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An Interesting Application of Linear Diophantine Equation

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Abstract:

Diophantine equations can be used in various fields. In this paper, I have tried to study the origin of Diophantine Equations and describe a solution to an interesting problem which I had encountered at my very early age but was unable to find a solution till not studying the Diophantine equation.

Keywords: Application of Linear Diophantine equation, Linear Diophantine equation, Extended Euclidean Theorem.

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I. INTRODUCTION

Suppose a man went to a bank to get some cash. The cash amount consists of some dollars and some cents and the bank had only an infinite supply of one-dollar coin and one-cent coin. There when he went to the cashier to take his money, by mistake the cashier exchanged the number of dollars with the number of cents (more formally, if the initial amount is x_d dollar and x_c cent, the cashier has given him x_c dollar and x_d cent). But he did not count the total amount of money in the bank cashier. Then he moved to the market to buy some things there he spent 5 cents. After buying the things he counted his money in his pocket and he found that the total money in his pocket is twice his initial money. Now how to calculate the total amount of his initial money. (1 dollar = 100 cent)

The above problem can be solved using the linear Diophantine equation.

In mathematics, a Diophantine equation is a polynomial equation, usually in two or more unknowns, such that only the integer solutions are sought or studied (an integer solution is such that all the unknowns take integer values). A linear Diophantine equation equates the sum of two or more monomials, each of degree 1 in one of the variables, to a constant. An exponential Diophantine equation is one in which exponents on terms can be unknowns.

Diophantine problems have fewer equations than unknown variables and involve finding integers that work correctly for all equations. In more technical language, they define an algebraic curve, algebraic surface, or more general object, and ask about the lattice points on it.

The word Diophantine refers to the Hellenistic mathematician of the 3rd century, Diophantus of Alexandria, who made a study of such equations and was one of the first mathematicians to introduce symbolism into algebra. The mathematical study of Diophantine problems that Diophantus initiated is now called Diophantine analysis.

While individual equations present a kind of puzzle and have been considered throughout history, the formulation of general theories of Diophantine equations (beyond the theory of quadratic forms) was an achievement of the twentieth century.

A Linear Diophantine Equation (in two variables) is an equation of the general form:

$ax + by = c$ where a, b, c are given integers and x, y are unknown integers.

II. SOLVING METHOD

A. The degenerate case

A degenerate case that needs to be taken care of is when a=b=0. It is easy to see that we either have no solutions or infinitely many solutions, depending on whether c=0 or not. In the rest of this article, I will ignore this case.

B. Finding a Solution

To find one solution of the Diophantine equation with 2 unknowns, one can use the Extended Euclidean algorithm. First, let assume that a and b are non-negative. When we apply the Extended Euclidean algorithm for a and b, we can find their greatest common divisor g and 2 numbers x_g and y_g such that:

$ax_g + by_g = g$

If c is divisible by g = gcd(a,b) then the given Diophantine equation has a solution, otherwise, it does not have any solution. The proof is straightforward: a linear combination of two numbers is divisible by their common divisor.

Now supposed that c is divisible by g, then we have:

$a. x_g .c/g + b. y_g .c/g = c$

Therefore, one of the solutions of the Diophantine equation is:

$x_0 = x_g \cdot c/g$

$y_0 = y_g \cdot c/g$

The above idea still works when a or b or both of them are negative. We only need to change the sign of x_0 and y_0 when necessary.

Finally, we can implement this idea as follows (note that this code does not consider the case a=b=0):

From one solution (x_0, y_0) we can obtain all the solutions of the given equation.

C. Getting all Solutions

Let g = gcd(a,b) and let x_0,y_0 be integers which satisfy the following:

$a\cdot x_0 + b\cdot y_0 = c$

Now, we should see that adding b/g to x_0, and, at the same time subtracting a/g from y_0 will not break the equality:

$a\cdot(x_0+b/g) + b\cdot(y_0-a/g)$

$= a\cdot x_0 + b\cdot y_0 + a\cdot b/g - b\cdot a/g$

$= c$

Obviously, this process can be repeated again, so all the numbers of the form:

$x = x_0 + k\cdot b/g$

$y = y_0 - k\cdot a/g$

are solutions of the given Diophantine equation.

Moreover, this is the set of all possible solutions to the given Diophantine equation.

III. RESULTS & DISCUSSION

Let suppose initially he had x_d amount of dollars and y_c amount of cents.

So, according to the problem

$100y_c + x_d – 5 = 2* (100x_d + y_c)$

Or, $- 199x_d + 98y_c = 5$ ……. .(i)

The above equation is an example of Diophantine equation as it has two unknowns and also gcd(199,98) = 1 divides 5 so solution exists for this linear Diophantine equation.

Using extended Euclidean Theorem $199x_g + 98y_g$ = gcd(199,98) = 1(ignoring negative signs).

199*(33) + 98*(-67) = 1

199*33 * 5 + 98* (-67) * 5 = 5 (multiplying by 5)

199 * (165) + 98 * (-335) = 5

Comparing with equation (i)

x_d = -165 and y_c = -335

General solution x_d = 98k – 165 and y_c = 199k – 335

It can be easily concluded that for k >= 2 we can get infinity set of solution among them x_d = 31 and y_c = 63 is the solution for k = 2.

IV. CONCLUSION

As has been explained above, linear diophantine equation in two variables can be solved by euclidean algorithm and extended euclidean algorithm. When the solution has been searched for linear diophantine equation in two variables, as ax + by = c, first should be checked (a, b) | c. If (a, b) | c, then there is a solution. To find the solution for linear diophantine equation in n variables, as $a_1x_1 + a_2x_2 + \cdot\cdot\cdot + a_nx_n = c$,again should be checked $(a_1, a_2, \cdot\cdot\cdot ,a_n) \mid c$ by mathematical induction.

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