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Neutrosophic Soft Topological Spaces on New Operations

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Abstract:

In recent years, the neutrosophic soft set's researches have developed quite strongly. Its applications are also expanded in many real problems such as:engineering, computer science, economics, social science, medical science,… Therefore, we are interested in this field and wish to study more deeply on neutrosophic soft set to provide effective tools for handling uncertain data. So, in this paper, we first re-introduce the notion of union, intersection, AND, OR operations on neutrosophic soft set; check some basic their properties. Secondly, we construct neutrosophic soft topological space, define open set, closed set, and prove the relationship between neutrosophic soft topological spaces, fuzzy soft topological spaces, fuzzy topological spaces. And the author also gives some examples clarify the proved propositions; properties in this paper.

Keywords **—Neutrosophic soft set, neutrosophic soft topological spaces, fuzzy soft topological spaces, soft topological spaces.**

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I. INTRODUCTION

Data sources help us collect many helpful informationif we know how to exploit them. In the previous period, the unclear data increased the complexities and difficulties when the scientists analyzed information. With the rapid development of sciences, especially Mathematics, many effective tools and techniques which handle the actual data were born. They have overcome the defectsthat existed before. There are some of the theories: theory of probability, theory of fuzzy sets as mathematical tools for dealing with uncertainties. But these theories still had irresistible disadvantages were that they were not able to treat uncertain and inconsistent data in the belief system.Example, fuzzy set was developed by Zadeh, existed a difficulty:how to set the membership function in each particular case. The reason was the parameterization tool.

In 1999, Molodtsov gave the first results on soft set theory, which provided a free tool from the parameterization. In 2005, Smarandache generalized the concept of neutrosophic set, brought effective methods to solve uncertain problems in some fields: philosophy, physic, medicine science, logic, statistics,…In 2013, Maji combined the neutrosophic set with soft sets, made a mathematical model "Neutrosophic Soft Sets" and presented its application with a decision making problem. Based on these new concepts, mathematicians extended their studies towards the construction of topological spaces by giving special operations and new definitions. We can mention authors such as: Chang (1968), Cagman (2011), Bera and Mahapatra (2017), Mayramov and Gunduz (2014), Ozturk (2019)…

In Ozturk's paper, the authors gave intersection, union, difference, AND, OR operations on neutrosophic soft set. Then, Ozturk investigated their properties, constructed neutrosophic soft topological spaces, checked the relationship between the topologies: neutrosophic soft topology, fuzzy soft topology, fuzzy topology. A question arises is that when we change the definition of operations on neutrosophic soft set, if the properties were preserved. And if the relationship between the mentioned topological spaces still has been kept onnew operations. This paper will give new operations on neutrosophic soft set and answer the above questions.

II. PRELIMINARIES

At first, we recall some necessary definitions related to soft set, neutrosophic set, neutrosophic soft set in previous studies.

Definition 2.1 ([2])A neutrosophic set A on the universe set X is defined as:

$$A=\left\{\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle : x\in X\right\},$$

where $T,I,F:X\to[0,1]$ and $0\le T_A(x)+I_A(x)+F_A(x)\le 3$.

Definition 2.2 ([1]) Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all subsets of X. Then for $A\subseteq E$, a pair (G,A) is called a soft set over X, where G is a mapping given by $G:E\to P(X)$, i.e,

$$(G,A)=\left\{(e,G(a)) : e\in A, G:A\to P(X)\right\}.$$

The notion of neutrosophic soft set in Deli and Broumi's paper [3] was given below:

Definition 2.3 ([3])Let X be an initial universe set and E be a set of parameters. Let $N(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set (G,E) over X is a set defined by a set valued function G representing a mapping $G:E\to N(X)$. In other word, the neutrosophic soft set is a parameterized family of some elements of the set $N(X)$ and therefore it can be written as a set of ordered pairs,

$$(G,E)=\left\{\begin{matrix}\left(e,\left\langle x,T_{G(e)}(x),I_{G(e)}(x),F_{G(e)}(x)\right\rangle : x\in X\right)\\ : e\in E\end{matrix}\right\}$$

, where $T_{G(e)}(x)$, $I_{G(e)}(x)$, $F_{G(e)}(x)\in[0,1]$ respectively called the truth – membership, indeterminacy – membership, falsity – membership function of $G(e)$ and $0\le T_{G(e)}(x)+I_{G(e)}(x)+F_{G(e)}(x)\le 3$.

Definition 2.4 ([5]) Let (G,E) be neutrosophic soft set over the universe set X. The complement of (G,E) is denoted by $(G,E)^C$ and is defined by:

$$(G,E)^C=\left\{\begin{matrix}\left(\begin{matrix}e,\left\langle x,F_{G(e)}(x),1-I_{G(e)}(x),T_{G(e)}(x)\right\rangle\\ : x\in X\end{matrix}\right)\\ : e\in E\end{matrix}\right\}.$$

And clearly, $\left((G,E)^C\right)^C=(G,E)$.

