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Hydrocarbon Reservoir Petrophysical Characterization with Statistical Simulations: A Case Study from the Gulf of Guinea

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Abstract:

Estimating the initial hydrocarbon in place and the reserve of oil fields requires the petrophysical characterization of reservoir formations through direct and indirect methods. Most of the existing methods are deterministic and do not take into the random nature of logging operations results. Our study aims to propose a rule of thumb, applying statistical simulations (Laplace-Gauss simulations) to existing deterministic methods for hydrocarbon reservoirs petrophysical characterization. It helps in computing P10 (high estimate), P50 (moderate estimate) P90 (low estimate) and confidence intervals of reservoir petrophysical parameters instead of determining single values thereof. A case study has been carried out on a Gulf of Guinea oil reservoir for shale volume and effective porosity determination from well logs. The results show that the expected values of layers A1-1, A1-2 and A1-3 shale volume have 90 percent of chance to be respectively higher than 29.56, 34.18 and 29.89 percent while the ones of the effective porosity of layers A2-1, A2-2 and A2-3 have 95 percent of chance to be respectively in the intervals [26.01%; 26.95%], [25.93%; 26.87%], [28.12%; 29.03%]. We recommend the use of Laplace-Gauss simulations for reservoir petrophysical characterization since it provides high, moderate, low estimates and confidence intervals instead of single values of the parameters of interest.

Keywords **—Hydrocarbon reservoir, Petrophysical characterization, Statistical simulation, High estimate, Moderate estimate, Low estimate, Confidence interval, Gulf of Guinea.**

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1. INTRODUCTION

Crude oil and natural gas are the major energy sources since the end of nineteenth century middle. According to BP Statistical Review of World Energy [4] the ratio of world oil and gas energy consumption to the overall energy consumption has raised from 27 percent in 1950 to 58 percent in 2019. Although the duty of the humanity to move on clean energy, the energy industry continues developing technologies and approaches enabling the research and the development of oil and gas-based energy. The Gulf of Guinea offshore counts great geological provinces in which a huge number of oil and natural gas fields has been discovered and particularly in the seaward of Cote d'Ivoire, Ghana and Nigeria's coasts.

In order to know the initial hydrocarbon (oil or natural gas) in place in oil fields, the reserve thereof and for other purposes, geoscientists and petroleum engineers must carry out the petrophysical characterization of reservoir formations through direct and indirect methods (Djoï, 2012)[5]. Several deterministic methods, that is, providing single values of the parameters of interest, help in determining the petrophysical properties of reservoirs. The random nature of logging operations results has been quite proven. Indeed, the natural response of underground formations to the detectors or their feedback to tools signals have never been the same in a given formation. For instance, the fact that the number of gamma rays that reach the detector vary with time although the stationarity in the borehole explain the randomness of radioactivity disintegration (Zakki, 1994)[19]. Furthermore, the choice of measurement points in a wellbore over a logging operation is random. As a result, the random aspect observed in well log data must be taken into account at their interpretation stage.

This study aims to propose a rule of thumb, applying statistical simulations (Laplace-Gauss simulations) to existing deterministic methods for hydrocarbon reservoirs petrophysical characterization. It will develop the techniques ofcomputing P10 (high estimate), P50 (moderate estimate), P90 (low estimate) and confidence intervals of reservoir petrophysical parameters instead of single valuesof the parameters of interest.

A case study will be performed on a Gulf of Guinea oil reservoir for shale volume and effective porosity determination from well logs.

2. Petrophysical characterization

Petrophysical formationevaluation can be generally defined as the practice of determining both the physical and chemical properties of rocks and the fluids they contain (HLS Asia Limited, 2007)[11]. Djoï (2012) [5] stated that these parameters can be determined by analyzing samples or cuttings collected in wells (direct method) or through well logs interpretation (indirect methods). Log in the oil industry means a recording against depth of any of the characteristics of the underground rock formations traversed by a measuring apparatus in the well-bore (Oberto, 1983)[15]. Log data analysis consists of interpreting the recorded information to get the qualitative and quantitative characteristics of formations and their contents (Schlumberger, 2009)[18]. The common petrophysical parameters of reservoir formations computed from well log data are, but not limited to, shale volume, porosity, permeability, fluids (water, oil and gas) saturation, and net pay thickness (Ekwere, 2004)[7]. Tools are developed for tens of formation physical and chemical properties well logging. However, we are going to spotlight the properties that are logged in the case study wells for shale volume and effective porosity computation (gamma ray and sonic attenuation) as well as the development of these two petrophysical properties.

The gamma ray (GR) log is a continuous recording of the intensity of the natural radiations emanating from the formations penetrated by the borehole versus depth (Oberto, 1983)[15]. The sources of natural radioactivity are the isotopes of ^{40}K, ^{232}Th and ^{238}U contained in the formation minerals (Zakki, 1994)[19]. The intensity of radiations from potassium are much more important than thorium and uranium and is characteristic of clay presence in formations. Thorium are present in two main types of sedimentary rocks (salt or anhydrite and limestone) while sandstone are wealthy in uranium (Djoï, 2012)[5]. As a result, high values of GR indicate the presence of shale and lower values characterize formations with less clay content. Indeed, GR value is related to the amount of radioactive items in the formation. According to Oberto (1983)[15] the GAPI unit used corresponds to microgram equivalent of uranium per tons (µg Ra-eq/t): the conversion relationship is 16.5 GAPI = 1 µg Ra-eq/t).

