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Strongly Soft G-Closed Sets in Soft Bi-Cech Closure Space

R. Vijaya Chandra[*], B. Bhuvaneshwari[**]

[*]*(Department of Mathematics, Navarasam Arts and Science College for Women, Erode, Tamil Nadu, India*
Email: risrchandra@gmail.com)
[**]*(Department of Mathematics, Navarasam Arts and Science College for Women, Erode, Tamil Nadu, India*
Email: bhuvaneshwari1212@gmail.com)

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Abstract:

In this paper, we introduce some basic concepts of soft Bi-Cech closure space and we study the basic properties of strongly soft generalized closed sets in soft Bi-Cech closure space.

Keywords: **Soft Bi-Cech closure space, Strongly soft g-closed set, Soft semi-open, Soft regular-open, Soft pre-open**

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I. INTRODUCTION

The concept of Cech closure space was introduced by E. Cech[1]. In Cech's approach the operator satisfies idempotent condition among kuratowski axioms. By this condition don't need to hold for each set A of M. At the point, when the condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space.

In this paper, we introduce the stronger form of soft generalized closed sets in soft Bi-Cech closure space and also we investigate some of their basic properties.

II. PRELIMINARIES

In this section, we see the fundamental definitions of soft Bi-Cech closure space.

2.1 Definition: Two functions b_1 and b_2 described from a soft power set $P(M_{X_A})$ to itself over M is called Cech Closure Operator if it satisfies the properties

i) $b_1(\phi_A) = \phi_A$ and $b_2(\phi_A) = \phi_A$
ii) $X_A \subseteq b_1(X_A)$ and $X_A \subseteq b_2(X_A)$
iii) $b_1(X_A \cup Y_A) = b_1(X_A) \cup b_1(Y_A)$ and $b_2(X_A \cup Y_A) = b_2(X_A) \cup b_2(Y_A)$ for any X_A and $Y_A \subset M$

Then (X_A, b_1, b_2, A) *or* (X_A, b_1, b_2) is called a Soft Bi-Cech Closure space.

2.2 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft $b_{i=1,2}$ closed (soft closed) if $b_{i=1,2}(P_A) = P_A$. Clearly, P_A is a soft closed subset of a Soft Bi-Cech Closure space (X_A, b_1, b_2) iff P_A is both soft closed subset of (X_A, b_1) and (X_A, b_2).

Let P_A be a soft closed subset of a Soft Bi-Cech Closure space (X_A, b_1, b_2).The following conditions are equivalent

i) $b_2 b_1(P_A) = P_A$
ii) $b_1(P_A) = P_A and b_2(P_A) = P_A$

2.3 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft $b_{i=1,2}$ open (soft open) if $b_{i=1,2}(p_A^C) = P_A^C$

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2.4 Definition: A soft set $Int_{b_{i=1,2}}(P_A)$ with respect to the closure operator $b_{i=1,2}$ is defined as $Int_{b_{i=1,2}}(P_A) = X_A - b_{i=1,2}(X_A - P_A) = \left[b_{i=1,2}(P_A^C)\right]^C$, here $P_A^C = X_A - P_A$

2.5 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is called soft $b_{i=1,2}$ neighbourhood of e_x if $e_x \in Int_{b_{i=1,2}}(P_A)$

2.6 Definition: If (X_A, b_1, b_2) be a Soft Bi-Cech Closure space, then the associate soft Bi-topology on X_A is $\tau_{i=1,2} = \{P_A^C : b_{i=1,2}(P_A) = P_A\}$

2.7 Definition: Let (X_A, b_1, b_2) be a Soft Bi-Cech Closure space. A be a Soft Bi-Cech Closure space (Y_A, b_1^*, b_2^*) is called a soft subspace of (X_A, b_1, b_2) if $Y_A \subseteq X_A$ and $b_{i=1,2}^*(P_A) = b_{i=1,2}(P_A) \cap Y_A$ $\forall$ soft subset $P_A \subseteq Y_A$

2.8 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft generalized closed (soft g-closed) set if $b_{i=1,2}[P_A] \subseteq Y_A$, whenever $P_A \subseteq Y_A$ and Y_A is soft open subset of (X_A, b_1, b_2)

III. STRONGLY SOFT GENERALIZED CLOSED SET

In this section, we introduce strongly soft g-Closed sets in Soft Bi-Cech Closure space and investigate a few simple properties.