Definition 2.5 ([4])Let (G_1,E) and (G_2,E) be two neutrosophic soft sets over the universe set X. (G_1,E) is said to be neutrosophic soft subset of (G_2,E), denoted by $(G_1,E)\subseteq(G_2,E)$ if $T_{G_1(e)}(x)\le T_{G_2(e)}(x)$; $I_{G_1(e)}(x)\le I_{G_2(e)}(x)$; $F_{G_1(e)}(x)\ge F_{G_2(e)}(x)$ $\forall e\in E$, $\forall x\in X$.

We say (G_1,E) equal to (G_2,E) if (G_1,E) is neutrosophic soft subset of (G_2,E) and (G_2,E) is neutrosophic soft subset of (G_1,E). It can be written by $(G_1,E)=(G_2,E)$.

III. NEW OPERATIONS ON NEUTROSOPHIC SOFT SETS

In this section, we re-define the operations of union, intersection, difference on neutrosophic soft sets. The author defines them differently from Ozturk's paper. Furthermore, basic properties of these operations will be presented.

Definition 3.1 Let (G_1,E) and (G_2,E) be two neutrosophic soft set over universe set X. Then, their union is denoted by $(G_1,E)\cup(G_2,E)=(G_3,E)$ and is defined by:

$$(G_3,E)=\left\{\begin{matrix}\left(\begin{matrix}e,\left\langle x,T_{G_3(e)}(x),I_{G_3(e)}(x),F_{G_3(e)}(x)\right\rangle\\ : x\in X\end{matrix}\right)\\ : e\in E\end{matrix}\right\},$$

where

$$T_{G_3(e)}(x)=\min\{T_{G_1(e)}(x)+T_{G_2(e)}(x),1\},$$
$$I_{G_3(e)}(x)=\min\{I_{G_1(e)}(x)+I_{G_2(e)}(x),1\},$$
$$F_{G_3(e)}(x)=\max\{F_{G_1(e)}(x)+F_{G_2(e)}(x)-1,0\}.$$

Definition 3.2 Let (G_1,E) and (G_2,E) be two neutrosophic soft set over the universe set X. Then their intersection is denoted by $(G_1,E)\cap(G_2,E)=(G_3,E)$ and is defined by:

$$(G_3,E)=\left\{\left\{e,\left\langle x,T_{G_3(e)}(x),I_{G_3(e)}(x),F_{G_3(e)}(x)\right\rangle : x\in X\right\} : e\in E\right\},$$

where

$$T_{G_3(e)}(x)=\max\{T_{G_1(e)}(x)+T_{G_2(e)}(x)-1,0\},$$
$$I_{G_3(e)}(x)=\max\{I_{G_1(e)}(x)+I_{G_2(e)}(x)-1,0\},$$
$$F_{G_3(e)}(x)=\min\{F_{G_1(e)}(x)+F_{G_2(e)}(x),1\}.$$

Definition 3.3 Let (G_1,E) and (G_2,E) be two neutrosophic soft set over the universe set X. Then "(G_1,E) difference (G_2,E)" operation on them is denoted by $(G_1,E)\setminus(G_2,E)=(G_3,E)$ and is defined by $(G_1,E)\cap(G_2,E)^C$ as follows:

$$(G_3,E)=\left\{\left\{e,\left\langle x,T_{G_3(e)}(x),I_{G_3(e)}(x),F_{G_3(e)}(x)\right\rangle : x\in X\right\} : e\in E\right\},$$

where

$$T_{G_3(e)}(x)=\max\{T_{G_1(e)}(x)+F_{G_2(e)}(x)-1,0\},$$
$$I_{G_3(e)}(x)=\max\{I_{G_1(e)}(x)-I_{G_2(e)}(x),0\},$$
$$F_{G_3(e)}(x)=\min\{F_{G_1(e)}(x)+T_{G_2(e)}(x),1\}.$$

Definition 3.4 Let $\{(G_1,E)\,|\,i\in I\}$ be a family of neutrosophic soft sets over the universe set X. Then

$$\bigcup_{i=1}^{\infty}(G_i,E)=\left\{\left\{e,\left\langle \begin{array}{l} x,\inf\left\{\sum_{i=1}^{\infty}T_{G_i(e)}(x),1\right\},\\ \inf\left\{\sum_{i=1}^{\infty}I_{G_i(e)}(x),1\right\},\\ \sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}F_{G_i(e)}(x)-n+1,0\right\}\right\}\end{array}\right\rangle : x\in X\right\} : e\in E\right\},$$

$$\bigcap_{i=1}^{\infty}(G_i,E)=\left\{\left\{e,\left\langle \begin{array}{l} x,\sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}T_{G_i(e)}(x)-n+1,0\right\}\right\},\\ \sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}I_{G_i(e)}(x)-n+1,0\right\}\right\},\\ \inf\left\{\sum_{i=1}^{\infty}F_{G_i(e)}(x),0\right\}\end{array}\right\rangle : x\in X\right\} : e\in E\right\}.$$

