

Energy Efficiency of Model Predictive Control Over proportional-Integral-Plus Towards Control of Air Conditioner

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ABSTRACT

The rate at which air conditioners are used all over the world is increasing, hence, increasing the amount of research to enhance the energy efficiency of the air conditioner. This article studies the energy efficiency of model predictive control (MPC) and compares it to the energy efficiency of the proportional-integral-plus (PIP) to keep the room at the required temperature without wasting energy by expending the least amount of energy possible to control the air conditioner and keep the room at the required temperature. Hence, the comparison is based on energy consumption, minimization and efficiently controlling the air conditioner to keep the temperature of the closed edifice at the required level. The statistical methods used in this analysis are the measurement of the system efficiency through Integration of the Square Error in a given time ($\int S_e$) which analyses how the control system follows the set point, the Integrated Absolute Control ($\int \bar{A}_c$) which determines energy expended by the control effort, the Variance of input ($V\bar{A}_i$) which determines the level of fluctuations in the control system and Energy Efficiency (\bar{E}_e) which determines the amount of energy saved by the control system.

Keywords: Control, Predictive, Proportional, Integral, Energy, Efficiency, Variance, Square, Error, Model, Absolute

INTRODUCTION

Man's need to live in an environment with the required temperature and ventilation, encouraged this research into conditions which create discomfort to enable the efficient control of these conditions. The research indicated that effective and efficient control is only possible in a closed environment because of inconsistencies in the weather condition. The regulation of temperature and ventilation of this closed environment or building motivated the development of the air conditioner. In recent times, workplaces are usually provided with air conditioners which regulate the temperature of the workplace and provides a conducive working temperature and ventilation. A mixture of natural ventilation with the aid of fan cooling, can add to the air conditioner to offer great minimization in the energy used up in the control of the air conditioner in appropriate climates, since fans consume relatively lesser energy

when compared to the air conditioner. The use of the fan-assisted cooling minimizes the adverse effect of energy consumption on the climate, by reducing greenhouse which is caused by the flaring of CO₂. [2]. dehumidifying, deodorizing, cleaning, heating, cooling, and humidifying the air in recent times are done by the air conditioner. In order to control a system, the uncertainties, non-linearity and constraints in the system have to be accounted for which influences the closed loop control of the system [2]. A lot of air conditioners work by mixing two air streams, which is moved by a fan to an air handling unit. A flow of water through the air handling unit is used to cool the air in the air handling unit and supplied to the closed edifice (building). The mixing is done by combining the two air streams. The mixing operation is presumed to be adiabatic (occurring without loss or gain of heat) if the difference in temperature between the surrounding and the air that is

conditioned is negligible. Thus, changes in kinetic energy and potential energy are very small, which implies that the mixing operation has no work interactions. The air handler that regulates the air before it is mixing it with natural air, acts as a heat exchanger, which transfers the heat from the air flow to the water flowing through the handler. A difference equation can be used to depict the characteristics of the heat exchanger [3]. Which implies that the air conditioning system perplexes the working principle and design process for lump systems [4]. Another obstacle to overcome in real time, is the constraint as a result of the exothermic reactions from the number of humans using the building and also heat from heat producing devices and equipment, the average weather condition, the size of the building and the materials used in the construction of the building. [5]. A building specially designed to keep the ambient constant is a high-performance building and it is better than a conventional building because, by keeping the regulated temperature and ventilation constant at the desired level, energy is saved. Hence, reducing the energy wasted by the air conditioner to frequently regulate the temperature and ventilation of the closed environment. Therefore, high-performance building is of greater economic value than the conventional building since, it contributes greatly to the cost minimization by reducing the energy consumed by the air conditioner. High-performance buildings keep the regulated temperature constant by minimizing heat lost or gained from the surrounding based on the material used to construct the building and the design of the building. This reduces the energy wasted by the air conditioner in maintaining the temperature of the building at the required level [6]. The control model for air conditioners usually applies estimation technique. Since high-order estimation that will produce the desired result for control design is not practically executable, low-order estimation is applied. But estimated modeling brings about a bad or unsuitable match amongst the estimated model and the real model, this brings about non-linearities in the system. Doubts in the system can also be brought about by changes in the operational conditions as a result of heat produced by the occupants and equipment used in the building which causes dynamic changes in the system. The dynamics of the system can also be influenced by the air and water flow temperature [7].

LITERATURE REVIEW

Simulation and modeling of systems are used by various engineers to solve problems on design and execution of various system models. Hence, the air conditioner can also be modelled by simulating the control of the air condition based on the regulation of the temperature and ventilation of the closed building. These practical methods and also the analysis and

modeling and control of temperature and ventilation of the air conditioner gives a vital practical implementation that helps the engineer to perform experiment on any system by modeling their operation using a computer system. Modeling and simulation also motivated the use of a lot of methods to provide optimal control of the temperature and ventilation of the closed building by the air conditioner, making the ambient conducive for the occupants. A lot of engineers have done research on optimizing the air conditioner for energy efficiency and also the possibility of using lower quality of energy supply. High-performance building, enhanced design, efficient control system, energy consumption reduction and the use of an energy supply that is inexhaustible are vital aspects considered when designing and construction an air conditioner [8].

