

# An Approach for Engineering Tuning of PID Controller with a Highly Oscillating Second Order Process

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## Abstract:

An approach is proposed for engineering tuning of PID controller with a highly oscillating second order process. A proposal is made to solve the problem by solving the characteristic equation of the closed system process-controller. As a result of the high order system analysis, the PID controller tuning parameters were calculated. The obtained values of the tuning parameters completely satisfy the set quality requirements of the high order system. A 0% overshoot value is obtained, which is the quality requirement. From the obtained transient processes of the closed system of process-controller by default and by interference confirm the set quality requirement, namely 0% overshoot. Therefore, the proposed tuning for a PID controller with a highly oscillating second order process is suitable for use in the analysis of high order dynamic systems.

**Keywords —PID controller, tuning, dynamic system, highly oscillating, second order process**

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## I. INTRODUCTION

In many industrial processes there are high fluctuations that are undesirable and a good tuning of the controller should eliminate this problem. Hassaan (2013) uses a PIDF controller to solve this problem, as high fluctuations are largely eliminated. The transient of the control process it uses has an overshoot of over 85 %, this value being reduced by the use of a PIDF controller. It adjusts the controller by minimizing the amount of absolute error of the control system [1]. Hassaan (2014) argues that the use of a PID controller eliminates problems with highly oscillatory processes, but with less efficiency than other types of PID controllers [2]. Hassaan (2014) proposes tuning up a PD controller with second order processes, many of the control processes being approximated to second order models. Here, the attenuation coefficient varies from 0.05 to 10, as the tuning is to minimize the square error integral [3]. Raut et al. (2014) presents the rules for tuning a PID controller for a higher order system [4]. Ray et al. (2017) presents an engineering approach to reduce oscillations in a high order system [5]. Sudha et al. (2016) presents a

classic PID controller for process control in aircraft systems. Here the PID controller is developed based on the dynamic and mathematical modeling of the flying system. Various tuning methods have been used and simulation of the flight system has been carried out [6]. Lorenzini et al. (2018) propose the setting of a PID controller based on forced oscillations for objects without finite frequency [7].

High order dynamic systems can be obtained in the following cases [8,9,10]:

process two aperiodic links operating with I-controller;

process two aperiodic links working with PI-controller;

process two aperiodic links, working with PID-controller with perfect differentiation;

process three aperiodic links operating with a P-controller;

process three aperiodic links, working with a PD-controller with perfect differentiation;

process oscillating link operating with PD-controller with perfect differentiation;

process oscillating link operating with a real-time differentiation first-order PD-controller;

process oscillating link operating with PID-controller with ideal differentiation.

### PROBLEMS WITH THE TUNING OF CONTROLLERS IN HIGH ORDER SYSTEMS

Higher order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to their complexity, few authors have attempted to do theoretical research on them [1-9]. The complexity is that the roots of the characteristic equation of the closed ACS ( automatic control system) is three, and it is not clear how the third real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

### POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

In analyzing high order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof. Vishnegradski's diagram [8].

Other possible options for solving this task are by using Ziegler & Nicols first method, Koppelovich's nomograms andnomograms given in [10]. These are methods for determining the parameters for adjusting the controllers by known data for the transitional characteristic of the control process [8,9,10].

*The purpose of this paper is to offer an engineering tuning of a proportional-integral-differential PID-controller with a highly oscillating second order process by solving the characteristic equation of the closed system (process-controller).*

### PROPOSAL FOR SOLVING THE PROBLEM BY SOLVING THE CHARACTERISTIC EQUATION

Figure 1 shows the structural diagram of a ACS comprising a oscillating second order process and a PID-controller.

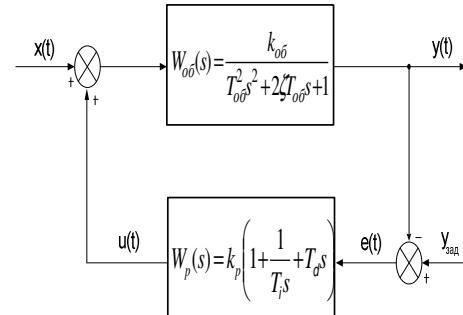


Fig. 1. A system with a oscillating second order process and a PID-controller

The transfer function of the closed system (fig.1) regarding the assignment is the type

$$\begin{aligned}
 W_{3ad}(s) &= \frac{Y(s)}{Y_{3ad}(s)} = \frac{W_{o\bar{o}}(s)W_p(s)}{1+W_{o\bar{o}}(s)\cdot W_p(s)} = \\
 &= \frac{\frac{k_{o\bar{o}}}{(T_{o\bar{o}}^2 s^2 + 2\zeta T_{o\bar{o}} s + 1)} \cdot k_p \left( \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)}{1 + \frac{k_{o\bar{o}}}{(T_{o\bar{o}}^2 s^2 + 2\zeta T_{o\bar{o}} s + 1)} \cdot k_p \left( \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)} = \\
 &= \frac{T_i T_d s^2 + T_i s + 1}{\frac{T_o^2 T_i s^3 + (2\zeta T_{o\bar{o}} + k_{o\bar{o}} k_p T_d) T_i}{k_{o\bar{o}} k_p} s^2 + \frac{(k_{o\bar{o}} k_p + 1) T_i}{k_{o\bar{o}} k_p} s + 1}. \quad (1)
 \end{aligned}$$