Definition 3.5 Let (G_1,E) and (G_2,E) be two neutrosophic soft set over the universe set X. Then "AND" operation on them is denoted by $(G_1,E)\wedge(G_2,E)=(G_3,E\times E)$ and is defined by:

$$(G_3,E\times E)=\left\{\left\{(e_1,e_2),\left\langle x,T_{G_3(e)}(x),I_{G_3(e)}(x),F_{G_3(e)}(x)\right\rangle : x\in X\right\} : (e_1,e_2)\in E\times E\right\}$$

where

$$T_{G_3(e_1,e_2)}(x)=\max\{T_{G_1(e_1)}(x)+T_{G_2(e_2)}(x)-1,0\},$$
$$I_{G_3(e_1,e_2)}(x)=\max\{I_{G_1(e_1)}(x)+I_{G_2(e_2)}(x)-1,0\},$$
$$F_{G_3(e_1,e_2)}(x)=\min\{F_{G_1(e_1)}(x)+F_{G_2(e_2)}(x),1\}.$$

Definition 3.6 Let (G_1,E) and (G_2,E) be two neutrosophic soft set over the universe set X. Then

"OR" operation on them is denoted by $(G_1,E)\vee(G_2,E)=(G_3,E\times E)$ and is defined by:

$$(G_3,E\times E) = \left\{ \begin{array}{l} \left((e_1,e_2), \left\langle x, T_{G_3(e)}(x), I_{G_3(e)}(x), F_{G_3(e)}(x) \right\rangle \atop : x\in X \right) \\ :(e_1,e_2)\in E\times E \end{array} \right\}$$

where

$$T_{G_3(e_1,e_2)}(x)=\min\left\{T_{G_1(e_1)}(x)+T_{G_2(e_2)}(x),1\right\},$$

$$I_{G_3(e_1,e_2)}(x)=\min\left\{I_{G_1(e_1)}(x)+I_{G_2(e_2)}(x),1\right\},$$

$$F_{G_3(e_1,e_2)}(x)=\max\left\{F_{G_1(e_1)}(x)+F_{G_2(e_2)}(x)-1,0\right\}.$$

Definition 3.7

1. A neutrosophic soft set (G,E) over the universe set X is said to be null neutrosophic soft set if $T_{G(e)}(x)=0; I_{G(e)}(x)=0;$ $F_{G(e)}(x)=1$ for all $e\in E$, for all $x\in X$. It is denoted by $0_{(X,E)}$.
2. A neutrosophic soft set (G,E) over the universe set X is said to be absolute neutrosophic soft set if $T_{G(e)}(x)=1; I_{G(e)}(x)=1; F_{G(e)}(x)=0$; for all $e\in E$, for all $x\in X$. It is denoted by $1_{(X,E)}$.

Obivious that, $0^C_{(X,E)}=1_{(X,E)}$ and $1^C_{(X,E)}=0_{(X,E)}$.

Proposition 3.1Let $(G_1,E),(G_2,E)$ and (G_3,E) be neutrosophic soft sets over the universe set X. Then

1. $(G_1,E)\cup[(G_2,E)\cup(G_3,E)]$
$=[(G_1,E)\cup(G_2,E)]\cup(G_3,E)$
$=(G_1,E)\cup(G_2,E)\cup(G_3,E)$,
$(G_1,E)\cap[(G_2,E)\cap(G_3,E)]$
$=[(G_1,E)\cap(G_2,E)]\cap(G_3,E)$
$=(G_1,E)\cap(G_2,E)\cap(G_3,E)$,
2. $(G_1,E)\cup 0_{(X,E)}=(G_1,E)$;
$(G_1,E)\cap 0_{(X,E)}=0_{(X,E)}$
3. $(G_1,E)\cup 1_{(X,E)}=1_{(X,E)}$;
$(G_1,E)\cap 1_{(X,E)}=(G_1,E)$.

Proof. 1. We prove:

$$(G_1,E)\cup[(G_2,E)\cup(G_3,E)] =[(G_1,E)\cup(G_2,E)]\cup(G_3,E).$$

$\forall e\in E$ and $\forall x\in X$, on the right hand, let $(G,E)=[(G_1,E)\cup(G_2,E)]\cup(G_3,E)$.

$$T_{G(e)}(x)=\min\left\{ \begin{array}{l} T_{G_3(e)}(x)+\min\left\{ \begin{array}{l} T_{G_1(e)}(x)+T_{G_2(e)}(x) \\ ,1 \end{array} \right\} \\ ,1 \end{array} \right\},$$

$$I_{G(e)}(x)=\min\left\{ \begin{array}{l} I_{G_3(e)}(x)+\min\left\{ \begin{array}{l} I_{G_1(e)}(x)+I_{G_2(e)}(x) \\ ,1 \end{array} \right\} \\ ,1 \end{array} \right\},$$

$$F_{G(e)}(x)=\max\left\{ \begin{array}{l} F_{G_3(e)}(x) \\ +\max\left\{ \begin{array}{l} F_{G_1(e)}(x)+F_{G_2(e)}(x)-1 \\ ,0 \end{array} \right\}-1 \\ ,0 \end{array} \right\}$$