3.1 Definition: Let P_A be a soft subset of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be

1. Soft Semi-Open if $P_A \subseteq b_{i=1,2}\left[Int_{b_{i=1,2}}(P_A)\right]$ and a Soft Semi-Closed if $Int_{b_{i=1,2}}\left(b_{i=1,2}[P_A]\right) \subseteq P_A$
2. Soft Regular-Open if $Int_{b_{i=1,2}}\left(b_{i=1,2}[P_A]\right) = P_A$ and a Soft Regular-Closed if $P_A = b_{i=1,2}\left[Int_{b_{i=1,2}}(P_A)\right]$
3. Soft Pre-Open if $P_A \subseteq Int_{b_{i=1,2}}\left(b_{i=1,2}[P_A]\right)$ and a Soft Pre-Closed if $b_{i=1,2}\left[Int_{b_{i=1,2}}(P_A)\right] \subseteq P_A$
4. Soft α-Open if $P_A \subseteq Int_{b_{i=1,2}}\left(b_{i=1,2}\left[Int_{b_{i=1,2}}(P_A)\right]\right)$ and soft α-Closed if $b_{i=1,2}\left[Int_{b_{i=1,2}}\left(b_{i=1,2}(P_A)\right)\right] \subseteq P_A$
5. $Soft\ Semi\ Pre-Open$ (Soft β-Open) if $P_A \subseteq b_{i=1,2}\left[Int_{b_{i=1,2}}\left(b_{i=1,2}(P_A)\right)\right]$and soft β-closed if $Int_{b_{i=1,2}}\left(b_{i=1,2}\left[Int_{b_{i=1,2}}(P_A)\right]\right) \subseteq P_A$

i) The smallest Soft Bi-Cech $Semi-Closed$ set containing P_A is called Soft Bi-Cech Semi-Closureof P_A with respect to $b_{i=1,2}$ and it is denoted by $b_{i=1,2(s)}(P_A)$

ii) The largest Soft Bi-Cech $Semi-Open$ set contained in P_A is called Soft Soft Bi-Cech Semi-Interior of P_A with respect to $b_{i=1,2}$ and it is denoted by $Int_{b_{i=1,2(s)}}(P_A)$

iii) The Smallest Soft Bi-Cech $Pre-Closed$ set containing P_A is called Soft Bi-Cech Pre-Closure of P_A with respect to $b_{i=1,2}$ and it is denoted by $b_{i=1,2(P)}(P_A)$

iv) The largest Soft Bi-Cech $Pre-Open$ set contained in P_A is called Soft Bi-Cech Semi-Closureof P_A with respect to $b_{i=1,2}$ and it is denoted by $Int_{b_{i=,2(p)}}(P_A)$

v) The smallest Soft Bi-Cech α-Closed set contained in P_A is called Soft Bi-Cech α-Closure of P_A with respect to $b_{i=1,2}$ and it is denoted by $b_{i=1,2(\alpha)}(P_A)$

vi) The largest Soft Bi-Cech α-Open set contained in P_A is called Soft Bi-Cech α-Interior of P_A with respect to $b_{i=1,2}$ and it is denoted by $Int_{b_{i=1,2(\alpha)}}(P_A)$

vii) The smallest Soft Bi-Cech Semi Pre-Closed set contained in P_A is called Soft Bi-Cech Semi Pre-Closure of P_A with respect to $b_{i=1,2}$ and it is denoted by $b_{i=1,2(sp)}(P_A)$

viii) The largest Soft Bi-Cech Semi Pre-Open set contained in P_A is called Soft Bi-Cech Semi Pre-Interior of P_A with respect to $b_{i=1,2}$ and it is denoted by $Int_{b_{i=1,2(sp)}}(P_A)$

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3.2 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft semi-generalized closed (soft sg-closed) set if $b_{i=1,2(s)}[P_A] \subseteq Y_A$, whenever $P_A \subseteq X_A$, Y_A is soft semi open in X_A

3.3 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft generalized semi-closed (soft gs-closed) set if $b_{i=1,2(s)}[P_A] \subseteq Y_A$, whenever $P_A \subseteq X_A$, Y_A is soft open in X_A

3.4 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft generalized semi Pre-closed (soft gsp-closed) set if $b_{i=1,2(sp)}[P_A] \subseteq Y_A$, whenever $P_A \subseteq X_A$, Y_A is soft open in X_A

3.5 Definition: Let (X_A, b_1, b_2) be a Soft Bi-Cech Closure space. A soft subset $P_A \subseteq X_A$ is called soft regular generalized closed (soft rg-closed) set if $b_{i=1,2}[P_A] \subseteq Y_A$, whenever $P_A \subseteq Y_A$ and Y_A is soft regular open subset of (X_A, b_1, b_2)

3.6 Definition: A soft subset P_A of a Soft Bi-Cech Closure space (X_A, b_1, b_2) is said to be soft generalized Pre-closed (soft gp-closed) set if $b_{i=1,2(p)}[P_A] \subseteq Y_A$, whenever $P_A \subseteq X_A$, Y_A is soft open in X_A

3.7 Definition: Let (X_A, b_1, b_2) be a Soft Bi-Cech Closure space. A soft subset $P_A \subseteq X_A$ is called strongly soft generalized closed (strongly soft g-closed) set if $b_{i=1,2}\left[Int_{b_{i=1,2(p)}}(P_A)\right] \subseteq Y_A$, whenever $P_A \subseteq Y_A$ and Y_A is soft open subset of (X_A, b_1, b_2).

