

Use of Operations Research in Lottery, Sports Betting & Gambling in Casinos

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Abstract:

The following study article is based on a topic for which there is no substantial previous analysis. The primary goal of this research article is to highlight one such problem that has resulted from greed and ambition. This study article attempts to identify gambling, lottery, and betting on the basis of various techniques such as Bayes' Theory, Logistic Regression, and so on, via significant use of Operations Research and some analysis that resulted from the findings. The goal of this study paper is to comprehend the use of analytical approaches in casino gambling, online sports betting, and lottery systems, as well as to develop a decision-making process based on a mathematical approach. The study suggests that OperationsResearch investigates the many possibilities that exist in the fields of the aforementioned issues. The absence of data not only impedes research but also has an impact on the accuracy of projections. The increased availability of data will, in turn, enhance the accuracy and dependability of the suggested models, making them suitable for generalisation to a considerable extent.

Keywords-Operations research, Casino gambling, Game theory, Slot floor optimisation, Sports betting odds, Win probability, Lottery strategies, Combinatorics

I. INTRODUCTION

"They say that 'The house always wins' but what if it can be beaten?"

Gambling, sports and lottery betting is a very renowned industry all over the world. The pandemic led to the emergence of a new section of this industry that was not given much importance before that is the online sector. The market size of the online gambling industry and global casinos reached 227 billion U.S dollars in 2020. (Lock, May 31, 2021)The lottery and Sports Betting market is poised to grow by \$

194.14 billion during 2021-2025.(Global Lottery Market 2021-2025, March 2021). Such statistical figures have attracted a number of researchers over the years leading to a number of fascinating OR studies related to this industry.

Gambling is the staking of a valuable item, generally money, on the outcome of a game or event. Gambling is to go to a physical location, such as a casino or a sportsbook, where customers can gamble in person. (S.Lock, 2021) Lottery betting is betting on the result of a lottery draw. Sports betting refers to a wager of

any kind on a sports activity. The other methods of these three are their online formats, which allow customers to wager through websites and applications.

The gambling industry which involves casino gambling, sports betting and lotteries to name a few, is a rapidly growing industry especially the online sector which was positively impacted by the covid-19 pandemic is expected to grow the most in the coming years. Despite the legal challenges, the online betting sector is very popular in India. People find it exciting and tend to go overboard with it. Strategies based on favorite numbers or sometimes simply due to lack of knowledge can push people into this trap and they might end up losing a lot of money. There are a lot of mathematical calculations done on a large scale for example in casinos. After all, we don't just say that 'the house always wins but what if it's not true and the house can be beaten? For that to be possible, even we need to make some mathematical calculations and take decisions based on math and logic. The application of OR in this industry is not widely researched but has a lot of potential not just for the management but also for consumers. Our main focus is on the consumers and the maximization of their benefit. The application of mathematical models to decision-making is an integral part of Operation research and we aim to explore that for this industry through our research paper.

Looking at the research gaps; In a casino, there are several games available such as blackjack, roulette, slots, poker and so on. Every game differs with respect to the number of players and the playing style. The randomness of the occurrence of an event in the game also differs. Several strategies are adopted while attempting each kind of gaming operation. Research has not been made for every game and every strategy resulting in several gaps. The research available online is quite old dating back to the early 2000s having the latest piece of research. Hence, there's a lack of data available. In recent times, there are several strategies that are suggested on the web but their optimality is not tested and every conclusion includes a high dependence on choice and perception which is true to some extent.

Coming to the contribution of this research paper, here, we analyze and examine the optimality in an attempt to mathematically substantiate our choice of strategies.

Talking about the implication, currently, a few Operations Research techniques are applied with respect to these industries. Mathematical models, artificial intelligence and machine learning have been ruling over the betting markets. Game theory is a branch of applied mathematics that provides tools for analysing situations in which parties make decisions to be used in gambling and especially in poker. Games such as Roulette make use of probability theory and try to predict the odds of them winning as it is based on the randomness of numbers. Logistic regression for sports betting and statistics and combinatorics for lottery betting have been used. Development of a new strategy, examination of an existing strategy and a mathematical approach towards the decision-making process to help maximize pay-out which involves a combination of the existing strategies have been studied to increase the efficiency and optimality of the methods.

