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A STUDY ON M/M/1 QUEUING SYSTEM USING PENTAGONAL FUZZY NUMBERS

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ABSTRACT:

In this paper, we analyzed thesingle server queuing system with pentagonalfuzzy numbers to deal with real word scenarios. System characteristic such as the expected number of customer in the queue and expected average waiting time in the queue are calculated using the alpha clipping. We can also analyze the performance measures in pentagonal fuzzy numbers. Various numerical examples using the arrival rate and service rates are shown as pentagonal fuzzy number to indicate reliable performance.

Keyword: Pentagonal fuzzy number, alpha cuts, customer, waiting time, entry and service rate.

INTRODUCTION:

Queuing theory is the branch of operational research, the objective of queuing theory is to understand queuing system behavior in order to predict the performance, control and optimize its performance. Li, R.J., and Lee, E.S.,(1989) defined the "Analysis of Fuzzy Queues".The origin of queuing theory can be traced to the early 1990s in a study of the Copenhagen telephone exchange by Anger Krarup Erlang, a Danish engineer, statistician, and mathematician. His work led to the Erlang theory of efficient networks and the fields of telephone analysis.

This analysis of queue system is based upon by the mode by which a unit join a queue, the rule by which they join the service and the time it take to serve the unit.Buckely. J.J, (1990) described the "Elementary Queuing Theory based on probability Theory".Queuing theory aims to design balanced system that serves customer quickly and efficiently but do not cost too much to be sustainable. Negi, D.S., and Lee, E.S.,(1992) explained the Analysis and Stimulation of Fuzzy Queue, Fuzzy sets and systems. His extensive studies of wait time in automated telephone services and his proposal for more efficient networks were widely adopted by telephone companies.Chen,S.P., (2005) described "Parametric Nonlinear Programming Approach to Fuzzy Queues with Bulk, Service". Again Chen, S.P., (2006) explained "A Mathematics Programming Approach to the Machine Interference Problem with Fuzzy Parameters".

The queuing model is widely applied to service organizations as well as manufacturing companies. Thus R.Srinivasan (2014) defined the "Fuzzy Queuing Model using DSW algorithm" and S.

Shanmugasundaram and B.B. Venkatesh (2015) explained "Fuzzy multi server Queuing system Model through DSW algorithm". Get a variety of customer services with different types of service and arrival between them, according to a specific queuing discipline in the frame work of queuing theory compliance with a specific distribution requires time and service time. S. Thamotharan (2016) discussed about "A Study on Multi Server Fuzzy Queuing Model in Triangular and Trapezoidal Fuzzy Numbers using α cuts".

The analysis of the M/M/1 queue with two vacation policies using pentagonal fuzzy numbers have been studied by G. Kannadasan and D. Deviin 2019. In practical the queuing model the arrival rate and service rate are uncertain. Uncertainties are solved using fuzzy set theory. Therefore, the classical queuing model is more applicable if the fuzzy model is widely used.

In this paper we first develop a theory on fuzzy set,alpha cut and pentagonal fuzzy number with membership function. In section 2 it deals with the solution procedure. Interval Analysis for arithmetic and numerical example are described in section 3 and 4. Conclusion is given in the section 5.

1.THEORY ON FUZZY SETS:

Fuzzy set theory not only provides a meaningful and powerful representation of uncertainty measurements, a meaningful representation of the measurement of ambiguous concepts expressed in natural language. Because every crisp set is fuzzy the mathematical incorporation of classical set theory into fuzzy sets rather than vice versa. As naturals the idea of embedding real numbers to the complex plane. So the idea of ambiguity is an improvement not a replacement.

1.1 FUZZY SET:

A fuzzy set T of Y is defined by the membership function T: $Y \to [0,1]$. Where Y is a non-empty set.

1.2 ALPHA CUT:

Alpha cut of fuzzy set $\tilde{T}$clear set $T\alpha$ contains all elements of the universal set Y with membership degree T. It is greater than or equal to the specified α value. Therefore,

$T\alpha = \{y \epsilon Y: \mu_{\tilde{A}}(y) \geq \alpha, 0 \leq \alpha \leq 1\}$

1.3 PENTAGON FUZZY NUMBERS WITH MEMBERSHIP FUNCTION:

A fuzzy number T(p, q, r, s, t) where $p \geq q \geq r \geq s \geq t$ is said to be

$$\mu_{\tilde{A}}(y)=\begin{cases} 0, for\ y < p \\ \frac{y-p}{q-p}, for\ p \le y \le q \\ \frac{y-q}{r-q}, for\ q \le y \le r \\ 1, for\ y = r \\ \frac{s-y}{s-r}, for\ r \le y \le s \\ \frac{t-y}{t-s}, for\ s \le y \le t \\ 0,\ if\ y > t \end{cases}$$

2.SOLUTION PROCEDURE:

Consider the first-come, first-served rule for a single-server M/M/1 queuing system. Arrival time P and service time Q is described by the fuzzy set,

$$P = \{(p, \tilde{\mu}_P(p)/p \epsilon Y\}$$

$$Q = \{(q, \tilde{\mu}_Q(q)/q \epsilon Z\}$$

Where P is the set of inter arrival time and Q is the set of service times.