On the left hand, let

$$(G',E)=(G_1,E)\cup[(G_2,E)\cup(G_3,E)].$$

$$G'_{T(e)}(x)=\min\left\{ \begin{array}{l} T_{G_1(e)}(x) \\ +\min\left\{T_{G_2(e)}(x)+T_{G_3(e)}(x),1\right\} \\ ,1 \end{array} \right\},$$

$$G'_{I(e)}(x)=\min\left\{ \begin{array}{l} I_{G_1(e)}(x) \\ +\min\left\{I_{G_2(e)}(x)+I_{G_3(e)}(x),1\right\} \\ ,1 \end{array} \right\},$$

$$G'_{F(e)}(x)=\max\left\{ \begin{array}{l} F_{G_1(e)}(x) \\ +\max\left\{ \begin{array}{l} F_{G_2(e)}(x)+F_{G_3(e)}(x)-1 \\ ,0 \end{array} \right\} \\ -1 \\ ,0 \end{array} \right\}$$

We have

$$\min\left\{T_{G_3(e)}(x)+\min\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x),1\right\},1\right\}$$
$$=\min\left\{T_{G_3(e)}(x)+T_{G_1(e)}(x)+T_{G_2(e)}(x),1\right\},$$
$$\min\left\{T_{G_1(e)}(x)+\min\left\{T_{G_2(e)}(x)+T_{G_3(e)}(x),1\right\},1\right\}$$
$$=\min\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x)+T_{G_3(e)}(x),1\right\},$$
$$\max\left\{\begin{array}{l}F_{G_3(e)}(x)\\+\max\left\{\begin{array}{l}F_{G_1(e)}(x)+F_{G_2(e)}(x)-1\\,0\end{array}\right\}-1\\,0\end{array}\right\}$$
$$=\max\left\{F_{G_3(e)}(x)+F_{G_1(e)}(x)+F_{G_2(e)}(x)-2,0\right\},$$
$$\max\left\{\begin{array}{l}F_{G_1(e)}(x)\\+\max\left\{\begin{array}{l}F_{G_2(e)}(x)+F_{G_3(e)}(x)-1\\,0\end{array}\right\}-1\\,0\end{array}\right\}$$
$$=\max\left\{F_{G_3(e)}(x)+F_{G_1(e)}(x)+F_{G_2(e)}(x)-2,0\right\}.$$

Thus

$$(G_1,E)\cup\left[(G_2,E)\cup(G_3,E)\right]$$
$$=\left[(G_1,E)\cup(G_2,E)\right]\cup(G_3,E).$$

Remark 3.1 Generally,

$$(G_1,E)\cup\left[(G_2,E)\cap(G_3,E)\right]$$
$$=\left[(G_1,E)\cup(G_2,E)\right]\cap\left[(G_1,E)\cup(G_3,E)\right],$$
$$(G_1,E)\cap\left[(G_2,E)\cup(G_3,E)\right]$$
$$=\left[(G_1,E)\cap(G_2,E)\right]\cup\left[(G_1,E)\cap(G_3,E)\right],$$

is not true for new operations.

Proposition 3.2Let $(G_1,E),(G_2,E)$ be two neutrosophic soft sets over the universe set X. Then,

$$\left[(G_1,E)\cup(G_2,E)\right]^C=(G_1,E)^C\cap(G_2,E)^C;$$
$$\left[(G_1,E)\cap(G_2,E)\right]^C=(G_1,E)^C\cup(G_2,E)^C.$$

Proof. For all $e\in E$ and $x\in X$,

$$\left[(G_1,E)\cup(G_2,E)\right]^C$$
$$=\left\{\left\langle\begin{array}{l}x,\max\left\{F_{G_1(e)}(x)+F_{G_2(e)}(x)-1,0\right\},\\1-\min\left\{I_{G_1(e)}(x)+I_{G_2(e)}(x),1\right\},\\\min\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x),1\right\}\end{array}\right\rangle\right\}.$$

And,

$$(G_1,E)^C\cap(G_2,E)^C$$
$$=\left\{\left\langle\begin{array}{l}x,\max\left\{F_{G_1(e)}(x)+F_{G_2(e)}(x)-1,0\right\},\\\max\left\{1-I_{G_1(e)}(x)-I_{G_2(e)}(x),0\right\},\\\min\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x),1\right\}\end{array}\right\rangle\right\}.$$