Younger generations i.e., millennials and generation Z do not consider this industry a taboo anymore like latter times, several countries are changing their policies and laws and becoming more open towards gambling, sports betting and lottery betting. Hence, we, who behold the future generations must spread information and awareness about this industry and continue analyzing it.

II. LITERATURE REVIEW

Operations Research is used in casino gambling for a variety of purposes. It can be used for consumer gratification, administration of worksite, improvement of casino traffic and product procurement and financing. Using OR concepts to analyse this data with respect to casino management may aid in better catering to their player base. Unfortunately, little research has been conducted in this sector. (Operations Research Society, 2021)

When coming to how operations research is relevant to slot operations on casino floors, several former writers have also acknowledged the importance of scientific judgment methodologies managing slot operations. (Fier,

2003) Research by (Kilby, Fox, & Lucas, 2005) talks about slot machines through various directions but provides no unique operations research model for these in a casino. (Ghaharian & A.K.Singh, 2011) highlights how a mathematical way is needed to systematically approach the slot machine area. It uses the Linear Programming Problem in a way that the probability of a gambler gaining an immense pool of wealth suddenly reduces such that the casino floor can maintain utmost revenue. Researches by (Lucas & Brewer, 2001) show effects of temporal, operational, and marketing variables hypothesised to affect slot handle were investigated using a regression model. With certified secondary data and variables derived from previous research, a simultaneous regression analysis was performed. However, it could be ascertained that the model failed to create a substantial effect for food variables.

Looking at online gambling, ranging from virtual casino table games to poker, bingos, lotteries, sports wagering, and horse betting, has grown in popularity over the years as a result of internet connectivity. (Wood & Williams, 2012) When comparing online versus land-based casinos, researchers have identified that the burden of analysing the former is lesser compared to the latter. (Operations Research Society, 2021)

Another significant part of the industry is Sports Betting. The online sports betting industry is a rapidly growing global sector. Research has suggested a newer form of sports betting called in-play sports betting which is done on events within a sports event. (Griffiths, 2019). This has led to a structural change in the mechanics of the sports betting industry by allowing larger wagers to be placed during a single sporting event. (Auer, 2013). However, several pieces of research have resulted in saying that in-play sports betting may offer more of a risk to gamblers as it is a high-speed continuous betting and needs instant decisions. (Hing, 2014). Online betting has led to machine learning models in order to predict the outcomes of the match. They use certain algorithms such as logistic regression and Bayes Method to analyse past performances and setting a benchmark for the result which leads to the creation of "odds"

in cricket. (Kampakis, 2015). Horseracing follows the pari-mutuel system as well which allows bettors to place wagers on a set of horses in a given race. These amounts form a pool from which a specific amount is considered as expenses such as track maintenance. The remaining amount is allocated to bettors on the winning bet based on their odds. To put any gambling strategy into action, a statistical model that can predict the real winning probability for each horse is necessary. (Chapman, 1986)

Coming to lottery betting, the lottery industry remains one of the industries that keep progressing to meet the advancing technology, especially when operated on an online platform. (Wiltshire, 2020) Efficiency for lotto games like 6/49 need not be assured as they are influenced by luck and skill. Most of the strategies like selecting the same combination over and over again involved are flawed but lotto games can be potentially exploited using unpopular numbers which do not have an effect on the probability of winning but on the payoff upon winning. (Ziemba, 2008)

It is believed that there's a negative expected return in 'gambling' as compared to a positive expected return in 'investing'. But in the analysis of the Massachusetts number game by Chernoff's [1980, 1981], he demonstrated that by focusing on the unpopular numbers, a positive expected return from bets is possible. (Hausch & Ziemba, 1995). Numerous estimation methods have been utilised to calculate the best numbers: regression of the log of payoffs on the winning numbers, simple counts of the frequency with which the numbers are selected and a constrained maximum likelihood model. All render the same result, namely that 15 to 20 of the numbers are quite unpopular. (Thaler & Ziemba, 1988)

In comparison with financial markets, lottery betting markets have barely received any attention. Given the transaction costs involved in this market (50%), average losses are large. Profitable wagering systems developed require exceedingly long-time horizons but this has not put an end to the findings. (Hausch & Ziemba, 1995)

The gap that we found is that the gambling and betting industry has a large scope in terms of

operations research but has not yet been investigated upon. We hope to delve more into it through our research. OR into the business is important if these industries want to build a stronger foundation in terms of management and efficiency in the future.