***3.8 Example*:** Assume that the initial universal set $M = \{a_1, a_2\}$ and $E = \{e_1, e_2, e_3\}$ be the parameters. Let $A = \{b_1, b_2\} \subseteq E$ and $X_A = \{(b_1, \{a_1, a_2\}), (b_2, \{a_1, a_2\})\}$.

Then $P(M_{X_A})$ are $X_{1A} = \{(b_1, \{a_1\})\}$,
$X_{2A} = \{(b_1, \{a_2\})\}$,
$X_{3A} = \{(b_1, \{a_1, a_2\})\}$,
$X_{4A} = \{(b_2, \{a_1\})\}$,
$X_{5A} = \{(b_2, \{a_2\})\}$,
$X_{6A} = \{(b_1, \{a_1, a_2\})\}$,
$X_{7A} = \{(b_1, \{a_1\}), (b_2, \{a_1\})\}$,
$X_{8A} = \{(b_1, \{a_1\}), (b_2, \{a_2\})\}$,
$X_{9A} = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}$,
$X_{10A} = \{(b_1, \{a_2\}), (b_2, \{a_2\})\}$,
$X_{11A} = \{(b_1, \{a_1\}), (b_2, \{a_1, a_2\})\}$,
$X_{12A} = \{(b_1, \{a_2\}), (b_2, \{a_1, a_2\})\}$,
$X_{13A} = \{(b_1, \{a_1, a_2\}), (b_2, \{a_1\})\}$,
$X_{14A} = \{(b_1, \{a_1, a_2\}), (b_2, \{a_2\})\}$,
$X_{15A} = X_A$, $X_{16A} = \phi_A$

An operator $b_1: P(M_{X_A}) \to P(M_{X_A})$ and $b_2: P(M_{X_A}) \to P(M_{X_A})$ are defined from the soft power set $P(M_{X_A})$ to itself over X as follows $b_1(X_{1A}) = X_{8A}$, $b_1(X_{2A}) = X_{9A}$, $b_1(X_{4A}) = X_{7A}$, $b_1(X_{5A}) = X_{10A}$, $b_1(X_{7A}) = X_{11A}$, $b_1(X_{8A}) = X_{14A}$, $b_1(X_{9A}) = X_{13A}$, $b_1(X_{10A}) = X_{12A}$, $b_1(X_{3A}) = b_1(X_{6A}) = b_1(X_{11A}) = b_1(X_{12A}) = b_1(X_{13A}) = b_1(X_{14A}) = b_1(X_A) = X_A$, $b_1(\phi_A) = \phi_A$, $b_2(X_{1A}) = b_2(X_{2A}) = b_2(X_{3A}) = X_{3A}$, $b_2(X_{4A}) = b_2(X_{6A}) = X_{6A}$, $b_2(X_{5A}) = X_{5A}$, $b_2(X_{8A}) = b_2(X_{10A}) = b_2(X_{14A}) = X_{14A}$, $b_2(X_{7A}) = b_2(X_{9A}) = b_2(X_{11A}) = b_2(X_{12A}) = b_2(X_{13A}) = b_2(X_A) = X_A$, $b_2(\phi_A) = \phi_A$.

Here $\phi_A, X_{4A}, X_{5A}, X_{6A}, X_{7A}, X_{8A}, X_{9A}, X_{10A}, X_{11A}, X_{12A}, X_{13A}, X_{14A}, X_A$ are called strongly soft g-closed sets in X_A. Then (M, b_1, b_2, A) or (X_A, b_1, b_2) is called Soft Bi-Cech Closure space.

***3.9 Theorem*:** In a Soft Bi-Cech Closure space (X_A, b_1, b_2), every soft closed set is strongly soft g-closed

***Proof*:** From the definition (2) of soft closed set, the proof is obvious.

***3.10 Theorem*:** If P_A be a soft subset of X_A is both soft open and strongly soft g-closed, then P_A is soft closed.