- If $I_{G_1(e)}(x)+I_{G_2(e)}(x)\geq 1$,
$$1-\min\left\{I_{G_1(e)}(x)+I_{G_2(e)}(x),1\right\}=1-1=0;$$
$$\max\left\{1-I_{G_1(e)}(x)-I_{G_2(e)}(x),0\right\}=0.$$
- If $I_{G_1(e)}(x)+I_{G_2(e)}(x)<1$,
$$1-\min\left\{I_{G_1(e)}(x)+I_{G_2(e)}(x),1\right\}$$
$$=1-I_{G_1(e)}(x)-I_{G_2(e)}(x);$$
$$\max\left\{1-I_{G_1(e)}(x)-I_{G_2(e)}(x),0\right\}$$
$$=1-I_{G_1(e)}(x)-I_{G_2(e)}(x).$$

Therefore,

$$\left[(G_1,E)\cup(G_2,E)\right]^C=(G_1,E)^C\cap(G_2,E)^C.$$

Proposition 3.3 Let $(G_1,E),(G_2,E)$ be two neutrosophic soft sets over the universe set X. Then,

$$\left[(G_1,E)\vee(G_2,E)\right]^C=(G_1,E)^C\wedge(G_2,E)^C;$$
$$\left[(G_1,E)\wedge(G_2,E)\right]^C=(G_1,E)^C\vee(G_2,E)^C.$$

Proof. It is similar to Proposition 3.2.

IV. NEUTROSOPHICSOFT TOPOLOGICAL SPACES ON NEW OPERATIONS

In this part, we will construct the neutrosophic soft topology based on the new operations of the neutrosophic soft union and intersection; the neutrosophic soft null and absolute set above. Propositions and theorems presented below, are proved as the same way as Ozturk's paper [6].

The most special proposition is that the neutrosophic soft topology induce component topologies: fuzzy soft topologies, fuzzy topologies. These are what we emphasize. Because not all operations given on the neutrosophic soft also guarantee the successful induction when we construct neutrosophic soft topological space. Therefore, pointing out the operations different from those defined in Ozturk's paper will be helpful in generalizing the operations on this set, helping the researches of the neutrosophic soft set.

Definition 4.1 Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X and $\overset{NSS}{\tau} \subset NSS(X,E)$. Then we say that $\overset{NSS}{\tau}$ is a neutrosophic soft topology on X if

1. $0_{(X,E)}$ and $1_{(X,E)}$ belong to $\overset{NSS}{\tau}$.
2. The union of any number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$.
3. The intersection of finite number of neutrosophic soft sets in $\overset{NSS}{\tau}$ belongs to $\overset{NSS}{\tau}$.

Then $\left(X, \overset{NSS}{\tau}, E\right)$ is said to be a neutrosophic soft topological space over X. Each elements of $\overset{NSS}{\tau}$ is said to be neutrosophic soft open set.

Definition 4.2 Let $NSS(X,E)$ be a neutrosophic soft topological space over X and (G,E) be a neutrosophic soft set over X. Then (G,E) is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Proposition 4.1 Let $\left(X, \overset{NSS}{\tau}, E\right)$ be a neutrosophic soft topological space over X. Then

1. $0_{(X,E)}$ and $1_{(X,E)}$ are neutrosophic soft closed sets over X.
2. The union of any number of neutrosophic soft closed sets is a neutrosophic soft closed set over X.
3. The intersection of finite number of neutrosophic soft closed sets is a neutrosophic soft closed set over X.

Definition 4.3 Let $NSS(X,E)$ be the family of all neutrosophic soft sets over the universe set X.

1. If $\overset{NSS}{\tau} = \left\{0_{(X,E)}, 1_{(X,E)}\right\}$, then $\overset{NSS}{\tau}$ is said to be the neutrosophic soft in discrete topology and $\left(X, \overset{NSS}{\tau}, E\right)$ is said to be a neutrosophic soft indiscrete topological space over X.
2. If $\overset{NSS}{\tau} = NSS(X,E)$, then $\overset{NSS}{\tau}$ is said to be the neutrosophic soft discrete topology and $\left(X, \overset{NSS}{\tau}, E\right)$ is said to be a neutrosophic soft discrete topological space over X.

Proposition 4.2Let $\left(X, \overset{NSS}{\tau_1}, E\right)$ and $\left(X, \overset{NSS}{\tau_2}, E\right)$ be two neutrosophic soft topological spaces over the same universe set X. Then $\left(X, \overset{NSS}{\tau_1} \cap \overset{NSS}{\tau_2}, E\right)$ is neutrosophic soft topological space over X.

Remark 4.1 The union of two neutrosophic soft topologies over X may not be a neutrosophic soft topology on X.