III. RESEARCH OBJECTIVES

1. To understand the use of analytical methodologies in casino-based gambling
2. To examine the effectiveness of OR strategies used in casino gambling.
3. To find out the efficiency of sports betting using OR methods.
4. To understand the use of algorithms involved in online betting and in-play sportsbetting.
5. To develop a mathematical approach towards the decision-making process to maximize pay-out in lotteries.
6. To formulate an optimal lottery strategy using mathematical tools.
7. To study optimality and practicality of strategies adopted in these markets using operations research tools.

IV. ANALYSYS & FINDINGS

A. Logistic Regression

Logistic regression is a statistical analysis approach for predicting a data value based on previous data set observations. It enables a machine learning application to categorise incoming data using an algorithm based on past data. Logistic regression may also help with data preparation by allowing data sets to be placed into specified buckets. (Burns 2019)

The probability of winning or losing in any situation is calculated using Logical Regression. In recent years, the sports betting business has adopted the same methodology. An online platform would have a complete data set containing all of the information about the teams involved and the players playing.

Let us take the Indian Premier League as an example. The application of Logical Regression would assist bookmakers and bettors in evaluating each game during the season. Every year, hundreds of individuals try to analyse and predict the winner. These analyses differ from person to person; some may examine the players'

previous performances, while others may evaluate the match circumstances.

The use of Logical Regression in the sports industry has resulted in 76% of the predictions being accurate. So, how does betting work in the biggest T-20 league in the world. When you bet online, you must first deposit cash before you can start making bets. This will make it easier for you to bet properly and safely. After you've set a realistic budget, you'll need to look into the odds. Odds are important since they influence the chance and reward associated with winning a bet. In order to apply Logistic Regression to the online betting industry, the following features of a normal bet are extracted:

- 1) Bettor
- 2) Amount of Bet Placed
- 3) Odds
- 4) Total Outgoing Amount

To get a better idea of how this would work in the real world, let us take a look at the following dummy data for some IPL matches and the odds that were provided before the match started.

Table 1: Predicted Match Odds

Match	Favourites	Odds	Total Bets (Crores)	Result	Income
MI vs CSK	MI	1.35	1.25	CSK	1.25
RCB vs SRH	RCB	1.20	1.20	RCB	0
DC vs KKR	DC	1.10	1.00	DC	1.00
RR vs PKS	PKS	1.15	1.11	RR	0
RCB vs MI	MI	1.35	1.33	RCB	0
SRH vs CSK	CSK	1.17	1.15	CSK	1.15
KKR vs PKS	KKR	1.26	1.25	PKS	0
DC vs RR	DC	1.05	0.95	DC	0.95

(Source: Author)

Here, we can see that the amount of wager placed are directly co-related to the odds that are given. It is inevitable since, we can say that the opportunity to earn more would encourage more bets to be placed. The trend would also suggest that if the favourite team ends up winning the match, the odds for their following match will reduce, as they are expected to perform and win.

The application of Logistic Regression in such cases would help bookies examine the probability of whether the team would win/lose in their next game.

We assume a linear relationship between the predicted variable (win/lose) and the odds of the event. This relation can be given in the following mathematical formula:

$$P = \frac{e^{a+bX}}{1 + e^{a+bX}} \quad (1)$$

B. Bayes Theorem

Bayes Theorem or Bayes' rule or Bayes' Law was given by Thomas Bayes in the 18th-century. It helps us to calculate conditional probability and further examine current expectations.