The transfer function of the closed system (fig.1) regarding the disturbance is the type

$$\begin{aligned}
 W_{3ad}(s) &= \frac{Y(s)}{Y_{3ad}(s)} = \frac{W_{o\bar{o}}(s)}{1+W_{o\bar{o}}(s)\cdot W_p(s)} = \\
 &= \frac{\frac{k_{o\bar{o}}}{(T_{o\bar{o}}^2 s^2 + 2\zeta T_{o\bar{o}} s + 1)}}{1 + \frac{k_{o\bar{o}}}{(T_{o\bar{o}}^2 s^2 + 2\zeta T_{o\bar{o}} s + 1)} \cdot k_p \left( \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)} = \\
 &= \frac{T_i}{k_p} \frac{s}{\frac{T_o^2 T_i s^3 + (2\zeta T_{o\bar{o}} + k_{o\bar{o}} k_p T_d) T_i}{k_{o\bar{o}} k_p} s^2 + \frac{(k_{o\bar{o}} k_p + 1) T_i}{k_{o\bar{o}} k_p} s + 1}. \quad (2)
 \end{aligned}$$

I propose that the analysis of high order dynamic system be carried out with a successively connected oscillating and aperiodic link, i.

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_0 s + 1} \cdot \frac{1}{Ts + 1}. \quad (3)$$

Assuming that the time constant of the aperiodic link (first order low pass filter) is equal to the time constant of the oscillating link, i.  $T = T_0$  is obtained

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_0 s + 1} \cdot \frac{1}{T_0 s + 1}. \quad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_o^2 s^2 + 2\xi T_0 s + 1)(T_0 s + 1) = T_o^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1. \quad (5)$$

If we equal the corresponding coefficients in front of  $s^3$ ,  $s^2$  etc. from the characteristic equation (5) to the coefficients of  $s^3$ ,  $s^2$  etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{3ad}(s) = k_{3ad} \cdot \frac{T_i T_d s^2 + T_i s + 1}{T_o^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1}, \quad (6)$$

where  $k_{3ad} = 1$  is called a coefficient of the system assignment.

The transmission function of the closed disturbance system will have the final appearance

$$W_x(s) = k_x \cdot \frac{T_o s}{T_o^3 s^3 + (2\xi + 1)T_0^2 s^2 + (2\xi + 1)T_0 s + 1}, \quad (7)$$

$$\text{where } k_x = \frac{T_i}{k_p} \cdot \frac{1}{T_0} = \frac{T_i}{k_p} \cdot \sqrt[3]{\frac{k_{o\bar{o}} k_p}{T_{o\bar{o}} T_i}} = \sqrt[3]{\frac{T_i^2 k_{o\bar{o}}}{k_p^2 T_{o\bar{o}}^2}}$$

called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of  $s$  in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o = \sqrt[3]{\frac{T_{o\bar{o}}^2 T_i}{k_{o\bar{o}} k_p}}. \quad (8)$$

Similarly, the attenuation coefficient  $\xi$  is determined. For it two expressions of  $s^2$  and  $s$  of (6) are obtained, ie.

The first expression that can be determined  $\xi$  is

$$(2\xi + 1)T_0^2 = \frac{(2\xi T_{o\bar{o}} + k_{o\bar{o}} k_p T_d) T_i}{k_{o\bar{o}} k_p}. \quad (9)$$

If we only express  $\xi$  we obtained

$$\xi = \frac{k_{o\bar{o}} k_p T_i T_d - T_0^2 k_{o\bar{o}} k_p}{2(T_0^2 k_{o\bar{o}} k_p - T_{o\bar{o}} T_i)}. \quad (10)$$

The second expression from which can be determined  $\xi$  is

$$(2\xi + 1)T_0 = \frac{(k_{o\bar{o}} k_p + 1) T_i}{k_{o\bar{o}} k_p}. \quad (11)$$

If we express only  $\xi$  it is obtained

$$\xi = \frac{T_i (k_{o\bar{o}} k_p + 1) - T_0 k_{o\bar{o}} k_p}{2T_0 k_{o\bar{o}} k_p}. \quad (12)$$