The sonic log is simply a recording versus depth of the time, t, required to a sound wave to traverse 1 ft of formation (Schlumberger, 2009)[18]. Conventional sonic tools measure the reciprocal of the velocity of the compressional wave. This parameter is called transient time, Delta t (DT) or slowness, and is expressed in microsecond per foot (µs/ft (Zakki, 1994)[19]). The interval transient time for a given formation depends upon its lithology and the porosity (Djoï, 2012)[5].

It's necessary for us to make a brief insight on the two parameters to be calculated from log data in our case study that are shale volume and effective porosity. Shale volume (V_{sh}) of a sedimentary formation is simply the proportion of shale mineral in that formation (Mohammed, 2021)[12]. Accordingly, the rock can be differentiated as clean if $V_{sh}$is less than 10%, shaly if V_{sh} ranged from 10 to 33% and if the V_{sh} is more than 33%, it is considered to be shale (Mostafa and Walid, 2003)[13].

Porosity is defined as the ratio of the pore volume to bulk volume (equation 1), and may be expressed as either a percentage or a fraction (Toraeter and Abtahi, 2000)[17].

$$\boldsymbol{\phi = \frac{V_p}{V_b} = \frac{V_b - V_s}{V_b}} \qquad (1)$$

Where V_p is the volume of pores, V_b the bulk volume, and Vs the grains volume. Two types of porosity may be measured: total or absolute porosity and effective porosity. Total porosity is the ratio of all the pore spaces in a rock to the bulk volume of the rock. Effective porosity is the ratio of interconnected void spaces to the bulk volume (Ekwere, 2004)[7].Thus, only the effective porosity contains fluids that can be produced from wells.

3. MATEIALS AND METHODS

3.1. Materials

The proposedapproach has been tried on the log data from two (02) appraisal wells of a Gulf of Guinea offshore oil field for the computation of shale volume and effective porosity of the identified reservoir layers through a qualitative interpretation. On the basis of this, the recorded shale volume and porosity indicator logs (GR and DT) have been withdrawal from the composite log dataset of the wells. The computation for deterministic methods was performed using Microsoft Excel and MATLAB while R Studio (a statistical analysis software) has helped in the simulation process via R-programming for the following reasons: (a) high execution speed, (b) the availability of necessary libraries and tools for their open-source nature, (c) ease of implementation, and (d) flexibility of the system (Arnold and Rume, 2020)[1].

3.2. Methods

The essence of our approach is to take into account the random nature of well log data, by applying Laplace-Gauss simulations to the existing deterministic techniques for reservoir petrophysical characterization. The application of that method on reservoir shale volume and effective porosity estimation is just an example; it is an appropriate approach to be used for any other petrophysical parameter whosecalculation relies on the average of a dataset.

The technique starts with the calculation of the parameter of interest with a deterministic method and ends with its statistical simulations on the basis of the data set resulting from the deterministic method.

3.2.1. Deterministic Estimation of Shale Volume

According to Djoï(2012)[5], Schlumberger (2009)[18], Gutiérrez-Torres et al. (2019)[16] and Mohammed (2021)[12] calculations of shale volume can be obtained from Gamma Ray log, Spontaneous Potential log or Density-Neutron Model. In the best case, when all these rock properties are recorded, V_{sh} is computed for each log dataset and the ultimate shale volume of the formation is deduced. The following formulae (equations 2 and 3) are used for that purpose to obtain the shale volume. In the case where three different shale volumes are considered, the average value is obtained as follows (Gutiérrez-Torres et al. 2019)[16]. When only one shale volume indicator log is available, the case of the dataset of our case study for instance, the calculated value of V_{sh} is valid to characterize the formation.

$$\boldsymbol{V_{sh} = \frac{GR_{sh} + SP_{sh} + ND_{sh}}{3}} \qquad (2)$$

$$\boldsymbol{V_{sh} = min(GR_{sh}; SP_{sh}; ND_{sh})} \qquad (3)$$

Where GR_{sh}, SP_{sh} and ND_{sh} are respectively the shale volume computed from gamma ray, spontaneous potential and neutron-density.

Fadiya et al. (2018)[8] and Hamada (1996) [9]have revealed two models for V_{sh} estimation from the resistivity log. The resistivity shale volume must be integrated into the equations 2 and 3, if resistivity logs are available.

Several studies have highlighted that gamma ray shale volume must be computed from the gamma ray index model (Mahommed, 2021 and Zakki, 1994)[19]. These studies have defined the gamma ray index (I_{GR}) as follows (equation 4).

$$I_{GR} = \frac{GR_{log} - GR_{min}}{GR_{max} - GR_{min}} \quad (4)$$

Where GR_{log}, GR_{min} and GR_{max} are respectively the recorded gamma ray, the gamma ray of clean rocks (sandstones and limestones) and the gamma ray of shale.