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Proof: Since P_A is strongly soft g-closed, Then $b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq P_A$ ie., $b_{i=1,2}(P_A) = b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq P_A$ $\{\because \; P_A \subseteq b_{i=1,2}(P_A)\}$

Hence P_A is soft closed

3.11 Theorem: Let $P_A \subseteq X_A$. If P_A is strongly soft g-closed set, then $b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] - P_A$ has no non-empty soft closed subset

Proof: Let Q_A be a soft closed subset of $b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] - P_A$, then $Q_A \subseteq b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \cap X_A - P_A$ and $P_A \subseteq X_A - Q_A$.

Consequently,$Q_A \subseteq P_A - b_{i=1,2}[Int_{b_{i=1,2}}(P_A)]$ $\{\because \; Q_A \subseteq b_{i=1,2}[Int_{b_{i=1,2}}(P_A)]\}$

$Q_A \subseteq b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \cap (X_A - b_{i=1,2}[Int_{b_{i=1,2}}(P_A)]) = \phi_A$.

Thus, $Q_A = \phi_A$ $\therefore$ $b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] - P_A$ has no non-empty soft closed set

3.12 Theorem: If P_A be a soft subset of X_A is both strongly soft g-closed set and soft semi-open, then P_A is soft g-closed

Proof: Suppose P_A is both strongly soft g-closed and soft semi-open, then $b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq Q_A$. Whenever $P_A \subseteq Q_A$ and Q_A is soft open subset of X_A.$P_A \subseteq b_{i=1,2}[Int_{b_{i=1,2}}(P_A)]$, $\{\because \mathrm{P_A is soft semi-open}\}$,

Then $b_{i=1,2}(P_A) \subseteq b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq Q_A$

Hence P_A is soft g-closed

3.13 Theorem: Let $P_A \subseteq R_A \subseteq X_A$ and if P_A is strongly soft g-closed in X_A, then P_A is strongly soft g-closed relative to R_A

Proof: Let $P_A \subseteq R_A \subseteq X_A$ and suppose that P_A is strongly soft g-closed in P_A. Let $P_A \subseteq R_A \cap Q_A$, where Q_A is soft open in P_A.

Since P_A is strongly soft g-closed in X_A,

$P_A \subseteq Q_A \Rightarrow b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq Q_A$

ie., $R_A \cap b_{i=1,2}[Int_{b_{i=1,2}}(P_A)] \subseteq R_A \cap Q_A$, where $R_A \cap b_{i=1,2}[Int_{b_{i=1,2}}(P_A)]$ is closure of interior of $P_A$with respect to $b_{i=1,2}$ in R_A. Hence P_A is strongly soft g-closed relative to R_A.

IV. CONCLUSION

In this paper, we introduced strongly soft g-closed sets in Soft Bi-Cech Closure space. Also we investigated the behavior relative to union, intersection of strongly soft g-closed sets. Further we can extend this to fuzzy soft topological spaces.

V. REFERENCES

[1] E. Cech, “Topological Spaces”, Inter Science Publishers, John Wiley and sons, Newyork, 1966.

[2] K. Chandrasekha Rao and R. Gowri, on Bi-Closure spaces. Bulletin of pure and applied sciences, vol 25E, 171-175,2006.

[3] Chawalit Boonpok, “Generalized Closed sets in Cech closed spaces”, Acta Universittatis Apulensis, No.22, 2010,, pp.133-140.

[4] R. Gowri and G. Jagadeesan, “On Soft Cech Closure Spaces”, International Journal of Mathematics Trends and Technology, Vol.9, 2014, 32-37

[5] R. Gowri and G. Jagadeesan, “Soft Generalized Closed sets in soft Cech Closure Spaces”, Global Journal of Pure and Applied Mathematics, Vol 2, No.1, 2016, 909-916

[6] R. Gowri and G. Jagadeesan, “On Soft Bi-Cech closure spaces”, International Journal of Mathematical Archive-5(11), 2014, 99-105

[7] R. Gowri and G. Jagadeesan, “Strongly Soft G-Closed and strongly Soft ∂-Closed Sets in Soft Cech Closure Space”, IOSR Journal of Mathematics, Vol 12, 2016, PP 14-20.

 ©IJSRED: All Rights are Reserved

[8] N. Levine,"Generalized closed sets in topology", Rend.Circ.Mat.Palermo, Vol.19, No.2, 1970, pp.89-96.

[9] C. Loganathan, V. Pushpalatha, "Circulant Interval Valued Fuzzy Matrices" Annals of Pure and Applied Mathematics, Vol.16, 313-322, (2018)

[10] D.A. Molodtsov, "The Theory of Soft sets (In Russian)", URSS Publishers, Moscow

[11] D.N. Roth and J.W. Carlson, Cech Closure Spaces, Kyungpook Math.J, Vol.20, 11-30, (1980)