If the expressions (9) and (11) are divided into one another, it is obtained

$$T_o = \frac{2\xi T_{o\bar{o}} + k_{o\bar{o}} k_p T_d}{k_{o\bar{o}} k_p + 1}. \quad (13)$$

If the expressions (10) and (12) are equal to one another, i.

$$\frac{k_{o\bar{o}} k_p T_i T_d - T_0^2 k_{o\bar{o}} k_p}{2(T_0^2 k_{o\bar{o}} k_p - T_{o\bar{o}} T_i)} = \frac{T_i (k_{o\bar{o}} k_p + 1) - T_0 k_{o\bar{o}} k_p}{2T_0 k_{o\bar{o}} k_p} \quad (14)$$

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal, i

$$T_o = \sqrt[3]{\frac{T_{o\bar{o}}^2 T_i}{k_{o\bar{o}} k_p}} = \frac{2\xi T_{o\bar{o}} + k_{o\bar{o}} k_p T_d}{k_{o\bar{o}} k_p + 1}. \quad (15)$$

If an expression (15) is solved regarding the time constant of integration  $T_i$ , it is obtained

$$T_i = \frac{(2\xi T_{o\bar{o}} + k_{o\bar{o}} k_p T_d)^3 k_{o\bar{o}} k_p}{T_{o\bar{o}}^2 (k_{o\bar{o}} k_p + 1)^3}. \quad (16)$$

The time constant of differentiation  $T_d$  can be determined by an expression (10), ie.

$$T_d = \frac{2\xi(T_0^2 k_{o\bar{o}} k_p - T_{o\bar{o}} T_i)}{k_{o\bar{o}} k_p T_i} \quad (17)$$

The proportionality coefficient of the controller  $k_p$  can be determined by an expression (11), ie.

$$k_p = \frac{T_i}{k_{o\bar{o}} [(2\xi + 1)T_0 - T_i]} \quad (18)$$

The transitional process is given with a highly oscillating second order process. The following algorithm performs the following:

Take the transitional process of the control process that is smooth and normalizing.

After identification (fig.2), it is determined:  $k_{o\bar{o}} = 0,4$ ,  $T_{o\bar{o}} = 19.5$  sec and  $\zeta = 0,05$ . The transfer function of the control process is :

$$W_{o\bar{o}}(s) = \frac{0,4}{19,5s^2 + 0,44s + 1}.$$

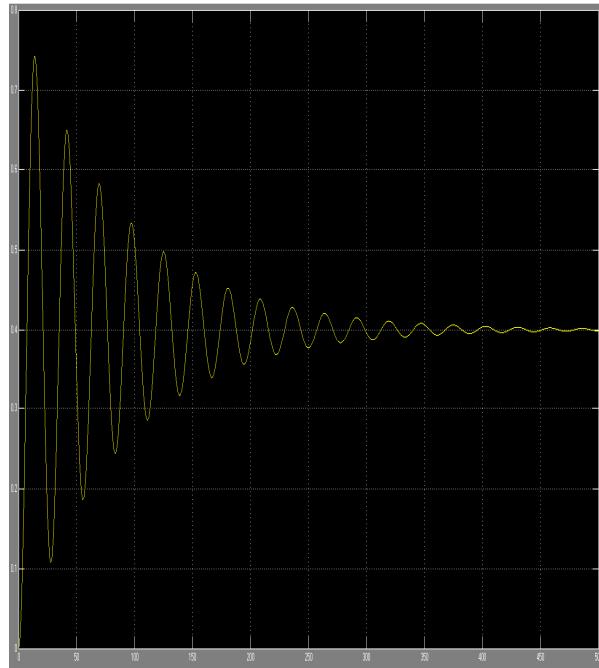


Fig. 2. Transitional process of the control process

By the expressions (16), (17) and (18), the tuning parameters of the PID-controller are calculated using the next procedure.

First, calculate the value of  $T_o$ , at a set value of  $T_{o\bar{o}}$  and  $k_{o\bar{o}}$ , where  $T_i$  and  $k_p$  are arbitrary values, ie

$$T_o = \sqrt[3]{\frac{T_{o\bar{o}}^2 T_i}{k_{o\bar{o}} k_p}} = \sqrt[3]{\frac{4,42^2 \cdot 4,7}{0,4 \cdot 2,2}} = 4,71 \text{ sec.}$$

It is then calculated  $k_p$  at the same setpoint of  $T_i$ , ie

$$k_p = \frac{4,7}{0,4 \cdot [(2,0,05 + 1) \cdot 4,71 - 4,7]} = 24,43.$$

Then calculate the value of  $T_d$  again at the same set value of  $T_i$

$$T_d = \frac{20,05(4,7^2 \cdot 0,42443 - 4,424,7) + 4,7^2 \cdot 0,42443}{0,42443 \cdot 4,7} = 5,15 \text{ sec.}$$

Finally  $T_i$  is calculated

$$T_i = \frac{(2,0,05 \cdot 4,42 + 0,42443 \cdot 5,15)^3 \cdot 0,42443}{4,42^2 \cdot (0,42443 + 1)^3} = 52,36 \text{ sec.}$$

sec.