The linear model considering the shale volume equal to the gamma ray index (equation 5) has been the only available model until the end of the 1960s (Djoï,2012)[5]. This model over estimates the shale volume of formations (Norbert, 2019[14] and Zakki, 1994[19]).

$$V_{sh} = I_{GR} \quad (5)$$

Non-linear models have been developed for shale volume computation (Mohammed, 2021[12] and Zakki, 1994[19]): Larionov models, Steiber model and Clavier model (figure 1). The following are the equations for these different non-linear models. For Larionov models, equation 6 is used for tertiary or younger rocks shale volume calculation while equation 7 helps for older rocks (Eje, 2018[6], Norbert, 2019 [14]and Mahommed, 2021[12]).

$$V_{sh} = 0.083(2^{3.7 I_{GR}} - 1) \quad (6)$$

$$V_{sh} = 0.33(2^{2 I_{GR}} - 1) \quad (7)$$

$$V_{sh} = I_{GR}/(3 - 2I_{GR}) \quad (8)$$

$$V_{sh} = 1.7 - [3.38 - (I_{GR} + 0.7)^2]^{1/2} \quad (9)$$

3.2.2. Deterministic Estimation of Effective Porosity

Three well logs are known as porosity indicator logs: sonic, neutron and density logs (Djoï, 2012)[5]. These logs can be used either individually or in combination for porosity estimation. However, the more the number of available porosity logs the more accurate the results. Since the sonic log is the porosity log in our data set, the techniques for porosity deterministic computation from sonic log will be developed in this study.

The total porosity from sonic log (ϕ_s) is calculated with equation 10 (Djoï, 2012)[5]. For high shaly formations a model involving a correction due to clay content (Karacan, 2010) [10] expresses sonic total porosity from equation 11.

$$\phi_S = \frac{DT - DT_f}{DT_f - GR_m} \quad (10)$$

$$\phi_S = \frac{DT - DT_f}{DT_f - GR_m} - V_{Sh}\frac{DT - DT_f}{DT_f - GR_m} \quad (11)$$

Where $V_{sh}$is the calculated shale volume, DT the sonic log value and DT_f and DT_m respectively the transient time of the fluid and matrix. Common values of the transient time of the fluid and matrix are used for calculations.

The effective porosity is calculated with the equation 12 (Djoï, 2012[5] and Karacan, 2010[10]).

$$\phi_e = \phi_S(1 - V_{sh}) (12)$$

Where ϕ_s and $\phi_e$are total and effective porosities and V_{sh} the shale volume of the formation.

3.2.3. Laplace-Gauss Simulations

The statistical simulations we are going to propose is a variant of Monte Carlo simulations, called Laplace-Gauss (or Gauss) simulation. Developed in 1949 by the American mathematicians John Von Neumann and Stanislav Ulam, Monte Carlo simulations is a series of techniques used to solve complex problems but very often deterministic by introducing random sampling (Baissa, K., 2012)[2]. The term 'Simulations' is related to random variables simulations, that is, the generation of a huge number thereof while Laplace-Gauss (or Gauss) name is due to the normal (Gauss) distribution of the amount to be simulated.

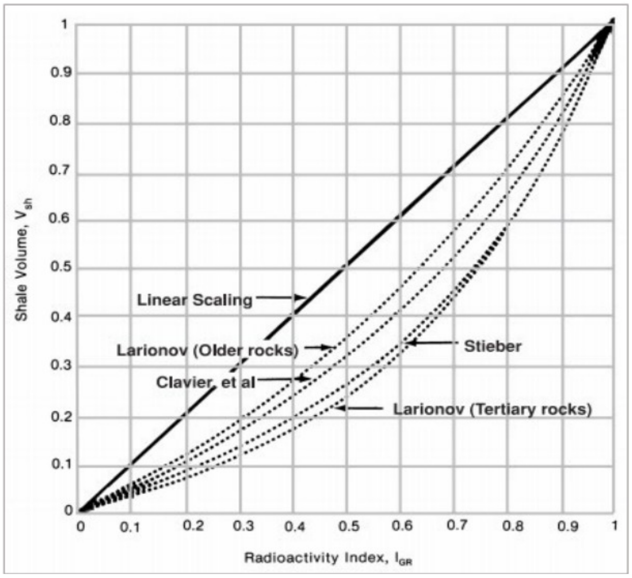

Fig. 1Linear and nonlinear Vsh modelsfrom GR index (Mohammed, 2021) [12]

The use of Laplace-Gauss simulations in our case study relies on the mathematical theorem called **Huge Numbers Law**. This law states that for an independent sample X_1, X_2, …, X_n of a random variable X, the empirical average $\bar{X}$ tends to the mathematic expectancy $E(X)$ of X when n is huge (Christian, 2015)[3]. That is, (equation 13):

$$\boldsymbol{E(X) = \lim_{n\to\infty} \frac{X_1 + X_2 + \cdots + X_n}{n}} \qquad (13)$$

In case of petrophysical parameters computation, the application of statistical simulation will take into account the random nature of formations responses to logging tools and the one linked to recording points choice. This will be performed in two steps: (1) probability distribution modelling and (2) petrophysical parameters simulation.

