RESEARCH ARTICLE OPEN ACCESS

Machine Learning based Multi-Period Portfolio Optimization Approach Using Convex Optimization and ESG Risk Ratings

Vedanth Venkatesh
Senior Student, Plano West Sr. High School, Plano, TX, USA
Email: vedanth.data@gmail.com

--************************----------------------------------

Abstract:

Since Markowitz formulated portfolio selection as an optimization problem trading off risk and return, portfolio optimization has been challenging in quantitative finance. With advances in computing power, data availability, and financial models, modern optimization techniques can now incorporate practical trading considerations like costs.

This paper examines Multi-period Portfolio Optimization (MPO) using convex optimization and Machine Learning (ML) modeling with ESG Risk ratings for Socially Responsible Investing. We focus on using ESG Risk ratings as a factor to obtain optimized portfolio returns with lower risk. Our hypothesis is that a portfolio of highly ESG risk-rated companies in a sector can outperform the Dow Jones Industrial Average index. Experiments found evidence that MPO with ESG Risk ratings as factors effectively enhances returns while reducing portfolio risk. An ESG-optimized portfolio outperformed the DJIA by 5% over 10 years with 10% lower volatility.

Incorporating social responsibility ratings into portfolio optimization shows promise for achieving excess risk-adjusted returns versus the broader market. This lends credence to the hypothesis that the ML model using ESG Risk rating as factors can be used to construct optimized portfolios that beat financial benchmarks.

Keywords—**PortfolioOptimization, Dynamic Programming, Machine Learning, Convex Optimization, Multi-Period, ESG Risk Ratings, Socially Responsible Investing**

--************************----------------------------------

I. INTRODUCTION

In financial terms, a portfolio is a collection of assets/investments. Due to changing market dynamics, diversification of portfolios is a crucial mechanism used to reduce risk on investments. Since Markowitz formulated portfolio selection as a mean-variance optimization (MVO) problem trading off risk and return over sixty years ago, the MVO approach has occupied a central role in constructing portfolios in both academic literature and industry (Markowitz 1952). The reasons for its success are diverse. The model was the first to quantify the benefits of diversification towards reducing portfolio risk. Further, it simplified the portfolio selection problem by introducing the concept of an efficient frontier. On this delimiting line or frontier, we can find the portfolio with the highest return for a given level of risk.

Despite its vast success, the model has its drawbacks[2]. To arrive at a mean-variance portfolio, an optimization problem is solved for one fixed period: hours, days, months, and years. However, an investor's end goal is broader than what could be achieved by a single mean-variance portfolio. Investor cares about maximizing their wealth over their entire investment period, which could last until a significant event or purchase, their lifetime, or many generations. Superimposing one static set of returns and risk completely ignores the time-varying properties of asset prices over a long period of time.

Multi-period portfolio optimization addresses this by finding the optimal dynamic asset allocation policy over multiple time periods[3]. By accounting for factors like transaction costs, price trends, and time-varying constraints, multi-period models can potentially improve on single-period MVO. Early research into multi-period portfolio optimization includes the seminal work of Mossin (1968) and Samuelson (1969) on consumption-investment problems. Merton (1969) analysed optimal consumption and portfolio policies over an investor's lifetime. Following these foundational papers, multi-period portfolio optimization has been an active research area in financial economics and operations research.

Recently, developments in computing capacity have renewed the interest in such models. For instance, we can cite the research by Boyd et al. (2017), Corsaro et al. (2021), Huang et al. (2021), Li et al. (2022). While multi-period optimization research continues, advances in computing power, machine learning, and optimization algorithms hold promise in overcoming limitations and realizing practical benefits. For example, deep neural networks can help estimate time-varying return distributions (Gu et al. 2020), while heuristic search techniques like genetic algorithms address complexity (Anagnostopoulos & Mamanis, 2011).