The formula of the same is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B|A)}{P(B)} \quad (2)$$

P(A) = The probability of A occurring

P(B) = The probability of B occurring

P(A|B) = The probability of A given B

P(B|A) = The probability of B given A

P(A∩B) = The probability of both A and B occurring

The theorem finds its application in various real-life situations, sports betting being one of them. The major hurdle while doing so is assigning probabilities to uncertain events. But with technological advancement this has been made possible and can help bettors and book-makers who try to predict the outcome of matches to make profits. (Pinnacle, 2014)

Considering the Premier League as an example to understand the Bayes theorem and how it can help in predicting the outcome of a game. These predictions in turn can assist in sports betting.

The taken data is of Manchester City Men's Team from the 2020/2021 session for this. The team played a total of 38 matches in the said session which included 19 home and 19 away matches. Out of which they were victories in 13 home and 14 away matches.

It is assumed:

P(A) = The probability of Manchester City winning a match = 27/38

P(B) = The probability of Manchester City playing a home match = 19/38

P(B|A) = The probability of Manchester City playing a home match when it wins = 13/27

The Bayes theorem when applied here reveals that the probability of Manchester City winning when it plays a home match which is P(A|B) is 68.42 %. Such a revelation can assist the parties involved in choosing their end during the betting process.

Although the given example considers home match as an event while calculating the concerning probability, it can also be replaced with other events. For example, weather data is often readily available and can be used to estimate the probability of a team winning a match. Such a kind of analysis is invaluable for bookies and bettors since it is expected that these gambles are calculated risks.

C. Game Theory & Probability Theory

Gambling with multiple players in a casino is considered as an instance of a zero game which means that the addition of ultimate payoff by the casino to its players in the game is zero. This implies that if one player is gaining, it will have a positive payoff and will lead to the loss of another player which will now have a negative payoff. Here, the strategies adopted by the numerous players do not affect the resources that are available in the game. (Nitisha)

This leads to the use of game theory in gambling which is a part of applied mathematics that provides methods for analysing situations in which parties known as players make decisions that depend on the decision or choices of the other. (Davis, Brams, & Steven, 2021) Taking Poker for an instance. Game theory helps in providing ideal optimal strategies to the players to prevent their exploitation by using certain mathematical precisions. (Li, 2018) Using the GTO strategy as a defensive mechanism when you just have a rudimentary understanding of the opponent's way of playing is a very good tactic. However, games such as roulette make use of the probability theory. Here there are only two outcomes conceivable because the game is completely random. Thus, the players will either win or lose. The chance of winning with a certain kind of gamble is computed by dividing

the likelihood of occurrence by the overall multitude of ways to win. (Casino Reports) Since every spin that takes place in a separate trial, mathematical precisions used to formulate strategies to win in roulette do not work and ultimately the casino will gain.

Another strategy used is called the Martingale money management strategy which aids in controlling the wagers made. Say if we have odds of 2.0 and we bet \$50, we lose and when we bet \$100, we lose again. The next bet is \$200 such that the total value that has been put at stake is $\$50+\$100+\$200=\350 . This time you win and get \$50. This payoff is the first amount that was betted. This is the Martingale system. (Consult) However, this is a very dangerous and risky strategy that works in the favor of those who have infinite money with them.

D. Linear Programming Problem

Linear programming Problem uses certain mathematical equations to express the problem of concern. The adjective linear means all the mathematical functions are linear in this model. (Merriam-Webster, 2021). The word programming is actually a synonym for planning. Thus, applied mathematics involves the design of activities to get an optimal result, i.e., a result that reaches the required goal best (according to the mathematical model) among all feasible alternatives (Hillier & Lieberman, 2010).