1) TEMPERATURE and VENTILATION CONTROL MODEL in THE SIMULATION of an AIR CONDITIONER

The importance of modeling and simulation for the optimization of the air conditioner control system in terms of energy efficiency and effective control of the temperature and ventilation of a closed building have encouraged engineers to increase research on the optimization of the air conditioning system. The control model design and simulation of the air conditioner to control the temperature and ventilation of a closed room are classified with respect to the solutions they provide to solve existing problems. Thus, they are classified into optimization of control system, energy efficiency control systems, and analysis and choosing of equipment and design tools for system optimization, since all tools are confined to a certain predefined functionality. Thus, there is motivation for improvement, since tools used for modeling and simulation of systems may not efficaciously handle all factors which act as a constraint and also deal with potential system design and construction intricacies. Hence, choice of the tools for simulation as well as modeling and simulation proficiency is important when designing and modeling air conditioner for the efficient control of temperature and ventilation. Methods applied in solving problems of temperature and ventilation of a simulated air conditioner model are categorized into concurrent modular resolution, modular resolution that is not dependent, and resolution with the aid of manipulated equation [9]. Designers and users' concepts of the air conditioner are perplexed by the degree of intricacies as a result of the growing need for them to understand the various aspects of identifying the systems parameter and the constraints involved in the control of temperature and ventilation of the air conditioner. The growing number of constraints needed to examine the

parameters of a model during design and development, makes it difficult to model and simulate a system. The designer has to integrate the interaction between the parts that make up the temperature and ventilation control air conditioner with the closed edifice. The model design and simulation become more intricate, since the designer has more constraints to deal with because of the performance of the building with respect to the heat generated by occupants and equipment and its ability to maintain the required temperature and ventilation level in spite of changing weather conditions also has to be considered in order to design and simulate an efficient model of the air conditioner [10]. Improvement on the design model, control efficiency and simulation of the air conditioner is currently happening and new models and methods are being introduced by engineers. Recently, a lot of new research methods have been used to design and model temperature and ventilation control air conditioners for optimization. The simulation mainly applied in the area of inexhaustible energy engineering and controlled environment (edifice) simulation of various solar design (TRNSYS) used modeling of air-cooled and water-cooled to study their various aspects of operation and result using different control methods [11] and [12]. The air conditioner also applied a cooling system that uses towers which is usually applied to regulate temperature and ventilation of buildings by expelling heat from buildings. The cooling tower expels heat by transferring mass of heat within ambient air and hot water droplets. Cooling towers are affordable and utilizes minimal amount of energy with respect to the total energy consumed by the system, but the energy utilized by other subsystems within the system is affected by their performance. Thus, improving the operation of the cooling tower will enhance its performance and also affect various subsystems within the system. [13], the effect of the cooling tower on the subsystems within the system have motivated studies in these areas. Austin proposed the application of the regressive methods to model parts of the systems and also in monitoring and improving the operation of the air conditioner [14]. The study on improving the operation of the cooling tower is still in its early stage and does not have accurate and effective model that is necessary for predicting and analyzing the energy efficiency.

II) EFFICIENT CONTROL OF HUMIDITY

Actualizing efficient control of humidity particularly in tropical regions is one of the factors militating against the development of temperature and ventilation control air conditioner. The control of humidity is usually accomplished by the efficient control of the cooling unit. The development of their air conditioner has to include humidity regulation to prevent de-humidification of the edifice, hence, inconveniencing the

occupants of the building and wasting the energy consumed by the air conditioner during the operation. Most air conditioners usually cool the air within the building while dehumidifying the air with twenty to thirty percent range of latent capability of the entire cooling capability [16]. Interaction between the air conditioner and humidity condition outside the building with the aid of percolation and ventilating system, and also the moisture produced within the building by the people using the building and their actions involving water is the biggest obstacle militating against the effective control of the humidity within a building. The amount of sweat produced by the people using the building which in turn produces moisture is based on their activeness, hence the latent load of an individual is within 105 Btu/hr and 250 Btu/hr [15]. Due to changes in pressure in the ambient around the building, outside air penetrates the building from cranny and influences the humidity. In order to reduce the inflow of outside air mercantile buildings require to function in positive pressure with respect to the air outside. The humidity of a building is also influenced by the ventilation of the outside air, since some outside air is needed based on the quality of air in the building and the design and size of the building. The occupant of the building experience some degree of inconvenience if outside air is allowed to enter the building when the temperature outside is warmer than the required temperature inside the building as a result of decrease in humidity. Adequate humidification is significant but recent research and design of air conditioner emphasize less humidification. Some air conditioner developer focuses on enlarging the cooling unit which reduces humidification as a result of excess evaporation [17].