Multi-period models help to find optimal adaptive policies tailored to an investor's objectives, constraints, and market views. Theoretical and empirical research has shown multi-period optimization can improve out-of-sample performance versus fixed portfolio weights (DeMiguel et al. 2009).

The aim of this study is to utilize the multi-period portfolio optimization approach, as defined by Steve Boyd in [5],[6],[7], uses dynamic programming and alsoinvestigates the effects of the ESG Risk ratings on the portfolio weight selection/allocation process and analyzes its impact in terms of annualized portfolio returns and risks.

This document is a template. An electronic copy can bedownloaded from the conference website. For questions onpaper guidelines, please contact the conference publicationscommittee as indicated on the conference website. Information about final paper submission is available from the conference website.

II. DEFINITIONS

A. Mean-Variance Optimization (MVO)

Mean-variance analysis is a method used in finance to evaluate asset risk and expected return. It looks at the trade-off between risk and return. The two main components are:

- Mean Expected Return - The investment's expected return or average return based on historical performance or forecasts.
- Variance of returns - A measure of the variability or volatility of returns. Lower variance means more stable, predictable returns.

The goal is to maximize the expected return for a given level of risk. The risk is measured by the variance of returns.

B. Convex Optimization in Machine Learning

Convex optimization plays a critical role in training machine learning models, which involves finding the optimal parameters that minimize a given loss function. Convex optimization problems can be broadly classified into Constrained and Unconstrained. Constrained convex optimization involves finding the optimal solution to a convex function subject to convex constraints. These constraints may include both equality and inequality constraints. The objective function may be subject to a constraint that requires it to lie exactly at a given point or within a specified range. An example of a constrained convex optimization problem is portfolio optimization, where the goal is to find the optimal allocation of investments subject to constraints on risk and return[6].

Convex optimization involves minimizing convex functions over convex sets. Convex functions and sets have useful mathematical properties that make these problems well-suited for efficient optimization algorithms. Specifically, local optima are guaranteed to be global optima for convex problems. This characteristic makes convex optimization invaluable for many machine-learning applications.

Gradient descent, a popular optimization technique, leverages convexity by following the negative gradient to iteratively reach an optimal solution. Beyond machine learning, convex optimization has diverse applications in fields like control systems, finance, and signal processing.

Portfolio optimization is one example where convexity ensures tractability in trading off risk versus return. Applying convex optimization requires reformulating the problem into a standard form solvable by generic algorithms[7]. This can be challenging and error-prone. Domain-specific languages (DSLs) for convex optimization simplify the process by allowing natural problem specifications and once defined, these are automatically converted into the required standard form. Examples of such DSLs include *CVX (Grant and Boyd, 2014), YALMIP (Lofberg, 2004), QCML (Chu et al., 2013), PICOS (Sagnol, 2015)*

C. Efficient Frontier

The efficient frontier is a set of portfolios that offer the highest return for a given level of risk. It's a financial tool that helps investors create investment portfolios with the best returns for a certain amountof risk. The efficient frontier is also known as the *portfolio frontier*.

The efficient frontier is a curved line because each increase in risk results in a smaller amount of returns. This curvature graphically shows the benefit of diversification and how it can improve a portfolio's risk versus reward profile.The efficient frontier was introduced by Harry Markowitz[1] in 1952. The mathematical theory to solve for an efficient frontier is straightforward. You can set the rate of return and find the minimum level of risk or set a level of risk and find the maximum rate of return.

D. Sharpe Ratio

One commonly used mean-variance metric is the Sharpe ratio. Sharpe ratio is a metric used to estimate the performance of an equity portfolio with respect to the risk-free rate of investment. Sharpe ratio is also defined as the ratio of the excess return of the portfolio to the portfolio volatility.It measures the excess return per unit of risk.

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

A higher *Sharpe ratio* for a portfolio means better risk-adjusted return for that portfolio.