A. Probability Distribution Modelling

For the simulation of a petrophysical parameter X, the determination of its probability distribution function (PDF) and cumulative density function (CDF) are needed. The first task is to determine these statistical functions of the petrophysical parameter random variable. Frequency analysis or CDF analysis coupled with a statistical test of distribution (Chi-square test for instance) is commonly used for probability distribution determination. Frequency and CDF analyses consist respectively of plotting the empirical frequency and CDF and checking which theoretical PDF and CDF comes much close to the empirical ones. Thereafter, Chi-square test can be carried out to check whether the theorical distribution explains really the empirical results.

The statistical theorem, called Central Limit Theorem (CLT), ceases the probability distribution determination when the parameter to be estimated is the average of a huge-size data set. That is the case of hydrocarbon reservoirs shale volume and effective porosity.

The**Central Limit Theorem**(CLT) states that for a huge number of independent random variables X_1, X_2, …, X_n identically distributed under a probability law of average μ and standard deviation σ, the random variable $\bar{X} = \frac{X_1+X_2+\cdots+X_n}{n}$ follows approximatively the normal (Gauss) law of meanμ and variance $\frac{\sigma^2}{n}$, even though the variables X_1, X_2, …, X_n do not follow the normal law (Christian, 2015)[3]. That is, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

A random variable X is said to follow a normal (Gauss) law of mean μ and variance σ^2 and is noted $X \sim N(\mu, \sigma^2)$, when its probability distribution function (PDF) is defined by the equation 14 (Christian, 2015)[3]; its cumulative distribution function (CDF) is given by equation 15 (Christian, 2015)[3].

$$\boldsymbol{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 dt}} \qquad (14)$$

$$\boldsymbol{(x) = P(X \le x) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{x} e^{-\left(\frac{t-\mu}{\sigma}\right)^2} dt} \qquad (15)$$

Figure 1 shows the patterns of the PDF and CDF of a normal law for different values of μ and σ^2.

For the purposes of our study, the size of deterministic estimation vectors of shale volume and effective porosity, that represent the samplesX_1,X_2, …, X_n of the corresponding random variables X, can be considered large, and then the CLT can be applied, when the thickness of the layer of interest is of the order of metre. Indeed, the recording interval of well logs are usually of the order of millimetres.

As stipulated, for any petrophysical parameter X to be simulated with Laplace-Gauss methods we proposed in this study, the deterministic estimation of that amount must rely on the average of a given vector $\hat{X} = (X_1, X_2, \cdots, X_n)$ of punctual computations of $\hat{X}$ represents a sample of the petrophysical parameter random variable X.

With regard to the Central Limit Theorem, the random variable $\bar{X} = \frac{X_1+X_2+\cdots+X_n}{n}$ characterizing a layer for a given petrophysical parameter X, follows the normal law of average $\mu_{\hat{X}}$ and variance $\frac{\sigma^2{}_{\hat{X}}}{n}$, that is, $\bar{X} \sim N\left(\mu_{\hat{X}}, \frac{\sigma^2{}_{\hat{X}}}{n}\right)$.

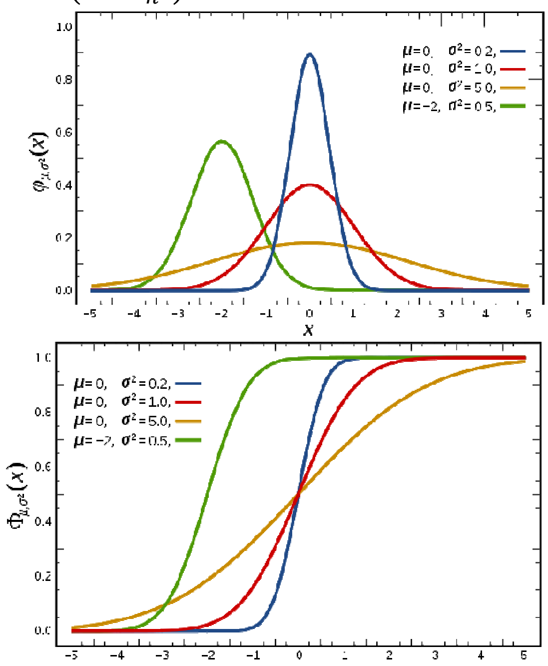

Fig. 2 (Top): PDF of normal law. (Bottom): CDF of normal law (Wikipedia, 2022)

B. Petrophysicl Parameters Simulations

Laplace-Gauss simulations of a petrophysical parameter X for an identified reservoir layer will result in generating a huge number (thousands or tens of thousands, or even millions) of its random variable $\bar{X}$. Since $\bar{X} \sim N\left(\mu_{\hat{X}}, \frac{\sigma^2{}_{\hat{X}}}{n}\right)$, $\hat{X} = (X_1, X_2, \cdots, X_n)$ being the sample got from individual deterministic computations of X, a huge-size vector $(\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N)$of the normal variable $\bar{X}$ will be generated with a random number generator. Almost all random number generators available in statistical analysis software are able to generate normal laws.