III) SYSTEMS THAT DE-HUMIDIFY AND RECOVER HEAT

Air to air heat exchangers is used to reduce the energy expended in cooling the building by bringing the outside air and the air inside the building into thermal contact, hence, transferring heat from the inside air to the outside air, which cools the inside air. This helps to reduce the energy expended to de-humidify the room, which is about one third of the total energy consumed by the air conditioner, since no moisture is wasted as a result of evaporation [15]. Conventional models used to dehumidify a room applied coils for evaporation which uses water-vapor condenser which has a demerit of producing a fungi habitat at the drain as a result of the moisture produced by the condenser. Wheels of enthalpy can be used to reduce the total heat content of a building by exchanging heat between outdoor air and expelled air with the aid of water flowing through them, which helps to de-humidify the building [18].

IV) DEDICATED SYSTEMS THAT RECOVER HEAT

This is a system that saves energy expended in removing heat by collecting and reprocessing expelled heat within the systems. The cooling unit in the air conditioner is responsible for expelling heat from the building when the temperature rises beyond the desired temperature.

METHODOLOGY

The system performance optimization and energy efficiency measured through Integration of the Square Error in a given time ($\int S_e$), the Integration of the Absolute Control ($\int \bar{A}_c$), the inconsistency in the system or Variance of Input ($V\bar{A}_i$) and Energy Efficiency (\check{E}_e). These statistical measures are applied to ascertain the optimization of the controllers applied to control the air conditioner for each control system model.

I) SYSTEM MODELING

Fig. 1 depicts the air conditioner plant setup

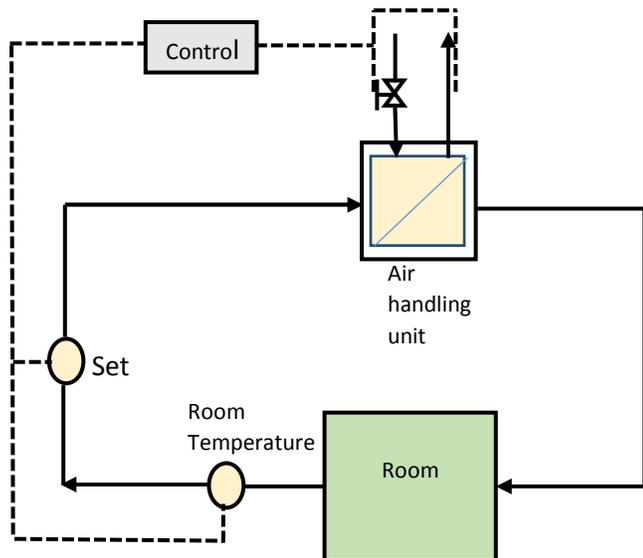


Fig.1 Schematic diagram of the Air Conditioner room

Air conditioners are used to control the humidity and temperature of a closed room. The air conditioner removes the heat from the air in the room and returns it to the room after it has undergone a cooling process. The aim of the control model is to regulate the temperature, humidity and ventilation of the room within the desired temperature while minimizing the energy expended and the control effort utilized. This is very significant because an inefficient control model wastes a lot of

energy and can damage control component as a result of high control effort.

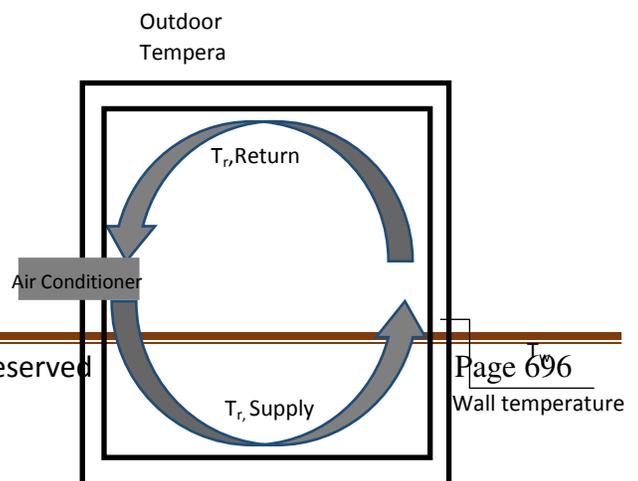
In an effort to increase the control efficiency, proportional-integral-plus (PIP) and the model predictive control (MPC) independently used to control an air conditioner for performance optimization and their results compared.

