

BASICS OF QUANTUM MECHANICAL SPIN AND Its CALCULATIONS

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Abstract:

In this article classical understanding of spin has been exploited to model and visualize the concept of quantum mechanical spin. Quantum mechanical spin holds significance in understanding the system of both the macro and quantum worlds. This article explains the existence of spin and its mechanism in particles studying the results obtained by Stern and Gerlach in 1922 AD through their experiment. The article explicitly explains the connection between spin and magnetism. Moreover, the study was made to ease the concept of Quantum mechanical spin for general understanding for readers and enthusiasts. Furthermore, the introductory calculations of spin of an electron will enlighten the readers towards the mathematical understanding of spin and the drawbacks of some of the proposed electron spin models that evaluate the value of angular momentum and magnetic moment.

Keywords —Magnetic moment, Angular momentum, spin , Wave function, Inhomogeneous magnetic field, Orbital, Bloch sphere, charge density

I. INTRODUCTION

The classical mechanics have always described the spin as an Angular momentum of macro objects. As compared to Quantum mechanics, Spin has bizarre definition and understanding in the quantum world. In quantum mechanics and particle physics, **spin** is an intrinsic form of **angular momentum** carried by elementary particles, composite particles (**hadrons**), and atomic nuclei. Spin is one of two types of angular momentum in quantum mechanics, the other being orbital angular momentum. The orbital angular momentum operator is the quantum-mechanical counterpart to the classical angular momentum of the orbital revolution and appears when there is a periodic structure to its wave function as the angle varies.[1] In practice, Spin is assigned a quantum number i.e. division of spin

angular momentum by reduced Planck constant(\hbar). Particles with half-integer spin ($1/2\hbar$, $3/2\hbar$, $5/2\hbar$, etc.) are called fermions. Particles with integer spin ($1\hbar$, $2\hbar$, $3\hbar$, etc.) are called bosons.[2]

II. BACKGROUND (ORIGIN OF EXISTENCE)

Spin was first discovered in the context of the emission spectrum of alkali metals. In 1924, Wolfgang Pauli introduced what he called a "two-valuedness not describable classically" associated with the electron in the outermost shell. This allowed him to formulate the Pauli Exclusion Principle, stating that no two electrons can have the same quantum state in the same quantum system. The physical interpretation of Pauli's "degree of freedom" was initially unknown. Ralph Kronig, one of Landé's assistants, suggested in early 1925 that it was produced by the self-rotation of the electron. When Pauli heard about the idea, he criticized it severely, noting that the electron's hypothetical surface would have to be moving faster than the speed of light in order for it to rotate quickly enough to produce the necessary angular momentum. This would violate the theory of relativity. Largely due to Pauli's criticism, Kronig decided not to publish his idea. [2]

In retrospect, the genesis of this bizarre spin mechanics dates back to 1922 when Stern and Gerlach delivered the first experimental proof of the fascinating degree of freedom of an electron spin. In the Stern-Gerlach experiment, a beam of silver atom is generated in an atomic beam furnace, it was then sent towards an inhomogeneous magnetic field. According to classical physics, the results would have been expected for the magnetic moments of silver atoms to be randomly oriented so that they should be deflected in the inhomogeneous magnetic field by different amounts depending on their orientation. However, the results of the researchers observed that the beam was split into two possible states which were later named spin up and spin down. If we look at the configuration of silver (Ag) atom then we can see the each silver atom has total of 47 electrons in which 46 of the electrons each spin up is paired with one spin down, spins neutralize each other and cancel out the each other but, still the unpaired spin either spin up or spin down state resulting in an anomalous magnetic behavior of an electron. We can represent the spin state using a Bloch sphere where the spin can point in any random direction, similarly, the spin of silver atoms are randomly distributed. Therefore, the silver atoms are not polarized.

III. SPIN MAGNETIC MOMENT

A spin magnetic moment is a magnetic moment caused by the spin of elementary particles. For example, the electron is an elementary spin-1/2 fermion. Quantum electrodynamics gives the most accurate prediction of the anomalous magnetic moment of the electron. [3] In light of the connection between spin and magnetic moment, it turns out particles behave like peculiar tiny magnets by generating weak magnetic fields. That is why objects from macro world, which are composed of many such “tiny magnets” are magnetic. But that does not explain why only a handful of materials are magnetic when all macroscopic materials are made up of these tiny magnets. How is that possible?