By application of the Huge Number Law, the average of the simulated vector $(\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N)$of the petrophysical parameter X, noted $\bar{X}_{LG}$ (LG for Laplace-Gauss), is the estimation of X with Laplace-Gauss simulations. Equation 16 gives the expression of $\hat{X}_{LG}$.

$$\bar{\boldsymbol{X}}_{\boldsymbol{LG}} = \frac{\bar{\boldsymbol{X}}_1 + \bar{\boldsymbol{X}}_2 + \cdots + \bar{\boldsymbol{X}}_N}{\boldsymbol{N}} \quad (\mathbf{16})$$

The following isthe algorithm of reservoir petrophysical parameters computation with Laplace-Gauss simulations.

Algorithm of Laplace-Gauss simulations:

Step 1: For a given reservoir petrophysical parameter X, determine the vector $\hat{X} = (X_1, X_2, \cdots, X_n)$ of X with a deterministic method.

Step 2: Generate a huge number values$\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N$of the normal law $\bar{X}$ of parameters $\mu_{\hat{X}}$ and $\frac{\sigma^2{}_{\hat{X}}}{n}$.

Step 3: Calculate $\bar{X}_{LG} = \frac{\bar{X}_1+\bar{X}_2+\cdots+\bar{X}_N}{N}$. $\bar{X}_{LG}$ is the estimated value of$X$ for the reservoir with Laplace-Gauss simulations.

When the a huge number of $\bar{X}$ is generated, $\bar{X}_{LG}$ converge to $\hat{X}$ got with the deterministic method since $\hat{X}$ is the mean of normal distribution simulatedin Laplace-Gauss approach.

Though, the main purpose of Laplace-Gauss simulations is to help in determining the following key statistics: (1) P10 (high estimate), P50 (moderate estimate) and P90 (low estimate) and (2) confidence intervals ofthe deterministic$\hat{X}$.

By definition, **P10**, **P50** and **P90** are the values of $\bar{X}$ for which any other value has respectively 10, 50 and 90% of chance to be higher than it, that is:

$\boldsymbol{P(\bar{X} \geq P10) = 10\%}$; $\boldsymbol{P(\bar{X} \geq P50) = 50\%}$ *and* $\boldsymbol{P(\bar{X} \geq P90) = 90\%}$.

Therefore, one has:

$P10 = G^{-1}(0.1)$, $P50 = G^{-1}(0.5)$ and $P90 = G^{-1}(0.9)$. With:$G = 1 - CDF_{\bar{X}}$.

The **confidence interval** (CI) of a random variable Xat the confidence level α% is the interval for which any realization of Xhas α% of change to lie in (Wikipedia): $P(X \in \mathrm{CI}) = \alpha/100$. This interval is generally, like in the case of a gaussian law, centred at the average of X.

The lower and upper bounds (respectively noted LB and UB) of the confidence interval of $\overline{X}$ at the confidence level of 95% can be computed with the formulae: $\boldsymbol{LB = G^{-1}(0.975)}$and $\boldsymbol{UB = G^{-1}(0.0275)}$.

The advantage of computing P10, P50 and P90 is that they give the expected minimum values of the parameter of interest with respectively low, moderate and high chances. As far as the confidence intervals are concerned, they give a range of values in which the parameter has a high chance to lie in.

4. RESULTS AND DICUSSION

As stipulated in the methodology section, the case study concerns two (2) appraisal wells (Well-1, and Well-2) of a Gulf of Guinea offshore field. Gamma ray (GR) and sonic (DT) logs data have been used for petrophysical parameters computation. Figure 3 shows the profile of these logs for both wells in the area of interest.

Since the aim of this study is not a qualitative analysis well logs, the results of such an analysis has been collected. These results show that three (3) oil soaked sandstone reservoir layers have been identified for each well: A1-1, A1-2 and A1-3 for well 1 and A2-1, A2-2 and A2-3 for well 2. Table 1 summarizes the location of these layers. In our study the shale volume and the effective porosity have been computed for these reservoir layers with both deterministic methods and Laplace-Gauss simulations.

4.1. Deterministic Estimation of Shale Volume

Since gamma ray (GR) log is the only shale volume indicator log available, we computed GR shale volume for each reservoir layer.

Regarding the geologists report stating that the formation are older than tertiary age, Larionov model for older rock has been used for shale volume computation with equations 5 and 7.