Slot Machines sustain their dominance in generating 50-80% of the casinos total revenues. As the marginally contribution of slot machines overpowers the contribution of other departments like Table games, Keno, Poker etc the optimization of slot machines becomes vital to maximising revenues from such a lucrative department. Through the application of linear programming, two mathematical models are proposed with the objective of maximizing slot floor performance under a set of constraints. (Ghaharian K. C., 2010)

One model focuses on maximising the total coin per day, Coin-in Per Unit Per Day (CPUPD) is calculated by dividing the total coin-in generated by a particular machine divided by the number of days that machine was on the floor. (Ghaharian K. C., 2010) The other model

focuses on maximising total win per day (WPD) less total promo liability per day (PLD) Win Per Unit Per Day (WPUPD) is determined by dividing the total win generated by a particular machine divided by the number of days that machine was on the floor and Promo Liability Per Unit Per Day (PLPUPD) refers to a dollar amount which is re-invested to the player. (Ghaharian K. C., 2010)

The type of constraints used like the capacity constraint, the set of constraints that deals with the decision variables and cost constraint targets different aspects of optimization which is essential to give an all-inclusive view into the application of Operations research. The contribution of each machine is evaluated and a configuration of the highest revenue generating machines maximise the slot floor's potential in relation to its operations. This relates perfectly to the application of operations research. The inclusion of shadow prices gives detailed insight into the type of slot floor which can be adopted to ensure full utilisation of the casino's resources.

E. Using Mathematics in Lottery Selection and Strategy Formulation

Before picking a lottery or selecting a strategy or even choosing not to play at all, you should calculate the odds. Winning the jackpot isn't something you should consider because it is a long shot, but you can try to increase your pay-out by reading the odds and selecting a suitable lottery. Here are the odds of one of the most famous online lottery games in India.

Table 2: Lottery Games Odds

6 + Joker	1 in 79,453,500	₹ 4 Crore
6	1 in 19,863,375	₹ 7,50,000
5	1 in 75,240	₹ 80,000
4	1 in 1,400	₹ 4,000
3	1 in 75	₹ 400
2	1 in 10	₹ 40
Joker Ball	1 in 5	Free Bet Bonus

(Source: Lotto India)

You win a pay-out larger than your entry fee of Rs.40 if you match between 2 and 5 numbers as well as the Joker Ball. With such a lottery design; even if you match just 2 numbers or even the joker ball which has very good odds, you still lose nothing or even get another shot at the lottery, and there will always be a chance of gaining more. Many people see this as a win-win situation go ahead with this lottery having a jackpot with odds of 1 in 79,453,500.

Practically speaking, lottery strategies can only help you improve your chances of winning and never guarantee a win because, after a considerable amount of research on this topic, I can safely state that the exact winning combination can never be predicted. But one can mathematically increase their chance of winning. After analysing various lottery strategies that have been used over a long period, according to me, a strategy that combines statistics and combinatorics is most likely to increase your chances of winning the Lotto India.

Statistics

Building your strategies around frequencies of hot, cold and most overdue numbers is suitable if you strike the right balance. The Lotto India (2021) page statistics were studied before the most recent draw of 7th September 2021. The

draw included 2 hot, 1 cold and 1 most overdue number which accounts for 4 numbers out of 7. It is to be noted that there are roughly 5-6 numbers in each of these categories which makes it a lot easier than picking from 50 numbers. The key is playing with the statistics with hot numbers and playing against the statistics with the most overdue. You can narrow it down to build a more focused strategy on just one of these but it makes your combination less diverse.

Combinatorics

There has been a lot of debate whether you can depend on statistics when it comes to lottery due to its randomness but combining it with combinatorics might be a good option.

Combinatorics is the mathematical tool used in a wheeling system that gives you all the possible combinations of the numbers that you insert. It should be noted that this strategy relies on buying more than one ticket. The more numbers you wheel, the more tickets you need to buy but it also considerably improves your winning odds. It is a simple method that provides maximum coverage of numbers and gives winning assurances. (Wilson, 2020)

For example, if you wheel 8 numbers for a 6-digit lottery, and you've 3 of the winning numbers in your set, you get nine tickets that have those three. The chances become better if you're able to guess more correctly. Other aspects like the amount of money you can spend should also be considered as it'll affect the number of tickets you buy and hence the numbers you wheel. One should carefully spend money on a lottery draw as in the end, a win is never guaranteed.