The reason is that the magnetic fields generated by individual particles (mostly electrons, whose magnetic fields are much stronger than those of protons or neutrons) often cancel out, which in turn makes most materials non-magnetic. For instance, if an atomic orbital is completely filled, electrons in this orbital have opposite spins that cancel out and hence, show no magnetic behavior. This means that no atom with filled or almost filled orbitals can be magnetic. In order to possess magnetic properties, an atom should have half-filled orbitals. So that the magnetic fields of individual electrons reinforce one another.

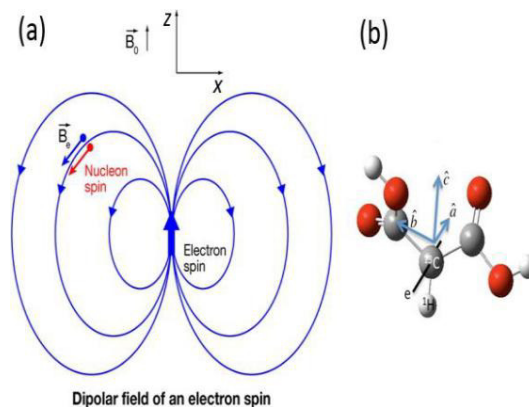


Fig. 1.1 <http://stonebrakerdesignworks.com>

However, not all materials made up of magnetic atoms exhibit magnetic properties. This is due to the configuration of individual atoms. Many materials have their atoms arranged so that their magnetic fields cancel out. Only a fraction of materials have the atoms

Arranged so that their magnetic fields mutually reinforce. This is why magnetic materials are so rare. The comparison of particles to tiny magnets is not accurately true. It's because the magnetic fields created by particles behave rather oddly. We can demonstrate this in a simple experiment. Say we have two axes: x and y (i.e. if we were to measure its spin in the direction of the x-axis, we would get 1). But what happens if we try to measure its spin (magnetic field) on the y-axis? If we took a classical magnet, pointed it in the direction x, and

Conducted the same experiment, we would measure no magnetic field pointing in the y-direction, of course (since the x-axis is perpendicular to the y-axis). However, particles behave in a completely different way. If we measure the spin of a particle in the y-direction, half the time we get to spin 1, the other half of the time we get -1. Even if we try to measure the spin in different directions, we always get either 1 or -1. However, the average of the values we get is always equal to the value we would expect to get with classical magnets. We can demonstrate this rule on our particle. The average of the values we got (half the time 1, half the time -1) is equal to zero, which is the value we would get with a normal magnet?

IV. SPIN CALCULATIONS AT QUANTUM SCALE

In Quantum mechanics, we deal with the calculations different from the problems we see in classical electron spin models. The mass origin and the radius of the electron are long-standing problems in classical and modern physics. If the electron's mass is totally due to electromagnetic origin, then the classical radius of the electron will be the order of 10-15 m. However, if we wish to get the spin value based on that hypothesis, the electron surface's rotational speed would be more than 100 c. [4], which seems unrealistic and unreasonable. The spin of an electron owes a relativistic effect as it is derived from the Dirac equation. Often electrons are regarded as the point particles in quantum field theory. However, if this has to be true, certain rotational effects would produce spin in an electron. It explicitly shows that the generated angular momentum and magnetic moment of an electron would always obey the classical relations. Therefore, the electron cannot be a point particle if it has to follow the quantum relations. And should have varying mass and charge density. At present, several proposed electron spin models suggest the values of angular momentum and magnetic moment to be correct. Still, only the handful of models

Based on external motion equations of the electrons could only give the valid value of angular momentum and magnetic motion. As of now, there's no unified model that can both elucidate the value of angular momentum and magnetic moment and the mass origin. Some of the proposed models have failed to give the correct value of angular momentum and magnetic moment. They could not even explain the forces that bind the electric charges together in the electron.

To describe the electron spin motion at the quantum mechanics level, we must get a spin wave function, just as the orbital wave function for the motion of the electron around the nucleus.[] To calculate the spin angular momentum and magnetic moment, we first make some assumptions for the electron structure. Enlightenment can be achieved from the mass distribution of quarks. Particle physics tells us that the mass of up/down current quark (bare quark) is only several Mev, while the quark's constituent mass is more than 300Mev. so most of the quark mass comes from the effective masses of the sea quarks (virtual quark-antiquark pairs) and the virtual gluons. Similarly, we think that the electron is composed of point electric charge at the center, virtual electron-positron pairs inside the electron and virtual photons both inside and outside the electron. [5]

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