E. ESG Risk Scores Framework

ESG (Environmental, Social, Governance) Risk scores are used by investors to evaluate companies on environmental, social, and governance factors. Here are some key points on ESG scoring frameworks:

- ESG scores summarize company performance on relevant ESG metrics like CO2 emissions, renewable energy use, labour practices, board diversity, etc.
- Many ESG rating agencies and data providers like *MSCI, Sustainalytics, Refinitiv, ISS ESG* provide ESG scores. Their models and underlying data utilized can vary.
- ESG scores are constructed using a combination of public disclosures, questionnaires, controversies research, proprietary models etc. Hundreds of data points may be used.
- ESG scores allow standardized comparison of company ESG performance across sectors and regions. But caution is required as scoring methods differ.

Institutional and retail Investors may use ESG scores to screen investments, integrate into analysis, engage with companies and benchmark ESG performance over time.

III. LITERATURE REVIEW

Mean-variance portfolio optimization assumes that all investors are risk averse and, hence they would prefer a high-return portfolio over a low-return one for a given level of risk. In other words, investors would always choose a low-risk portfolio over a high-risk one, for a given level of return.

Multi-period optimization (MPO) allows investors to make portfolio decisions across multiple time periods rather than optimizing narrowly over a single period. This helps address the shortcomings of traditional single-period models like mean-variance optimization (MVO). The objective for one period alone is oblivious to future constraints and return expectations. MPO incorporates inter-temporal dynamics.

For example, suppose long-term forecasts favor a large position in an asset, but near-term forecasts are pessimistic. With MPO, the solution could be gradually building the place over negative return periods. This long-term view is hard to capture in single-period optimization. Similarly, known future macroeconomic events like elections can be modelled by forecasting higher risk or adding constraints to reduce holdings ahead of time. This is preferable to sudden liquidation in one period, which risks higher transaction costs.

In addition, MPO allows directmodeling time-varying return behaviors like mean reversion. The investor gains a unified framework incorporating dynamic return forecasts, adaptive rebalancing, and future objectives. Rather than myopic period-by-period optimization, MPO finds an optimal policy mapping market states to actions over time.

Considering the random nature of financial markets, multi-period investment problems are usually solved by scenario approximations of stochastic programming models. Such programming implementations are computationally intensive and challenging.This research implements a specific case of the methods of Boyd et al. (2017), with an additional factor that considers ESG Risk ratings of individual stocks.

Multi-period portfolio optimization has been an active research area since the pioneering work of Mossin (1968), Samuelson (1969), and Merton (1969). This research has shown that short-term portfolio selection can differ considerably from long-term optimization. However, return predictions tend to revert to long-run averages for sufficiently long-term horizons as forecasting accuracy decreases. As Gârleanu and Pedersen (2013) and Boyd et al. (2017) discuss, the key becomes determining the optimal sequence of trades to execute over the following several periods.

Rather than approximating an infinite-horizon problem, looking a modest number of steps ahead seems appropriate. Return dynamics and new information make long-term forecasts unreliable. Optimizing over the following few periods balances tractability with incorporating inter-temporal effects.

Differences in short- and long-term forecasts and trading and holding costs can be adequately modelled in a multi-period framework. Multi-period optimization, naturally, leads to a dynamic strategy.

The multi-period portfolio optimization method leveraged in this paper was introduced by Boyd et al. (2017) and is based on model predictive control (MPC). The key idea is to incorporate new information by solving a finite-horizon optimization problem at each time step. Specifically, an optimization is performed at time $T$to determine the optimal policy for the following H periods. Only the actions for the immediate time $T+1$are implemented, and the process repeats at $T+1$ with updated data.

This receding horizon procedure simplifies the complete dynamic programming formulation, enabling fast adaptation to changing market conditions. MPC-based multi-period optimization has been applied to problems like portfolio selection, trade execution, and index tracking.