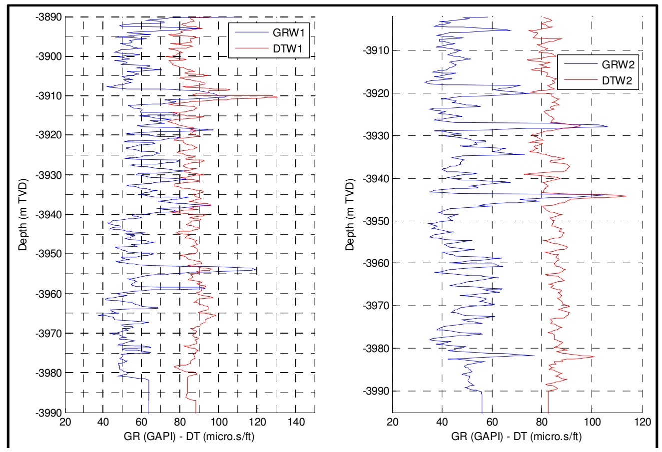

Fig. 2 (Left): Well 1 GR end DT profiles. (Right): Well 2 GR end DT profiles

TABLE I
LOCATION AND LITHOLOGY OF RESERVOIR LAYERS

Well	Layer	Location	Lithology
Well 1	A1-1	3890 - 3908 m TVD	Sandstone
	A1-2	3911 – 3953 m TVD	Laminated sandstone
	A1-3	3955 – 3994 m TVD	Sandstone
Well 2	A2-1	1902 – 1927 m TVD	Sandstone
	A2-2	1928,5 – 1943,5 m TVD	Sandstone
	A2-3	1946 – 1999 m TVD	Sandstone

The GR boxplots in figure 3 shows the statistics of that parameter for both wells and the minimal and maximal values used for the computations are GR-W1min = 10.73 GAPI, GR-W2min = 6.01 GAPI, GR-W1max = 162.11 and GR-W2max = 164.88 GAPI.

Table 2 shows the results of this deterministic estimation of shale volume and figure 4 shows its profile for each reservoir layer.

4.2. Deterministic Estimation of Effective Porosity

Neutron total and effective porosities (PHIEs) have been computed from the equations 10 and 13 since only sonic log was recorded.The matrix (sandstone) and fluid transient time used in the calculus are $DT_{sandstone}$ = 53.25 µs/ft and DT_{fluid} = 189 µs/ft.

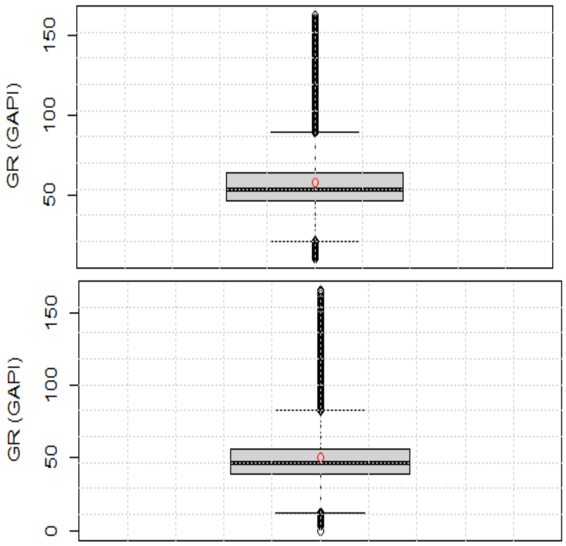

Fig. 3 (Top): GR Boxplot for Well 1. (Right): GR Boxplot for Well 2

Table 2 shows the results of this deterministic estimation of effective porosity and figure 4 shows its profile for each reservoir layer.

TABLE II
SHALE VOLUME AND EFFECTIVE POROSITY OF RESERVOIR LAYERS

Well	Layer	V_{sh} (%)	PHIE (%)
Well 1	A1-1	30.67	27.39
	A1-2	34.75	28.74
	A1-3	30.33	30.57
Well 2	A2-1	25.33	26.48
	A2-2	27.56	26.40
	A2-3	29.24	28.58

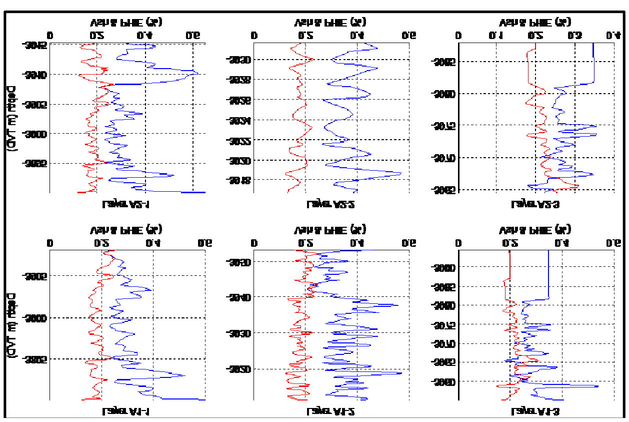

Fig. 4 V_{sh} and PHIE profiles for reservoir layers (V_{sh} in blue and PHIE in red)

4.3. *Laplace-Gauss Simulations*

A. *Probability Distribution Modelling*

In this section the layers computed shale volume samples vectors with the deterministic methods for the well 1 layers will be noted $\widehat{V_{11}}$, $\widehat{V_{12}}$, $\widehat{V_{13}}$ and $\widehat{V_{21}}$, $\widehat{V_{22}}$, $\widehat{V_{23}}$ for the well 2 while the effective porosity samples vectors will be respectively $\widehat{P_{11}}$, $\widehat{P_{12}}$, $\widehat{P_{13}}$ and $\widehat{P_{21}}$, $\widehat{P_{22}}$, $\widehat{P_{23}}$ for well 1 and well 2 layers.