V. CONCLUSION

As we endeavoured to plan a suitable result for the final analysis of this research paper, numerous problems occurred. In terms of statistics, it was extremely difficult to obtain data on gambling and betting, as they are either banned or deemed "taboo" in many nations. Gambling is prohibited in ten nations, including the United Arab Emirates, Japan, Singapore, Qatar, and Poland. (Lawyer Monthly 2018). The study article attempted to address an issue with very little research available online, yet it is

undeniably an intriguing topic because it concerns money.

The application of operations research is prominent in gambling and betting because it encompasses mathematical calculations and forecasting outcomes based on numbers rather than just luck. Evolution is inexorable, and the gambling business is no exception; the world has progressed from traditional casinos to generating money while betting online.

The goal of the study paper was to come up with new techniques that would assist comprehend the gambling and betting business better by analysing the mathematics underpinning gambling and betting. This study attempted to connect each and every finding to the previously mentioned research objectives. This study technique has sought to discover a unique way of analysing the gambling and betting business based on the models and coming up with a credible result, ranging from Logistic Regression to Linear Programming. Regardless, knowing the science underlying them is a long way off.

When Game Theory and Probability Theory were utilised as a technique, the major goal was to examine the effectiveness of Operations Research Strategies in casino gambling. This used the GTO technique, which was thought to be a defensive mechanism by someone who had been successful in their opponent's game plan, and it turned out to be an effective method. However, certain games, such as roulette, are wagered on using probability theory, which is nothing more than calculating the likelihood of winning. In terms of the sports betting business, two of the paper's techniques are specially designed to determine its efficiency using Operations Research.

Both Logistic Regression and Bayes' Method are used to determine the conditional probability of a bet winning or losing. The former use statistical analysis to forecast future data, whereas the latter bases additional inquiry on previous likelihood. Logistic Regression may be a key element in understanding the chances at which bets can be placed, and Bayes' Method identifies a probability's applicability in the present occurrence, and increased use of technology has made both of these techniques easier to utilise. The extended use of

mathematics in the lottery system was analysed using strategy formulation. Based on statistics, an individual would strategize their bets by studying the recent draws.

All-in-all this research paper has tried to grab a peep into the large world of gambling and betting, something which people considered easy money has turned out to be one of the most calculated risks in the general market. A larger variety of stats lives beyond this paper which is backed by some extreme mathematics.

VI. LIMITATIONS

As Application of Operations Research in Sports Betting, Lottery betting and Gambling in casinos is a novel concept, The secondary data available was limited and not easily accessible which restricted the scope of presentation in this research paper. Although there have been some studies on Sports betting, Lottery betting and Gambling In casinos, the research paper and other research material available were circumscribed which posed certain difficulties in forming objectives for the research paper. As the introduction of Operations Research as a subject was contemporary, the lack of prior exposure to Applications of Operations research made it tricky to navigate the applied techniques and better comprehend its implications. Operations Research develops models pertained to measurable factors. It cannot account for qualitative factors like Luck which is an undeniable part of Lottery Betting, Sports Betting and Gambling. Therefore, such qualitative factors can act as an impediment to the accuracy of the models proposed. Sports betting is based in a competitive environment, which makes it taboo to disclose important stats and details of the games. This hindered the availability of data and research for sports betting. However, from a panoramic perspective Operations research in Sports betting, Lottery betting and gambling in casinos have a bright potential for enhancement hence the limitations faced will subside over time.

VII. FUTURE RESEARCH DIRECTIONS

The study recommends that Operations Researchers should explore the various possibilities that are possible in the field of

Sports Betting, Lottery Betting and Gambling. The topic remains insufficiently discussed due to the issues surrounding it in various parts of the world.

It is important to note that while the study doesn't deny the existence of randomness when it comes to the outcome of events on which betting, or gambling takes place but it also believes that operational research techniques such as the ones discussed can play an instrumental role in increasing the accuracy of predictions. It is further recommended that more extensive data should be made accessible to the experimenters. The lack of data not only hinders research but also affects the quality of forecasts that are made. The increase in the data availability will in turn increase the accuracy and reliability of the models proposed which shall make them fit for generalization to a large extent.

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