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RESEARCH ARTICLE OPEN ACCESS

RP-161: Solving Some Special Standard Cubic Congruence Modulo an Odd Prime Multiplied by Eight

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ABSTRACT

In this paper, some special types of standard cubic congruence of composite modulus modulo an odd prime multiplied by eight is considered for finding their solutions. The author has provided special solutions to them.The formulation of solutions are established, tested and verified. The author selected the speciality of the positive integers appearing in the congruence. Formulation of the congruence increased the interest of the readers and the students towards the study of the congruence. Thus, it can be said that the formulation of solutions is the merit of the paper.

KEY-WORDS

Cubic Congruence, Cubic Residues, Incongruent Solutions, Odd Primes.

INTRODUCTION

A standard cubic congruence of composite modulus is a congruence of the type:

$x^3 \equiv a \ (mod\ m)$, a and m are composite positive integers.

For the solvability of the congruence, a must be cubic residue of m. In this paper, the author imposes some speciality to$a$, considering$m = 8p, p$ being an odd prime, the author has taken: $a = p, 3p, 5p, 7p$.

These generates four types of standard cubic congruence of composite modulus.

All of them have exactly one solution of each.

PROBLEM STAYEMENT

Here the problem is "To formulate the solutions of the congruence:

(1) $x^3 \equiv p \ (mod\ 8p), p\ odd\ prime$;
(2) $x^3 \equiv 3p \ (mod\ 8p), p\ odd\ prime$;
(3) $x^3 \equiv 5p \ (mod\ 8p), p\ odd\ prime$;
(4) $x^3 \equiv 7p \ (mod\ 8p), p\ odd\ prime$.

LITERATURE REVIEW

Cubic congruence is a part of Elementary Number Theory. Most of the book of Number theory discusses on quadratic congruence of prime and composite modulus. Now the aim of the author is to find the available literature of the problem framed above.Burton [1], Zukerman [2], Koshy [3], remained silent about the problem of this paper. Only Koshy had discussed a problem of the type:$x^2 \equiv a\ (mod\ 8p^2). Page - 542, example - 11.33.$

There he used Chinese Remainder Theorem [2] to find all the solutions.Also, in the book of Burton [1], page-18, line-23, proved that the square of any odd integer is of the form $8k + 1$ .Nothing more relevant to the said problem is found in the literature of mathematics. But only the authors published papers related to the problem are found [4], [5], [6].

ANALYSIS & RESULTS

Consider the congruence: $x^3 \equiv a\ (mod\ m)$.

Case-I: Let $a = p\ , m = 8p$.

Then congruence reduces to: $x^3 \equiv p\ (mod\ 8p)$.

Now, $p^3 - p = p(p^2 - 1) = p.8t \equiv 0\ (mod\ 8p)$ as $p^2 \equiv 1\ (mod\ 8)$.

Therefore, $p^3 \equiv p\ (mod\ 8p)$.

This showed that $x \equiv p(mod\ 8p)$ satisfied the congruence:$x^3 \equiv p\ (mod\ 8p)$.

Hence, $x \equiv p(mod\ 8p)$ is the solution of the said congruence.

Case-II: Let $a = 3p\ , m = 8p$.

Then congruence reduces to: $x^3 \equiv 3p\ (mod\ 8p)$.

So, $27p^3 - 3p \equiv 3p^3 - 3p = 3p(p^2 - 1) = 3p.8t \equiv 0\ (mod\ 8p)$.

Therefore, $27p^3 \equiv 3p\ (mod\ 8p)$ and hence $x \equiv 3p(mod\ 8p)$ is the solution of the congruence: $x^3 \equiv 3p\ (mod\ 8p)$.

Case-III: Let $a = 5p\ , m = 8p$.

Then the congruence reduces to: $x^3 \equiv 5p\ (mod\ 8p)$.

So, $125p^3 - 5p \equiv 5p^3 - 5p = 5p(p^2 - 1) = 5p.8t \equiv 0\ (mod\ 8p)$

Therefore, $125p^3 \equiv 5p\ (mod\ 8p)$ and hence $x \equiv 5p(mod\ 8p)$ is the solution of the congruence: $x^3 \equiv 5p\ (mod\ 8p)$.

Case-IV: Let $a = 7p\ , m = 8p$.

Then congruence reduces to: $x^3 \equiv 7p\ (mod\ 8p)$.

So, $343p^3 - 7p \equiv 7p^3 - 7p = 7p(p^2 - 1) = 7p.8t \equiv 0\ (mod\ 8p)$

Therefore, $343p^3 \equiv 7p \ (mod\ 8p)$ and hence $x \equiv 7p(mod\ 8p)$ is the solution of the congruence: $x^3 \equiv 7p \ (mod\ 8p)$.

ILLUSTRATIONS

Example-1: Consider the congruence $x^3 \equiv 17 \ (mod\ 136)$

It can be written as $x^3 \equiv 17 \ (mod\ 8.17)$

It is of the type $x^3 \equiv p \ (mod\ 8p)\ with\ p = 17, an\ odd\ prime.$

It has exactly one solution given by $x \equiv p \ (mod\ 8p)$

$$\equiv 17(mod\ 136).$$

Example-2: Consider the congruence $x^3 \equiv 51 \ (mod\ 136)$

It can be written as $x^3 \equiv 3.17 \ (mod\ 8.17)$

T is of the type $x^3 \equiv 3p \ (mod\ 8p)\ with\ p = 17$.

It has exactly one solution given by $x \equiv \ 3p \ (mod\ 8p)$

$$\equiv \ 51 \ (mod\ 136).$$

Example-3: Consider the congruence $x^3 \equiv 85 \ (mod\ 136)$

It can be written as $x^3 \equiv 5.17(mod\ 8.17)$

T is of the type $x^3 \equiv 5p \ (mod\ 8p)\ with\ p = 17.$

It has exactly one solution given by $x \equiv \ 5p \ (mod\ 8p)$

$$\equiv \ 85(mod\ 136).$$

Example-4: Consider the congruence $x^3 \equiv 119 \ (mod\ 136)$

It can be written as $x^3 \equiv 7.17(mod\ 8.17)$

T is of the type $x^3 \equiv 7p \ (mod\ 8p)\ with\ p = 17.$

It has exactly one solution given by $x \equiv \ 7p \ (mod\ 8p)$

$$\equiv \ 119 \ (mod\ 136).$$

CONCLUSION

Therefore, it can be concluded that the special standard cubic congruence modulo an odd prime multiplied by eight is correctly formulated. It has exactly one incongruent solution in each case. The solutions are given by:

$x^3 \equiv p \ (mod\ 8p)$has a unique solution: $x \equiv$ p (mod 8p);

$x^3 \equiv 3p \ (mod\ 8p)$has a unique solution$x \equiv \ 3p \ (mod\ 8p)$;

$x^3 \equiv 5p \ (mod\ 8p)$has a unique solution $x \equiv\ 5p\ (mod\ 8p)$;

$x^3 \equiv 7p \ (mod\ 8p)$has a unique solution $x \equiv\ 7p\ (mod\ 8p)$.

The formulations are elaborated using numerical examples.

MERIT OF THE PAPER

The problemsof the said cubic congruence is formulated for solutions. Formulation enabled the finding of the solutions orally. This is the merit of the paper.

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