Table 3 summarizes these statistics and length for all layers.

TABLE III
SHALE VOLUME AND EFFECTIVE POROSITY OF RESERVOIR LAYERS

Well	Layer	Sample length	Vsh (%)		PHIE (%)	
			Mean	Standard deviation	Mean	Standard deviation
Well 1	**A1-1**	119	30.67	0.0086	27.39	0.0025
	A1-2	275	34.75	0.0044	28.74	0.0017
	A1-3	258	30.33	0.0034	30.57	0.0016
Well 2	**A2-1**	165	25.33	0.0045	26.48	0.0024
	A2-2	98	27.56	0.0068	26.40	0.0024
	A2-3	154	29.24	0.0038	28.58	0.0023

The histogram of these vectors (figures 5 and 6) show that the corresponding random variable do not all follow normal distributions. However, we can notice that the dimensions of these samples are as great as the Central Limit Theorem can be applied to the corresponding random variables. **The following requirements for Laplace-Gauss simulation application are therefore satisfied**: (1) the parameters to be estimated are average of samples vectors and (2) the samples vectors sizes are large.

The **random variables of reservoir layers shale volumes** (noted $\overline{V_{11}}$, $\overline{V_{12}}$, $\overline{V_{13}}$, $\overline{V_{21}}$, $\overline{V_{22}}$, $\overline{V_{23}}$ for the well 1) **and porosities** (noted $\overline{P_{11}}$, $\overline{P_{12}}$, $\overline{P_{13}}$, $\overline{P_{21}}$, $\overline{P_{22}}$, $\overline{P_{23}}$ for the well 2) **follow then normal(Gauss) laws whose parameters are set in table 3.**

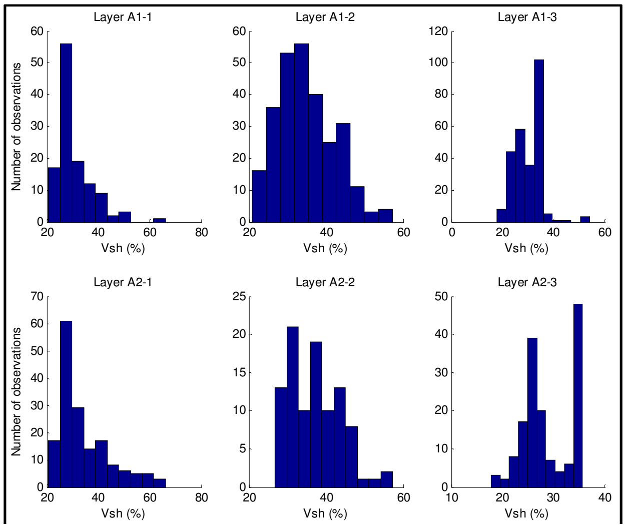

Fig. 5 Histogram of Vsh vectors for reservoir layers

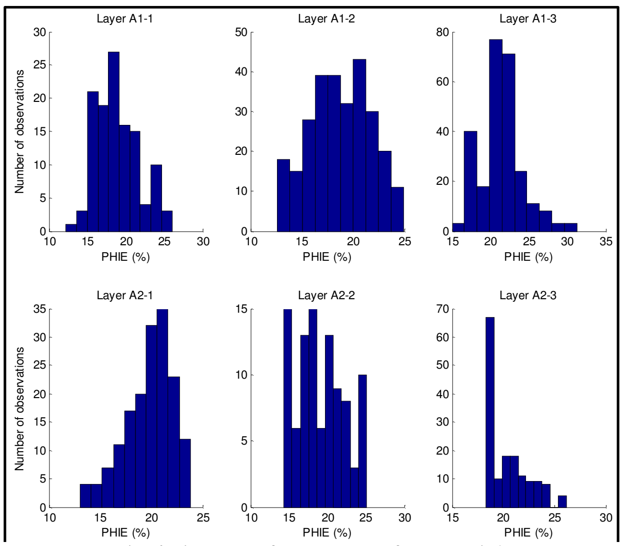

Fig. 6 Histogram of PHIE vectors for reservoir layers

B. Petrophysical Parameters Simulations

At this stage of the study, one million random values of the shale volumes and effective porosities random variables determined have been generated for all layers with the random number generator of R Studio software. P10, P50 and P90 have also been calculated and confidence intervals were set up for shale volumes and effective porosities of each layer with a confidence threshold of 5%.

