of Civil, Structural, Construction and Architectural Engineering Vol. 7, No. 1.

[18] Ajay G. Dahake, Dr. Yuwaraj M. Ghugal, 2012, Flexure of thick beams using refined shear deformation theory, International Journal of civil and structural engineering, Vol. 3, No. 2.

[19] Ajay G. Dahake, Dr. Yuwaraj M. Ghugal, November 2012, Flexure of Thick Simply Supported Beam Using Trigonometric Shear Deformation Theory, International Journal of Scientific and Research Publications, Vol. 2, Issue 11.

[20] Vaibhav B. Chavan, Dr. Ajay G. Dahake, Analysis of Thick Beam Bending Problem by Using a New Hyperbolic Shear Deformation Theory, International Journal of Engineering Research and General Science Volume 2, Issue 5, August-September, 2014.

[21] Ajay Dahake, Rupa Walunj, Flexure of Thick Cantilever Beam using Third Order Shear Deformation Theory, International Journal of Engineering Research and Development, Volume 6, Issue 10 (April 2013), PP. 09-14.

[22]G.R.Gandhe, P.M.Pankade, D.H.Tupe, P.G.Taur, Higher Order Computational Method for Static Flexural Analysis of Thick Beam, 2nd International Conference on Materials Manufacturing and Design Engineering, Elsevier, Procedia Manufacturing 20 (2018) 493–498.