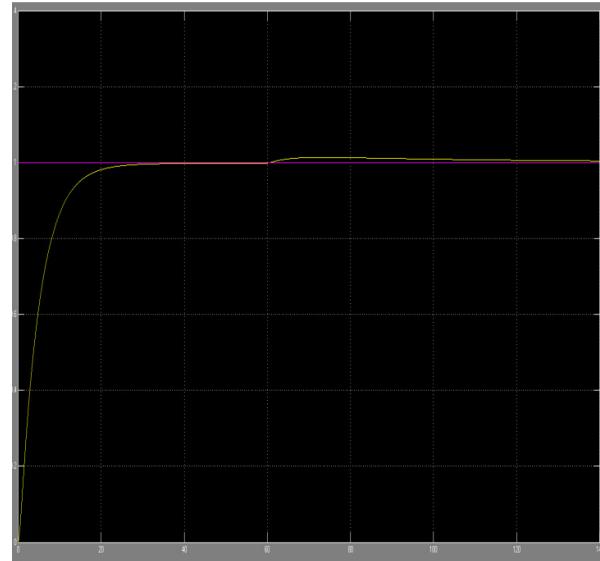


Fig. 3. Transitional processes by assignment and by disturbance.

## CONCLUSIONS

An approach is proposed for engineering tuning of PID controller with a highly oscillating second order process. A proposal is made to solve the problem by solving the characteristic equation of the closed system process-controller. As a result of the high order system analysis, the PID controller tuning parameters were calculated. The obtained values of the tuning parameters completely satisfy the set quality requirements of the high order system. A 0% overshoot value is obtained, which is the quality requirement. From the obtained transient processes of the closed system of process-controller by default and by interference confirm the set quality requirement, namely 0% overshoot. Therefore, the proposed tuning for a PID controller with a highly oscillating second order process is suitable for use in the analysis of high order dynamic systems.

## REFERENCES

- [1] G. Hassaan and A. Mohammed, M. Lashin, Tuning of a PID Controller Used With A Highly Oscillating Second Order Process, International Journal of Emerging Technology and Advanced Engineering, Volume 3, Issue 3, pp. 943-945, 2013.
- [2] G. Hassaan, Tuning Of A PID With First-Order-Lag Controller Used With A Highly Oscillating Second-Order Process, International Journal of Scientific & Technology Research, Volume 3, Issue 9, pp. 314-317, 2014.
- [3] G. Hassaan, Tuning of a PD-controller used with Second Order Processes, International Journal of Engineering and Technical Research, Volume 2, Issue 7, pp. 120-122, 2014.
- [4] K. Raut and S. Vaishnav, Performance Analysis of PID Tuning Techniques based on Time Response specification, International Journal of Innovative Research in Electrical, Electronics, Instrumentation and Control Engineering, Volume 2, Issue 1, pp. 616-619, 2014.
- [5] S. Ray and Md. Rahman and Md. Ahmed, Tuning of PI and PID Controller with STATCOM, SSSC and UPFC for Minimizing Damping of Oscillation, IOSR Journal of Electrical and Electronics Engineering, Volume 12, Issue 1, pp. 30-44, 2017.
- [6] G. Sudha and S. Deepa, Optimization for PID Control Parameters on Pitch Control of Aircraft Dynamics Based on Tuning Methods, International Journal of Applied Mathematics & Information Sciences, Volume 10, Issue 1, pp. 343-350, 2016.
- [7] C. Lorenzini and A. Bazanella, L. Pereira, PID Tuning Based on Forced Oscillation for Plants Without Ultimate Frequency, IFAC Conference, Volume 51, Issue 4, pp. 131-136, 2018.
- [8] N. Naplataanov, Theory of Automatic Control - Volume 1, Linear Systems, "Technika", Sofia, pp. 226-229, 1976 (in Bulgarian).
- [9] H. Hinov and S. Tsonkov et al. Automation of production - part 2. Automation of technological processes, "Technika", Sofia, pp. 170-173, 1978 (in Bulgarian).
- [10] I. Dragotinov and I. Ganchev, Automation of technological processes, UFT, Plovdiv, pp. 182-192, 2003 (in Bulgarian).
- [11] H. Patel and S. Chaphekar, Developments in PID Controllers: Literature Survey, International Journal of Engineering Innovation & Research, Volume 1, Issue 5, pp. 425-430, 2012.
- [12] J. Rana and R. Prasad, R. Agarwal, Designing of a Controller by Using Model Order Reduction Techniques, International Journal of Engineering Innovation & Research, Volume 5, Issue 3, pp. 220-223, 2016.