A vital advantage of the MPC approach is that convex programming can be leveraged, allowing the incorporation of transaction costs, risk constraints, and other real-world factors. In an empirical equity trading study, Boyd et al. (2017) demonstrates improved risk-adjusted returns versus single-period optimization. Performance bounds for finite-horizon dynamic portfolio policies can also be derived (Boyd et al. 2014). These bounds provide a benchmark to evaluate the suboptimal MPC policy. While optimality is not guaranteed, Boyd et al. show the MPC solution is typically near optimal in simulations.

Francesco Cesarone et al [13] examined a multi-objective optimization model for portfolio selection, in which they added to the classical Mean-Variance analysis a third non-financial goal represented by the ESG scores. The resulting optimization problem, formulated as a convex quadratic programming, consists of minimizing the portfolio variance with parametric lower bounds on the portfolio's expected return and ESG levels. Reference [13] describes analysis of multiple Mean Variance - ESG portfolios between 2006 and 2020, observations were made that only in the S&P500 and Dow Jones datasets, the most sustainable portfolio strategies show better financial performances than European FTSE stocks.

This paper examines the effects of different planning horizons H on portfolio performance. Though the MPC formulation considers future periods, there is no consensus on the ideal horizon. We evaluated horizons of one, two, and five periods using the MPC approach to provide insights into this modelling choice and observed few differences in results. Hence the time horizon of *two* was chosen for the experiments to optimize computation time.

This paper does not address a critical component in a trading algorithm- the projections or forecasts of future quantities. The methods described in Stephen Boyd [7] can be considered good ways to exploit predictions, no matter how they are made.

IV. DATA AND SOFTWARE LIBRARIES

F. Stock Prices Data

Historical stock data used in this article were downloaded from Yahoo Finance, which includes US-listed stocks. We selected the closing prices of these US stock assets for 10 years and calculated the monthly returns as a preliminary data treatment. Furthermore, we considered predominantly Dow 30 stocks as the universe for our experiments to meet the diversified portfolio requirement. We have included stocks with observations from 2010 to 2019, intentionally omitting 2020-2022 due to COVID-19 pandemic market impacts.

G. ESG Ratings Data

For ESG Ratings, we primarily used data from popular ESG Ratings sourced from publicly available **MSCI** and **Morningstar**® Sustainalytics ESG Risk Ratings for stocks.

H. Software Libraries

For this research and experimentation, we used the following critical libraries outside of other standard Python libraries for Data Science, ML, and Data visualization purposes.

H.A.1 CVXPY

CVXPY [18] is a Python programming library and new DSL for convex optimization. It is based on CVX (Grant and Boyd, 2014) but introduces new features such as signed disciplined convex programming analysis and parameters. CVXPY makes it easy to combine convex optimization with high-level features of Python such as parallelism and object-oriented approach to constructing optimization problems.[16]

H.A.2 CVXPORTFOLIO

CVXPORTFOLIO[9] is a Python programming library for portfolio optimization and available as GNU open source from Stanford University. It lets users quickly try optimization policies for financial portfolios by testing their past performance with a sophisticated market simulator.

This python package provides an object-oriented framework with classes representing return, risk measures, transaction costs, holding constraints, trading constraints, etc.Single-period and multi-period optimization models are constructed from instances of these classes. Each instance generates CVXPY expressions and conditions for any given period t, making combining the cases into a single convex model easy.

H.A.3 PyPortfolioOpt

PyPortfolioOpt is a library that implements portfolio optimization methods, including classical efficient frontier techniques, Black-Litterman allocation, andexponentially weighted covariance matrices. We used this library to calculate this research's MVO-based returns and weights[17].

V. MODEL AND METHODOLOGY

For this research paper and associated experiments, we leveraged the model and multiperiod portfolio methodology described by Stephen Boyd et al. [7] - *Section 2 and Section 5.*

In the absence of transaction costs while doing portfolio optimizations, a greedy strategy that only optimizes one period at a time is optimal. However, with transaction costs, current holdings affect whether a return prediction can be profitably acted on. We should consider if recent trades put us in a good position for future periods. While this idea can be incorporated into single-period optimization, it is more naturally handled with multi-period optimization. For example, single period optimization may recommend going very long in an illiquid asset, which could be costly to unwind later. Multi-period optimization better accounts for these transaction costs.