II) ROOM MODEL

The room is presumed to be an accurately mixed room, the diagram of the system based on air flow in and out of the room is shown in Fig. 2. The energy balance equations of the heat capacity of the room with respect to the temperature of the air within the room, the temperature of the walls enclosing the room and the temperature of the ambient outdoor air is expressed as [20].

$$Q_r = \frac{m_a c_a \partial T_{ra}(t)}{\partial t} - \left[\frac{m_w c_w \partial T_w(t)}{\partial t} - \frac{m_a c_a \partial T_{oa}(t)}{\partial t} \right] \tag{1}$$

Where Q_r is the heat capacity of the room, m_{ra} , m_w and m_{oa} are the mass of the air in the room, the mass of the walls enclosing the room and the mass of the ambient air surrounding the room respectively. The specific heat capacity of both the air in the room and the outdoor air is c_a and that of the wall is c_w . Also, the change in temperature of the room air, the outdoor air and the wall with time are respectively $\partial T_{ra}(t)$, $\partial T_{oa}(t)$ and $\partial T_w(t)$. The temperature of the room is influenced by the nature of the walls which determines the rate in which heat is transferred between the walls and the outside air. That is why high-performance buildings are very important because they are made of materials that are very poor conductors of heat and reduces the heat exchange between the walls and the outdoor air to the barest minimum.



squared error, the more the deviation of the control dynamics from the set point and the more control effort is utilized.

II) INTEGRAL of ABSOLUTE CONTROL (I_AC)

This is a measure of the absolute value of the controlled output and indicates the degree of control effort utilized by the control model. High values of Integral of Absolute control indicated that a lot of control effort was expended to control the system, hence expending more energy and exerting more pressure on the control component. It is expressed as

$$I_{A}C = \sum \int_i^n |u_i|(t)dt \tag{3}$$

III) VARIANCE of INPUT SIGNAL (V_Ai)

This measures the pressure on the various parts which make up the control system. It measures the deviations of the control dynamics from the set point and high values of variance of input signal indicates that there is a lot of fluctuations of the control dynamics from the set point, increasing the pressure on the various control parts of the control system. Hence the Variance of the input signal should be kept as small as possible. The Variance of Input Signal is expressed as

$$V_{A}i = \sum_i^n (u_{i+1} - u_i)^2 \tag{4}$$

Where u_{i+1} represents the current output of the system and u_i the previous output of the system. The number of discrete time sample is represented by n and i represents the number of data sampled at each discrete time. The lower the Variance of Absolute Control, the more consistent is the dynamics of the system to the set point.

IV) ENERGY EFFICIENCY (Ė_e)

Energy efficiency (EE) is a measure of the amount of energy utilized by the control system and is given by the expression

$$\check{E}e = \sum_i^n |Q_r| \tag{5}$$

Where Q_r is the heat energy equivalent of the energy used up by the air conditioner which is the heat transferred from the room air with consideration of the wall heat and outdoor air heat to the air conditioner, which is returned to the room after cooling (after the heat transfer) and is expressed as $Q_r = \frac{m_a c_a \partial T_{ra}(t)}{\partial t} - \left[\frac{m_w c_w \partial T_w(t)}{\partial t} - \frac{m_a c_a \partial T_{oa}(t)}{\partial t} \right]$. The smaller the value of the Energy Efficiency, the lesser the energy utilized by the air conditioner.

Fig. 2 Schematic diagram of the room temperature model

III) OPERATION OF THE AIR CONDITIONER

The air-conditioner apply chemicals that changes the state of gas to liquid and vice versa in a process that expels heat to the outside air. The air conditioner consists of the evaporator, condenser and compressor. It also consists of a gaseous fluid which flows to the compressor at low pressure. The compressor compresses this fluid, increasing the temperature and thus, the heat energy in the fluid. This high temperature fluid leaves the compressor at a high pressure and flows to the condenser where it is cooled and converted to liquid. This liquid fluid then flows to the evaporator which evaporates it to a low-pressure cool air and it is blown back into the room with the aid of a fan through ducts. This process continues until the heat sensor senses that the temperature of the room has reached the required temperature. The process starts all over again if the heat sensor senses that the temperature has risen beyond the required temperature. The work of the control model is to effectively control this process for optimization and energy efficiency.

STATISTICAL ANALYSIS OF PERFORMANCE

The following statistical measures were used to ascertain the efficiency of the control models used.

I) INTEGRAL of the SQUARED ERROR (I_Se)

The integration of the square error is a measure of system performance formed by integrating the square of the system error over a fixed interval of time; this performance measures the deviation of the control dynamics from the set point or true value which determines the optimization of the control model. The Integration of the Square Error is expressed as

$$I_{S}e = \sum \int_i^n e^2(t)dt \tag{2}$$

Where e is the error in the output and it is the difference between the expected output and the actual output, n is the number of discrete time samples taken and i is the amount of data per discrete time taken. The higher the Integral of the

Where n is the number of discrete time samples and i is the amount of data sampled.

PROPORTIONAL INTEGRAL PLUS (PIP) CONTROL
 The Proportional Integral Plus (PIP) controller is a state variable feedback (SVF) controller whose control law is based on non-minimal state-space model which is given by [22].

$$u(k) = -k^T x(k)$$

where k is the (n + m + τ - 1) dimensional SVF control gain vector expressed as

$$k = [f_0 \quad f_1 \quad \dots \quad f_{n-1} \quad g_1 \quad g_2 \quad \dots \quad g_{m+\tau}]$$

The PIP controller uses pole placement and the required closed loop polynomial is expressed as

$$D(z^{-1}) = 1 + d_1 z^{-1} + \dots + d_{n+m+\tau-1} z^{-(n+m)} \tag{12}$$

Based on pole placement, $d_1, d_2, \dots, d_{n+m+\tau-1}$ are chosen to ascertain that the closed loop poles are placed at the complex z plane to give a decent operation.