Example 4.1 Let $X = \{x_1, x_2, x_3\}$ be an universe set, $E = \{e_1, e_2\}$ be a set of parameters and $\overset{NSS}{\tau_1} = \left\{0_{(X,E)}, 1_{(X,E)}, (G_1,E), (G_2,E)\right\}$ and $\overset{NSS}{\tau_2} = \left\{0_{(X,E)}, 1_{(X,E)}, (G_3,E)\right\}$ be two neutrosophic soft topologies over X. And the neutrosophic soft sets $(G_1,E), (G_2,E)$ and (G_3,E) are defined as following:

$$(G_1,E) = \begin{Bmatrix} e_1 = \{\langle x_1, 0.7, 0.8, 0.3\rangle, \langle x_2, 0.5, 0.2, 0.6\rangle\}, \\ e_2 = \{\langle x_1, 0.4, 0.5, 0.8\rangle, \langle x_2, 0.3, 0.4, 0.2\rangle\} \end{Bmatrix},$$

$$(G_2,E)=\begin{cases}e_1=\{\langle x_1,0.3,0.2,0.7\rangle,\langle x_2,0.5,0.8,0.4\rangle\},\\ e_2=\{\langle x_1,0.6,0.5,0.2\rangle,\langle x_2,0.7,0.6,0.8\rangle\}\end{cases},$$

$$(G_3,E)=\begin{cases}e_1=\{\langle x_1,0.3,0.9,0.1\rangle,\langle x_2,0.7,0.5,0.6\rangle\},\\ e_2=\{\langle x_1,0.1,0.2,0.8\rangle,\langle x_2,0.4,0.3,0.5\rangle\}\end{cases}.$$

Because

$$(G_1,E)\cup(G_3,E)$$

$$=\begin{cases}e_1=\{\langle x_1,1,1,0\rangle,\langle x_2,1,0.7,0.2\rangle\},\\ e_2=\{\langle x_1,0.5,0.7,0.6\rangle,\langle x_2,0.7,0.7,0\rangle\}\end{cases}\notin \overset{NSS}{\tau_1}\cup\overset{NSS}{\tau_2},$$

so $\overset{NSS}{\tau_1}\cup\overset{NSS}{\tau_2}$ is not a neutrosophic soft topology over X.

Proposition 4.3Let $\left(X,\overset{NSS}{\tau},E\right)$ be a neutrosophic soft topological space over X and

$$\overset{NSS}{\tau}=\{(G_i,E):(G_i,E)\in NSS(X,E)\}$$
$$=\{[e,G_i(e)]_{e\in E}:(G_i,E)\in NSS(X,E)\}$$

where

$$(G_i,e)=\left\{\left\langle x,T_{G_i(e)}(x),I_{G_i(e)}(x),F_{G_i(e)}(x)\right\rangle:x\in X\right\}.$$

Then

$$\tau_1=\left\{\left[T_{G_i(e)}(X)\right]_{e\in E}\right\},\tau_2=\left\{\left[I_{G_i(e)}(X)\right]_{e\in E}\right\},$$
$$\tau_3=\left\{\left[F_{G_i(e)}(X)\right]^C_{e\in E}\right\},$$

define fuzzy soft topologies on X.

Proof.

1. $0_{(X,E)}\in\overset{NSS}{\tau}\Rightarrow 0\in\tau_1;0\in\tau_2;0\in\tau_3$.

$1_{(X,E)}\in\overset{NSS}{\tau}\Rightarrow 1\in\tau_1;1\in\tau_2;1\in\tau_3$.

2. Let $\left\{\left[T_{G_i(e)}(X)\right]_{e\in E}\right\}_{i=1}^{\infty}$ is a family of fuzzy soft sets in τ_1; $\left\{\left[I_{G_i(e)}(X)\right]_{e\in E}\right\}_{i=1}^{\infty}$ is a family of fuzzy soft sets in τ_2; $\left\{\left[F_{G_i(e)}(X)\right]^C_{e\in E}\right\}_{i=1}^{\infty}$ is a family of fuzzy soft sets in τ_3. And they satisfy $\{(G_i,E)\}_{i\in I}=\left\{\left[e,G_i(e)\right]_{e\in E}\right\}_{i=1}^{\infty}$ where $G_i(e)=\left\{\left\langle x,T_{G_i(e)}(x),I_{G_i(e)}(x),F_{G_i(e)}(x)\right\rangle:x\in X\right\}$ be a family of neutrosophic soft sets in $\overset{NSS}{\tau}$. So $\bigcup_{i=1}^{\infty}(G_i,E)\in\overset{NSS}{\tau}$. That is,

$$\bigcup_{i=1}^{\infty}(G_i,E)$$

$$=\left\{\left(e,\left\langle\begin{array}{l}x,\inf\left\{\sum_{i=1}^{\infty}T_{G_i(e)}(x),1\right\},\\ \inf\left\{\sum_{i=1}^{\infty}I_{G_i(e)}(x),1\right\},\\ \sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}F_{G_i(e)}(x)-n+1,0\right\}\right\}\end{array}\right\rangle:x\in X\right):e\in E\right\}$$