In addition, Multi-period optimization handles conflicting return predictions on different timescales better than single-period. If a short-term forecast is optimistic but long-term is damaging, single-period analysis would just average them, possibly missing the optimal action. With multi-period, if trading costs are high, no action is best since any trade would be reversed per the long-term view. If prices are low, follow the short-term view since unwinding is cheap. Multi-period optimization naturally captures the right actions based on timescale and costs.

In multi-period optimization, we choose the current trade vector z_t by solving an optimization problem over a planning horizon that extends H periods into the future:

$$t, t+1, \dots, t+H-1$$

We can develop a multi-period optimization problem: Let $z_t, \dots, z_{t+H-1}$ denote sequence of planned trades over the horizon and $w_t, w_{t+1}, \ldots, w_{t+H}$ time horizon.With the dynamics simplification, we arrive at the multi period portfolio problem, the below equation 5.1 from *Section 5*of [7]:

$$\begin{array}{ll} \text{maximize} & \sum_{\tau=t}^{t+H-1} \Big(\hat{r}_{\tau|t}^T (w_\tau + z_\tau) - \gamma_\tau \psi_\tau (w_\tau + z_\tau) \\ & \qquad - \hat{\phi}_\tau^{\text{hold}} (w_\tau + z_\tau) - \hat{\phi}_\tau^{\text{trade}} (z_\tau) \Big) \\ \text{subject to} & \mathbf{1}^T z_\tau = 0, \quad z_\tau \in \mathcal{Z}_\tau, \quad w_\tau + z_\tau \in \mathcal{W}_\tau, \\ & w_{\tau+1} = w_\tau + z_\tau, \quad \tau = t, \ldots, t+H-1, \end{array}$$

Fig. 1: Multi period portfolio model equation [7]

This simplified dynamic equation is a convex optimization problem, provided the transaction cost, holding cost, risk functions, and trading and holding constraints are all convex.

I. Constraints

I.A.1 Stock Holding Constraints

Holding constraints restrict the choice of normalized post-trade portfolio $\omega_t + z_t$. Having constraints may be surrogates for constraints on wt+1, which we cannot constrain directly since it depends on the unknown returns. Usually returns are small and ω_{t+1} is close to $\omega_t + z_t$so constraints on $\omega_t + z_t$are good approximations for constraints on ω_{t+1} . Some constraints always hold exactly precisely when they hold for $\omega_t + z_t$ .Holding constraints may be mandatory, imposed by law or the investor, or discretionary, included to avoid specific portfolios.

We used research-related holdingconstraints for this experiment. This constraint requires only long asset positions, $\omega_t + z_t \geqslant 0$, are heldif only the assets must be long, extended becomes $(\omega_t + z_t)$1:n ≥ 0. When a long-only constraint is imposed on the post-trade weight $\omega_t + z_t$, it automatically holds on to the next period value (1 + rt) ◦ (h_t + z_t), since 1 + $r_t \geq 0$. Such a portfolio is long-only when the asset holdings are all nonnegative, i.e., $(h_t)\_{i \geq 0}$ for i = 1, . . . ,n" This constraint ignores the cash, which is $n{+}1^{th}$ term of h_t.

I.A.2 Risk models

We used aresearchrelatedexperiment Full covariance matrix and fitting from the past data for this research-related experiment. We used a simple factor risk model estimated from past realized returns. We calculated it on the first day of each month and used it for the rest.

I.A.3 Cost Models

A reasonable model for the scalar transaction cost functions is:

$$x \to a\,|x| + b\sigma\,\frac{|x|^{3/2}}{V^{1/2}} + cx$$

where *a, b, σ, V,* and *c* are real numbers, and x is a dollar trade amount. The constants in the transaction cost model vary with asset and trading period, i.e., they are indexed by *i*and *t*. This *3/2* power transaction cost model is widely known and employed by practitioners.