The PIP is an enhancement of the PI (Proportional Integral), this is done by the addition of a higher order forward path ($1/G(z^{-1})$) and feedback compensators ($F_1(z^{-1})$). They are expressed as

$$F_1(z^{-1}) = f_1 z^{-1} + f_2 z^{-2} + \dots + f_{n-1} z^{-(n-1)} \tag{9}$$

$$G(z^{-1}) = 1 + g_1 z^{-1} + \dots + g_{m+\tau-2} z^{-(m+\tau)} \tag{10}$$

Hence, $F(z^{-1}) = f_0 + F_1(z^{-1})$ and the PIP transfer function for a closed loop is expressed generally as:

$$y(k) = \frac{k_1 B(z^{-1}) z^{-\tau+1}}{\nabla [G(z^{-1}) A(z^{-1}) + F(z^{-1}) B(z^{-1}) z^{-\tau+1}] + k_1 B(z^{-1}) z^{-\tau+1}} \tag{11}$$

Where k_1 is the integral of error gain and $\nabla = 1 - z^{-1}$ the difference operator

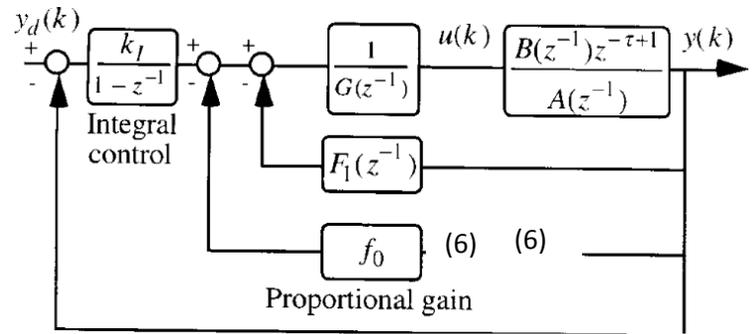


Figure 3 PIP control with feedback structure
 Figure 3 shows a PIP control with a feedback structure which checks the error and try to adjust the proportional gain for the error.

1) PIP CONTROL DESIGN

The expression of the transfer function in a discretized system is [19].

$$G(z) = \frac{Q(z)}{P(z)} \tag{8}$$

where Q(z) and P(z) are expressed as

$$Q(z) = q_0 + q_1 z^{-1} + \dots + q_r z^{-r} \tag{13}$$

$$P(z) = 1 + p_1 z^{-1} + \dots + p_e z^{-e} \tag{14}$$

The single-input single-output for a discrete time transfer function is expressed as [21].

$$y_k = \frac{B(z)}{A(z)} u_k \tag{15}$$

where B(z) and A(z) are expressed as

$$B(z) = b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-q} \tag{16}$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_p z^{-p} \tag{17}$$

And the backward shift operator, z^{-1} is expressed as

$$z^{-1} x_k = x_{k-1} \tag{18}$$

This gives rise to second order transfer function. The second order discrete system transfer function is shown in Figure 4

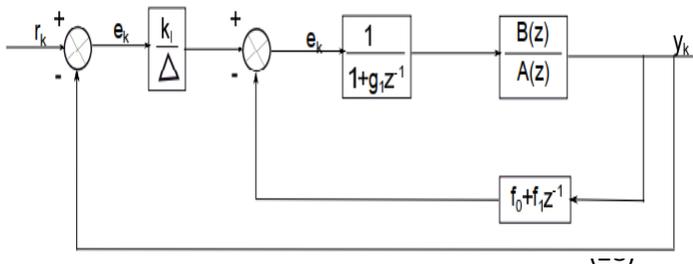


Figure 4 Schematic diagram of the controlled system

(29)

$$\frac{y_k}{u_k} = \frac{b_1 z^{-1} + \dots + b_q z^{-q}}{1 + a_1 z^{-1} + \dots + a_p z^{-p}}$$

the difference expression of (19) yields

$$y_k = -a_1 y_{k-1} - \dots - a_p y_{k-p} + b_1 u_{k-1} + \dots + b_q u_{k-q}$$

shifting the output of the system by one step gives

$$y_{k+1} = -a_1 y_k - \dots - a_p y_{k-p+1} + b_1 u_k + \dots + b_q u_{k-q+1}$$

making y_k the subject of the formula in (19) gives

$$y_{k+1} + a_1 y_k + \dots + a_p y_{k-p+1} = [b_1 + \dots + b_q z^{-q}] u_k$$

based on control law of the state variable feedback the input of the system is given as

$$u_k = -k^T x_k \quad (31) \quad (23)$$

Where k is the control gain vector and x_k is the vector of the states of the system given by

$$x_k = [y_k \quad y_{k-1} \dots y_{k-p+1} \quad u_{k-1} \quad u_{k-2} \dots u_{k-q+1}]^T \quad (24)$$

where z_k is the integral of error given as

(32)

$$z_k = z_{k-1} + [r_k - y_k]$$

$$z_k - z_{k-1} = r_k - y_k$$

$$\Delta z_k = r_k - y_k$$

where $\Delta z_k = 1 - z^{-1}$

$$z_k = \frac{r_k - y_k}{\Delta}$$

the gain vector, k^T is given (19)