Therefore,

$$\left\{\left[\left\langle\inf\left\{\sum_{i=1}^{\infty}T_{G_i(e)}(x),1\right\}\right\rangle:x\in X\right]_{e\in E}\right\}$$
$$=\bigcup_{i=1}^{\infty}\left\{\left[T_{G_i(e)}(X)\right]_{e\in E}\right\}\in\tau_1,$$

$$\left\{\left[\left\langle\inf\left\{\sum_{i=1}^{\infty}I_{G_i(e)}(x),1\right\}\right\rangle:x\in X\right]_{e\in E}\right\}$$
$$=\bigcup_{i=1}^{\infty}\left\{\left[I_{G_i(e)}(X)\right]_{e\in E}\right\}\in\tau_2,$$

$$\left\{\left[\left\langle\left[\sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}F_{G_i(e)}(x)-n+1,0\right\}\right\}\right]\right\rangle:x\in X\right]_{e\in E}\right\}^C$$

$$=\left\{\left[\left\langle\left[1-\sup_{n\to\infty}\left\{\max\left\{\sum_{i=1}^{n}F_{G_i(e)}(x)-n+1,0\right\}\right\}\right]\right\rangle:x\in X\right]_{e\in E}\right\}$$

$$=\left\{\left[\left\langle\left[\inf_{n\to\infty}\left\{\min\left\{n-\sum_{i=1}^{n}F_{G_i(e)}(x),1\right\}\right\}\right]\right\rangle\right]_{e\in E} : x\in X\right\}$$

$$=\bigcup_{i=1}^{\infty}\left\{\left[\left[I_{G_i(e)}(X)\right]^{C}_{e\in E}\right]\right\}\in\tau_3.$$

3. We have $(G_1,E)\cap(G_2,E)\in \overset{NSS}{\tau}$. That is,

$$(G_1,E)\cap(G_2,E)$$

$$=\left\{\left(e,\left\langle\begin{array}{l} x,\max\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x)-1,0\right\},\\ \max\left\{I_{G_1(e)}(x)+I_{G_2(e)}(x)-1,0\right\},\\ \min\left\{F_{G_1(e)}(x)+F_{G_2(e)}(x),1\right\}\end{array}\right\rangle : x\in X\right) : e\in E\right\}$$

Hence,

$$\left\{\left[\left\langle\max\left\{T_{G_1(e)}(x)+T_{G_2(e)}(x)-1,0\right\}\right\rangle : x\in X\right]_{e\in E}\right\}$$

$$=\left\{\left[T_{G_1(e)}(X)\right]_{e\in E}\right\}\cap\left\{\left[T_{G_2(e)}(X)\right]_{e\in E}\right\}\in\tau_1,$$

$$\left\{\left[\left\langle\max\left\{I_{G_1(e)}(x)+I_{G_2(e)}(x)-1,0\right\}\right\rangle : x\in X\right]_{e\in E}\right\}$$

$$=\left\{\left[I_{G_1(e)}(X)\right]_{e\in E}\right\}\cap\left\{\left[I_{G_2(e)}(X)\right]_{e\in E}\right\}\in\tau_2,$$

$$\left\{\left[\left\langle\left[\min\left\{F_{G_1(e)}(x)+F_{G_2(e)}(x),1\right\}\right]^{C}\right\rangle : x\in X\right]_{e\in E}\right\}$$

$$=\left\{\left[\left[F_{G_1(e)}(X)\right]^{C}\right]_{e\in E}\right\}\cap\left\{\left[\left[F_{G_2(e)}(X)\right]^{C}\right]_{e\in E}\right\}\in\tau_3.$$

Remark 4.2 Generally, converse of the above proposition is not true.

Example 4.2 Let $X=\{x_1,x_2\}$ be an universe set, $E=\{e_1,e_2\}$ be a set of parameters. And the neutrosophic soft sets $(G_1,E),(G_2,E)$ and (G_3,E) are defined as following:

$$(G_1,E)=\left\{\begin{array}{l} e_1=\left\{\begin{array}{l}\langle x_1,0.25,0.25,0.75\rangle,\\ \left\langle x_2,\frac{1}{3},\frac{1}{3},\frac{2}{3}\right\rangle\end{array}\right\},\\ e_2=\left\{\begin{array}{l}\langle x_1,0.25,0.25,0.75\rangle,\\ \left\langle x_2,\frac{1}{3},\frac{1}{3},\frac{2}{3}\right\rangle\end{array}\right\}\end{array}\right\},$$

$$(G_2,E)=\left\{\begin{array}{l} e_1=\left\{\begin{array}{l}\langle x_1,0.5,0.75,0.5\rangle,\\ \left\langle x_2,\frac{1}{3},\frac{2}{3},\frac{2}{3}\right\rangle\end{array}\right\},\\ e_2=\left\{\begin{array}{l}\langle x_1,0.5,0.75,0.5\rangle,\\ \left\langle x_2,\frac{1}{3},\frac{2}{3},\frac{2}{3}\right\rangle\end{array}\right\}\end{array}\right\},$$

$$(G_3,E)=\left\{\begin{array}{l} e_1=\left\{\begin{array}{l}\langle x_1,0.75,0.5,0.25\rangle,\\ \left\langle x_2,\frac{2}{3},\frac{1}{3},\frac{1}{3}\right\rangle\end{array}\right\},\\ e_2=\left\{\begin{array}{l}\langle x_1,0.75,0.5,0.25\rangle,\\ \left\langle x_2,\frac{2}{3},\frac{1}{3},\frac{1}{3}\right\rangle\end{array}\right\}\end{array}\right\}.$$