$\tilde{\mu}_P(p)$ membership is subject to inter-entry times.

$\tilde{\mu}_Q(q)$ membership is subject to service time.

The α cut of the arrival and service time is expressed as follows,

$$P(\alpha) = \{(p \epsilon Y)/, \tilde{\mu}_P(p) \ge \alpha\}$$

$$Q(\alpha) = \{(q \epsilon Z)/, \tilde{\mu}_Q(q) \ge \alpha\}$$

With this α cut we need to define the membership function of X (P,Q) as

$$\mu_{X(P,Q)}(y) = \begin{cases} 0, for\ y < p \\ \frac{y-p}{q-p}, for\ p \le y \le q \\ \frac{y-q}{r-q}, for\ q \le y \le r \\ 1, for\ y = r \\ \frac{s-y}{s-r}, for\ r \le y \le s \\ \frac{t-y}{t-s}, for\ s \le y \le t \\ 0,\ if\ y > t \end{cases}$$

This queue is based on first come first served discipline with arrival time and the service time follows a poisson and exponential distribution with the parameter λ and μ respectively, each is a fuzzy variable instead of a sharp amount.

The expected number of customer in the queue,

$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

The expected average waiting time in the queue,

$W_q = \frac{L_q}{\lambda}$

3. INTERVAL ANALYSIS FOR ARITHMETIC:

Let J_1 and J_2 be the two interval number defined by ordered pair of real numbers with lower and upper bounds.

$J_1 = [p, q], p \leq q; J_2 = [r,s], r \leq s$

The symbol $* = [+,-,\times,\div]$ define general arithmetic properties. The operation is

$J_1 * J_2 = [p,q]*[r,s]$

Another distance calculation depends on the magnitude and sign of elements p,q,r, and s.

$[p,q]+[r,s]=[p+r, q+s]$

$[p,q]-[r,s]=[p-s,q-r]$

$[p,q]*[r,s]=[\min(pr,ps,qr,qs),\max(pr,ps,qr,qs)]$

$[p,q]\div[r,s]=[p,q]*[\frac{1}{s}, \frac{1}{r}]$, [r,s] not equal to zero

$\alpha[p,q]=[\alpha p,\alpha q]$ for $\alpha>0$ and $[\alpha q,\alpha p]$ for $\alpha<0$.

4. NUMERICAL EXAMPLE:

Consider an M/M/1 queue where both the arrival and the service rate are fuzzy numbers denoted by λ=[5,6,7,8,9] and μ=[16,17,18,19,20].

The interval of aconfidence level α is [5+2α,9-2α] and [16+2α,20-2α].

$L_q = \frac{x^2}{y(y-x)}$

$W_q = \frac{L_q}{x}$

Where x=[5+2α,9-2α] and y=[16+2α,20-2α]. By taking the α values from [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]

A	L_q	L_q	W_q	W_q
0	0.0833	0.7232	0.0166	0.0803
0.1	0.9353	0.6459	0.1798	0.0733
0.2	0.1047	0.5781	0.0193	0.0294
0.3	0.1171	0.5183	0.0209	0.0617
0.4	0.1307	0.4653	0.0225	0.0567
0.5	0.1457	0.4183	0.0242	0.0522
0.6	0.1624	0.3762	0.0261	0.0482
0.7	0.1805	0.3387	0.0282	0.0445
0.8	0.2006	0.3050	0.0303	0.0412
0.9	0.2228	0.2747	0.0327	0.0381
1.0	0.2474	0.2474	0.0353	0.0353

We performed alpha cut of arrival rate, service rate and fuzzy expected number of jobs in queue at eleven distance of alpha level, (0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1). The alpha cut represents the probability that these four presentation criteria are within their respective ranges.

5. CONCLUSION:

In this paper, we investigate the M/M/1 model using pentagonal fuzzy numbers. We have obtained the membership function for the pentagonal fuzzy numbers and analyzed the interval for arithmetic. We calculated the expected number of customer in the queue and the expected average waiting time in the queue. We have obtained numerical result to all the performance measures for the fuzzy queues.

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