(19)

$$k^T = [f_0 \quad f_1 \dots f_{p-1} \quad g_1 \dots g_{q-1} \quad -k_I]$$

(20)

(20)

the integral gain is represented by k_I . Putting (24) and (29) in (22) yields

(21)

$$u_k = - \begin{bmatrix} f_0 & f_1 \dots f_{p-1} & g_1 \dots g_{q-1} & -k_I \end{bmatrix} \begin{bmatrix} y_k \\ y_{k-1} \\ \dots \\ y_{k-p+1} \\ u_{k-1} \\ \dots \\ u_{k-q+1} \\ z_k \end{bmatrix} \quad (22)$$

Multiplying the matrix and putting Z_k as in (28) we have:

$$u_k = -f_0 y_k - f_1 y_{k-1} - \dots - f_{p-1} y_{k-p+1} - g_1 u_{k-1} - \dots - g_{q-1} u_{k-q+1} + k_I \frac{(r_k - y_k)}{\Delta}$$

Putting (30) in (32) we have:

$$u_k [1 + g_1 z^{-1} + \dots + g_{q-1} z^{-(q-1)}] = -f_0 y_k - f_1 y_{k-1} - \dots - f_{p-1} y_{k-p+1} + k_I \frac{(r_k - y_k)}{1 - z^{-1}} \quad (33)$$

expressing in terms of u_k and multiplying by $1 - z^{-1}$ we have:

$$u_k = \frac{-f_0 y_k - f_1 y_{k-1} z^{-1} - \dots - f_{p-1} y_{k-p+1} z^{-(p-1)} + f_1 y_{k-1} z^{-2} + \dots + f_{p-1} y_{k-p+1} z^{-(p+1)} + k_I}{[1 + g_1 z^{-1} + \dots + g_{q-1} z^{-(q-1)}]} \quad (34)$$

Putting (33) in place of u_k in (22) gives the expression:

$$\frac{y_{k+1} + a_1 y_k + a_p y_{k-p+1} = [b_1 + \dots + b_q z^{-q+1}] [-f_0 y_k - f_1 y_k z^{-1} - \dots - f_{p-1} y_{k-p+1} + f_0 y_k z^1 + f_1 y_k z^2 + \dots + f_{p-1} y_k z^{-p+1} + k_I r_k - k_I]}{1 + g_1 z^1 + \dots + g_{q-1} z^{-q+1} - z^1 - g_1 z^2 - \dots - g_{q-1} z^{-q+1}}$$

When we multiply (33) by

$$1 + g_1 z^1 + \dots + g_{q-1} z^{-q+1} - z^1 - g_1 z^2 - \dots$$

and write the expression in terms of $\frac{y_k}{r_k}$ we have:

$$\frac{y_k}{r_k} = \frac{b_1 k_I + \dots + b_q k_I z^{-q+1}}{X_E} \tag{40}$$

The characteristic equation, X_E is expressed as

$$X_E = 1 + (a_1 - 1 + g_1 + b_1 f_0 + b_1 k_I) z^{-1} + (a_2 - a_1 + a_1 g_1 - g_1 + g_2 - b_2 f_0 + b_1 f_0 - b_1 f_1 - b_2 k_I) z^{-2} + \dots$$

The closed loop characteristic equation is a polynomial expressed as

$$D(z) = 1 + d_1 z^{-1} + d_2 z^{-2} + \dots + d_n$$

comparing (34) and (35) yields

$$\begin{aligned} z^0 : & 1 = 1 \\ z^{-1} : & a_1 - 1 + g_1 + b_1 f_0 + b_1 k_I = d_1 \\ z^{-2} : & a_2 - a_1 + a_1 g_1 - g_1 + g_2 - b_2 f_0 + b_1 f_0 - b_1 f_1 - b_2 k_I = d_2 \end{aligned}$$

rewriting in matrix form

$$\begin{bmatrix} b_1 & 0 & 1 & 0 & \dots & 0 & b_1 \\ b_2 - b_1 & b_1 & a_1 - 1 & 1 & \dots & 0 & b_2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ b_q - b_{q-1} & b_{q-1} - b_{q-2} & a_p - a_{p-1} & a_{p-1} - a_{p-2} & \dots & 1 & b_q \\ -b_q & b_q - b_{q-1} & -a_p & a_p - a_{p-1} & \dots & a_1 - 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & -a_p & 0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ g_1 \\ g_2 \\ \vdots \\ k_I \end{bmatrix}$$