I.A.4 Trading Frequency

We used *Monthly* as the trading frequency of portfolio stocks and simulated the portfolio performance to mimic an investment professional. Experiments can be done with *Quarterly* or *Annual* trading frequency to mimic an average retail investor.

I.A.5 Returns Forecast

We used standard Returns forecast for non-cash assetsreturns' forecasts $\hat{r}_t$ are the total average of past returns at each point in time and the risk model is the full covariance, also computed from the past returns.

I.A.6 Planning Horizon

This approach determines the steps in the future we plan for as horizon. As stated, [10], the longer horizon values did not yield better results. Hence,weused a horizon of'2' value for this research experiments.

I.A.7 Solver

For this exercise, we use the default generic solver Embedded Conic Solver (ECOS) as supported by CVXPY library. ECOS is a numerical software for solving convex second-order cone programs (SOCPs).

J. Hyperparameters

In summary, the simulations were carried out using the market data with parameters described below:

- Period: 10 years, from January 2010 through December 2019.
- Assets: the components of the Dow 30 index (collected open-source market data from Yahoo Finance);
- Risk-free rate/cash return: use the federal reserve overnight rate;
- Bid-ask spread: a_t = 0.05%;

- Holding costs: $s_t = 0.01\%$;
- Other parameters: $b_t = 1$, $c_t = 0$, $d_t = 0$

VI. EMPIRICAL ANALYSIS

As of July 2023, here are the Dow 30 stocks and their corresponding weights for reference purposes:

TABLE 1 STOCK COMPONENTSOF THE DOW JONES INDEX

As of July 31, 2023					**ESG Rating Source: MSCI**
#	**Company**	**Sector**	**Symbol**	**Weight (%)**	
1	United Health Group Inc	Healthcare	UNH	9.44864	Leader
2	Goldman Sachs Group Inc	Financial	GS	6.609519	Average
3	Home Depot Inc	Retail	HD	6.173264	Leader
4	Microsoft Corp	Technology	MSFT	6.113562	Leader
5	McDonald's	Food	MCD	5.446546	Average
6	Caterpillar Inc	Industrials	CAT	5.26744	Average
7	Visa Inc Class A Shares	Financial	V	4.468481	Average
8	Boeing Co	Manufacturing	BA	4.329987	Average
9	Amgen Inc	Pharma	AMGN	4.317635	Leader
10	Salesforce Inc	Technology	CRM	4.034285	Leader
11	Apple Inc	Technology	AAPL	3.577817	Average
12	Honeywell International Inc	Industrial	HON	3.565465	Leader
13	Johnson & Johnson	Pharma	JNJ	3.19359	Average
14	Travelers Cos Inc	Insurance	TRV	3.170383	Average
15	American Express Co	Financial	AXP	3.114424	Leader
16	Chevron Corp	Oil & Gas	CVX	2.988283	Average
17	Walmart Inc	Retail	WMT	2.980609	Average
18	Procter & Gamble Co	Consumer	PG	2.938313	Average
19	JPMorgan Chase & Co	Financial	JPM	2.926148	Average
20	Intl Business Machines Corp	Technology	IBM	2.703435	Leader
21	Nike Inc Cl B	Retail	NKE	2.033237	Average
22	3m Co W/d	Industrial	MMM	2.0061	Leader
23	Merck & Co. Inc.	Pharma	MRK	1.978776	Average
24	Walt Disney Co	Media	DIS	1.599977	Average
25	Coca Cola Co	Food	KO	1.153615	Leader
26	Dow Inc	Chemicals	DOW	1.027474	Leader
27	Cisco Systems Inc	Technology	CSCO	0.994722	Leader
28	Intel Corp	Technology	INTC	0.650265	Leader
29	Verizon Communications Inc	Telecom	VZ	0.61873	Leader
30	Walgreens Boots Alliance Inc	Retail	WBA	0.567263	Leader

We assumed these 30 stocks were part of Dow 30 index from 2010 onwards purely for experiment and simplification purposes.Reviewing the total returns of two popular indices in the US Stock markets, the average annualized returns (Table 2) is approximately ~12% for the last ten years, as of July30th 2023.