As highlighted in the methodology, the shale volumes and effective porosities computed from Laplace-Gauss simulations are fairly different from the ones we got from the deterministic methods. Table 4 shows P10, P50 and P90 of shale volume and effective porosity of the different reservoir layers and the confidence intervals thereof at confidence threshold of 5% (that is confidence level of 95%) are summarized in table 5. The profiles of shale volume P10, P50 and P90 of figure 7 and 8 indicate that they are closer to one another for layers A1-2, A1-3, A2-1 and A2-3 while those of and effective porosity show this pattern only for layers A1-2, A1-3.

TABLE IV
P10, P50 AND P90 OF RESERVOIR LAYERS VSH AND PHIE

Well	Layer	Vsh (%)			PHIE (%)		
		P10	**P50**	**P90**	**P10**	**P50**	**P90**
Well 1	**A1-1**	31.77	30.67	29.56	27.71	27.39	27.07
	A1-2	35.31	34.75	34.18	28.95	28.74	28.52
	A1-3	30.76	30.32	29.89	30.77	30.57	30.36
Well 2	**A2-1**	25.90	25.32	24.75	26.78	26.47	26.17
	A2-2	28.43	27.55	26.68	26.70	26.40	26.09
	A2-3	29.72	29.24	28.75	28.87	28.57	28.28

The results show clearly that any estimation or the expected value of layers A1-1, A1-2 and A1-3 shale volume has a 90% of chance to be higher than 29.56%, 34.18% and 29.89%. In the same way, the expected values of the effective porosity of layers A2-1, A2-2 and A2-3 have 95% of chance to be respectively in the interval [26.01%; 26.95%], [25.93%; 26.87%], [28.12%; 29.03%]. Similar conclusions can be made from the values and intervals of table 4 and 6.

TABLE V
CONFIDENCE INTERVALS OF RESERVOIR LAYERS VSH AND PHIE

Well	Layer	Vsh confidence interval (%)	PHIE confidence interval (%)
Well 1	**A1-1**	[28.98; 32.35]	[26.89; 27.88]
	A1-2	[33.88; 35.61]	[28.40; 29.07]
	A1-3	[29.66; 30.99]	[30.25; 30.88]
Well 2	**A2-1**	[24.44; 26.20]	[26.01; 26.95]
	A2-2	[26.22; 28.89]	[25.93; 26.87]

	A2-3	[28.49; 29.98]	[28.12; 29.03]

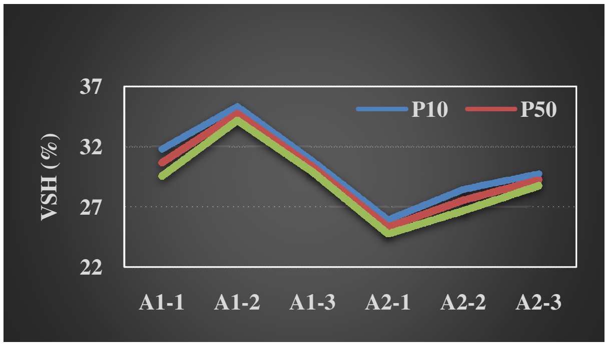

Fig. 7 V_{sh} P10. P50 and P90 profiles

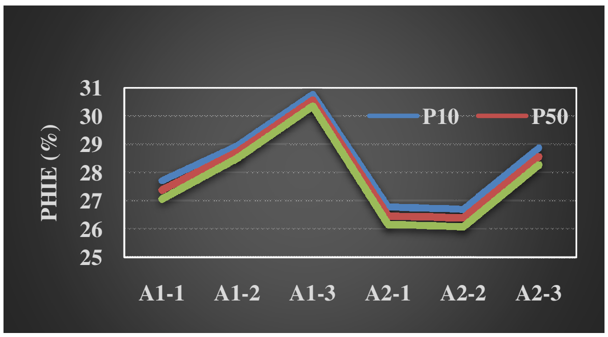

Fig. 8 PHIE P10, P50 and P90 profiles

5. CONCLUSIONS AND RECOMMANDATIONS

Estimating the initial hydrocarbon in place and the reserve of oil fields requires the petrophysical characterization of reservoir formations through direct and indirect methods. Our study has proposed Laplace-Gauss simulations for hydrocarbon reservoirs petrophysical characterization. It has shown that computing the low, moderate andhigh estimates and confidence intervals of reservoir petrophysical parameters is much more informative than estimating single values. The results of our case study carried out on a Gulf of Guinea oil reservoir for shale volume and effective porosity determination from well logs, show that the expected values of layers A1-1, A1-2 and A1-3 shale volume have 90 percent of chance to be respectively higher than 29.56, 34.18 and 29.89 percent while value of the effective porosity of layers A2-1, A2-2 and A2-3 have 95 percent of chance to be respectively in the intervals [26.01%; 26.95%], [25.93%; 26.87%], [28.12%; 29.03%].

We recommend the use of the Laplace-Gauss simulations for reservoir petrophysical characterization since it provides high, moderate and low estimates and confidence intervals instead of single valuesof the parameters of interest.

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