$$\left[\begin{array}{c} d_1 - a_1 + 1 \\ d_2 - a_2 + a_1 \\ \vdots \\ d_n - a_q + a_{q-1} \\ d_{n+1} - a_{q+1} + a_q \\ \vdots \\ d_{n+1+m} - a_{q+1+m} + a_{q+m} \end{array} \right] \tag{35}$$

$$\left[\begin{array}{c} d_2 - a_2 + a_1 \\ \vdots \\ d_n - a_q + a_{q-1} \end{array} \right] \tag{36}$$

$$\left[\begin{array}{c} d_{n+1} - a_{q+1} + a_q \\ \vdots \\ d_{n+1+m} - a_{q+1+m} + a_{q+m} \end{array} \right] \tag{37}$$

$$\left[\begin{array}{c} d_{n+1+m} - a_{q+1+m} + a_{q+m} \end{array} \right] \tag{38}$$

MPC

MODEL PREDICTIVE CONTROL with NON-MINIMAL STATE SPACE MODEL (NMSS-MPC)

This is a Model Predictive Control that applies the non-minimal state space model that allows the inbuilt managing of constraints. This makes it easier to model the control system, since the inbuilt constraint management eliminates the use of an observer. Constraints usually makes a system to be non-linear, hence effectively managing constraints is essential [22].

1) MODEL DESIGN

Using a system with p number of inputs and q number of outputs, the system discrete model is expressed as

$$G(z^{-1}) = \frac{b_q z^{-q} + b_{q+1} z^{-q-1} + \dots + b_{q+n_b-1} z^{-q-n_b+1}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_p z^{-p}} \tag{41}$$

The Model Predictive Control of the non-minimal state space (NMSS) is expressed as [20].

$$y_{k+1} = -a_1 y_k - a_2 y_{k-1} - \dots - a_p y_{k-p+1} + b_q u_{k-q} + \dots + b_1 u_{k-1}$$

When we multiply the left-hand and right-hand side of the equation by Δ we have:

$$\Delta y_{k+1} = -a_1 \Delta y_k - a_2 \Delta y_{k-1} - \dots - a_p \Delta y_{k-p+1} + b_q \Delta u_{k-q+1} + \dots + b_1 \Delta u_{k-1} \tag{43}$$

$$X_{m,k} = [\Delta y_k \quad \Delta y_{k-1} \quad \Delta y_{k-p+1} \quad \Delta u_{k-q} \quad \dots \quad \Delta u_{k-1}] \tag{44}$$

Hence, the state model is expressed as:

$$X_{m,k+1} = A_m X_{m,k} + B_m \Delta u_{k+1} = C_m \Delta y_{k+1} \tag{45}$$

Which is reduced to

$$X_k = [X_{m,k}^T \quad y_k]^T \tag{53}$$

$$\begin{bmatrix} \Delta y_{k+1} \\ \Delta y_k \\ \vdots \\ y_{k-p+1} \\ \Delta u_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-q-n_b} \\ \vdots \\ \Delta u_{k-q+1} \\ \vdots \\ \Delta u_{k-q-n_b+1} \\ \Delta u_{k-q-n_b+2} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_p & 0 & b_q & b_{q+n_b-1} \\ 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ -a_1 & -a_2 & \dots & -a_p & 0 & b_q & b_{q+n_b-1} \end{bmatrix} \begin{bmatrix} \Delta y_k \\ \Delta y_{k-1} \\ \vdots \\ \Delta y_{k-i} \\ \Delta u_{k-1} \\ \Delta u_{k-2} \\ \vdots \\ \Delta u_{k-q} \\ \vdots \\ \Delta u_{k-n_b} \\ \Delta u_{k-n_b+1} \\ y_k \end{bmatrix} + \begin{bmatrix} \Delta y_k \\ \Delta y_{k-1} \\ \vdots \\ \Delta y_{k-i} \\ \Delta u_{k-1} \\ \Delta u_{k-2} \\ \vdots \\ \Delta u_{k-q} \\ \vdots \\ \Delta u_{k-n_b} \\ \Delta u_{k-n_b+1} \\ y_k \end{bmatrix} \tag{54}$$

$$C = [C_m^T \quad 1] \tag{56}$$

The accuracy of the state is essential for accurate tracking of the set point, since the wrong state will result to inaccurate prediction of the actual output and inefficient control of the system.