Then,

$$\tau_1=\left\{\begin{array}{l}\{(0,0),(0,0)\},\{(1,1),(1,1)\},\\ \left\{\left(0.25,\frac{1}{3}\right),\left(0.25,\frac{1}{3}\right)\right\},\left\{\left(0.5,\frac{1}{3}\right),\left(0.5,\frac{1}{3}\right)\right\},\\ \left\{\left(0.75,\frac{2}{3}\right),\left(0.75,\frac{2}{3}\right)\right\}\end{array}\right\},$$

$$\tau_2=\left\{\begin{array}{l}\{(0,0),(0,0)\},\{(1,1),(1,1)\},\\ \left\{\left(0.25,\frac{1}{3}\right),\left(0.25,\frac{1}{3}\right)\right\},\\ \left\{\left(0.75,\frac{2}{3}\right),\left(0.75,\frac{2}{3}\right)\right\},\\ \left\{\left(0.5,\frac{1}{3}\right),\left(0.5,\frac{1}{3}\right)\right\}\end{array}\right\},$$

$$\tau_3 = \left\{ \begin{array}{l} \{(0,0),(0,0)\},\{(1,1),(1,1)\}, \\ \left\{\left(0.25,\frac{1}{3}\right),\left(0.25,\frac{1}{3}\right)\right\},\left\{\left(0.5,\frac{1}{3}\right),\left(0.5,\frac{1}{3}\right)\right\}, \\ \left\{\left(0.75,\frac{2}{3}\right),\left(0.75,\frac{2}{3}\right)\right\} \end{array} \right\},$$

are fuzzy soft topologies on X. But $\overset{NSS}{\tau} = \{0_{(X,E)}, 1_{(X,E)}, (G_1,E),(G_2,E),(G_3,E)\}$ is not a neutrosophic soft topology on X because $(G_1,E)\cup(G_2,E)\notin \overset{NSS}{\tau}$.

Proposition 4.4Let $\left(X,\overset{NSS}{\tau},E\right)$ be a neutrosophic soft topological space over X. Then

$$\tau_{1e} = \left\{\left[T_{G_i(e)}(X)\right] : (G_i,E)\in \overset{NSS}{\tau}\right\},$$

$$\tau_{2e} = \left\{\left[I_{G_i(e)}(X)\right] : (G_i,E)\in \overset{NSS}{\tau}\right\},$$

$$\tau_{3e} = \left\{\left[F_{G_i(e)}(X)\right] : (G_i,E)\in \overset{NSS}{\tau}\right\},$$

for each $e\in E$, define fuzzy topologies on X.

Proof.It can be implied from Proposition 4.3.

Remark 4.3 Generally, converse of the above proposition is not true.

Example 4.3 Let us consider the Example 4.2. Then,

$$\tau_{1e_1} = \left\{(0,0),(1,1),\left(0.25,\frac{1}{3}\right),\left(0.5,\frac{1}{3}\right),\left(0.75,\frac{2}{3}\right)\right\},$$

$$\tau_{2e_1} = \left\{(0,0),(1,1),\left(0.25,\frac{1}{3}\right),\left(0.75,\frac{2}{3}\right),\left(0.5,\frac{1}{3}\right)\right\},$$

$$\tau_{3e_1} = \left\{(0,0),(1,1),\left(0.25,\frac{1}{3}\right),\left(0.5,\frac{1}{3}\right),\left(0.75,\frac{2}{3}\right)\right\},$$

are fuzzy topologies on X. Similarly, $\tau_{1e_2}, \tau_{2e_2}, \tau_{3e_2}$ are also fuzzy topologies, but $\overset{NSS}{\tau} = \{0_{(X,E)}, 1_{(X,E)}, (G_1,E),(G_2,E),(G_3,E)\}$ is not a neutrosophic soft topology on X because $(G_1,E)\cup(G_2,E)\notin \overset{NSS}{\tau}$.

V.CONCLUSIONS

In this paper, we define some new operations of the neutrosophic soft set. Finally, we have checked the properties of new neutrosophic soft topological spaces and the relationship between neutrosophic soft topological space and component topological spaces: fuzzy topological space, fuzzy soft topological space. By giving operations different from Ozturk's paper, the authors hope that we will define general operations on neutrosophic soft set and construct successfully neutrosophic soft topological spaces on them and keep the relationship between neutrosophic soft topology and component topologies.

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