The state space model is given by:

$$\begin{aligned} X_{k+1} &= AX_k + B\Delta u_k \\ y_k &= CX_k \end{aligned} \tag{57}$$

let R_s^T be a vector which consists of the set point data, hence, the cost function, J of the control system can be expressed as

$$J(N_1, N_2) = E \left(\sum_{j=N_1}^{N_2} [y(t+j) - \omega(t+j)]^2 + \sum_{j=N_1}^{N_2} \lambda(j) [\Delta u(t+j)]^2 \right) \tag{42}$$

Where $N_1, N_2, \lambda(j), \omega$ and respectively represents the minimum cost horizon, the maximum cost horizon, control-weighting pattern, the set point and time taken within the duration of the cost function. Based on the current system information, the state variable prediction is expressed as

$$\begin{aligned} x(k+1|k) &= Ax(k) + B\Delta u(k) \\ x(k+2|k) &= Ax(k+1|k) + B\Delta u(k+1) \end{aligned} \tag{52}$$

With respect to (3.12) and (3.13), the predicted variables of the output result, is expressed as:

$$y(k+1|k) = CAx(k) + CB\Delta u(k) \tag{46}$$

$$y(k+2|k) = CA^2x(k) + CAB\Delta u(k) + CB\Delta u(k+1) \tag{52}$$

$$y(k+2|k) = CA^2x(k) + CAB\Delta u(k) + CB\Delta u(k+1) \tag{53}$$

The matrix form of (52) and (53) is expressed as

$$Y = Fx(k) + G\Delta U$$

With F and G expressed as:

$$F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}; \quad G = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \dots & CA^{N_p-N_c}B \end{bmatrix} \tag{47}$$

Where N_p is the prediction horizon which is the number of samplings done in advance for tracking the output of the system, and N_c is the control horizon which is the number of samplings done in advance to control the input signal. The cost function based on the first derivative is expressed as:

$$\frac{\delta J}{\delta \Delta U} = -2G^T(\omega - Fx(k)) + 2(G^T G + \lambda I)\Delta U \tag{49}$$

The smallest condition is derived when $\frac{\delta J}{\delta \Delta U}$ moves towards zero, thus:

$$\Delta U = (G^T G + \lambda I)^{-1} G^T (\omega - Fx(k)) \tag{58}$$

COMPARISON of PIP CONTROL and MPC CONTROL

Fig. 1.4 shows the plot of the Proportional Integral Plus control of the temperature of the closed room with respect to time. And it can be observed from the plot that the control model did not perfectly track the set point or true value. This usually leads to more energy being expended and also more pressure on control components due to excessive vibration about their mean position which can lead to wear or tear of control components. Figure 1.5 shows a plot of the Model Predictive Control based on temperature control for a given amount of time. It can be observed that the MPC produced a better result of the set point tracking when compared to the PIP. Hence, the MPC is a more efficient control model, since it is more energy efficient and puts less pressure on control components when compared to the PIP.

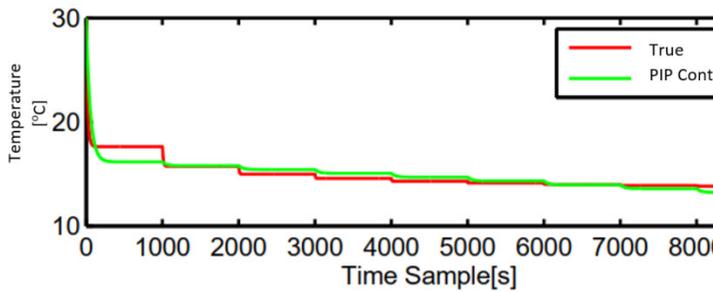


Figure 1.4 Plot of PIP Control of the Air Conditioner

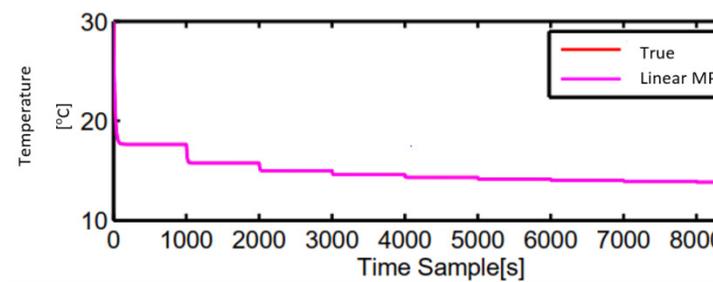


Figure 1.5 Plot of MPC Control of the Air Conditioner

Table 1.1 shows the comparison between PIP control and MPC with respect to energy efficiency, set point tracking, control effort utilized and pressure based on vibration which leads to wear or tear of control components. In order to achieve this, the Integral of the Squared Error ($\int Se$), Integral of Absolute Control ($\int \bar{A}c$), Variance of Input Signal ($V\bar{A}_i$) and Energy Efficiency (\bar{E}_e) are the statistical measures that were used to analyze the control performance.

	$\int Se$	$\int \bar{A}c$	$V\bar{A}_i$	\bar{E}_e
PIP	2.1377	6893.445	2.4412	1395031735.4755
MPC	0.040175	8668.6079	0.055133	4535062.4043

Table 1.1

CONCLUSION

PIP control was compared to MPC in the control of the air conditioner with respect to Integral of Square Error, Integral of Absolute Control, Variance of Input Signal and Energy Efficiency statistical measures. It was noticed that the PIP Control

expended more energy and put more pressure on the control components in terms of wear and tear of moving control components in comparison to the MPC. thus MPC is a better control model than the PIP control model.

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