TABLE 2 ANNUALISED RETURNS OF POPULAR US INDEXES

	Annualized Returns (%)			
	1 Year	3 Year	5 Year	10 Year
Total Return S&P 500	**13.02**	**13.72**	**12.20**	**12.66**
Total Return Dow Jones Industrial Average	**10.62**	**12.65**	**9.30**	**11.19**

We considered multiple portfolios made of 2023 Dow 30large cap stocks (Table 1) for our experiments. The key reason for this decision was that all Dow 30 stocks are well-established, diversified, large corporations with long track records in their respective industries. Typically, investors consider these types of companies when they want to invest to reduce risk and maximize returns. Also, these 30 companies have MSCI ESG Rating of either *LEADER* or *AVERAGE* values.

K. Exploratory Data Analysis

Before defining our experiment portfolios, we analysed the Dow 30 stocks using the MVO technique. Using the *pxpyportfolio*Python library, we generated charts on Adjusted Close price of all Dow 30 stocks between 2010-2019.

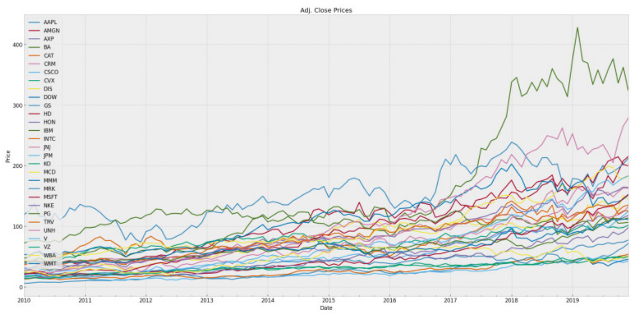

Figure 2a Adjusted Close Price chart for Dow 30 stocks over 2010-2019 period

To understand the stock price movements better, Figure 2b is a zoomed version of Figure 2a for the period Jan 2016 - Dec 2019.

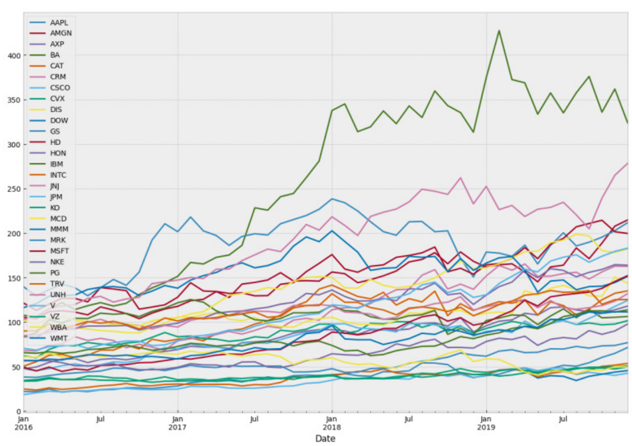

Figure 2b Adjusted Close Price chart for Dow 30 stocks over 2010-2019

From this chart, we can observe that except PG and JNJ, all other stocks had mixed returns year on year between 2016 - 2019. This chart also confirms that the Dow 30 set of storescontains all blue-chip stocks from diverse sectors to reduce risks, if a portfolio is constructed from this universe.

In mean-variance optimization (MVO), the variance-covariance matrix is used with expected returns to determine the optimal asset allocation. It helps identify the asset mix that maximizes returns for a given level of risk or minimizes risk for a desired level of returns. By quantifying risk contributions and considering correlations between assets, the covariance matrix helps assess diversification benefits.

Most stock prices were independent when we generated the covariance matrices (full and semi) for Dow 30 stocks with Adjusted Closing Price values as shown in Figure 3a and 3b.

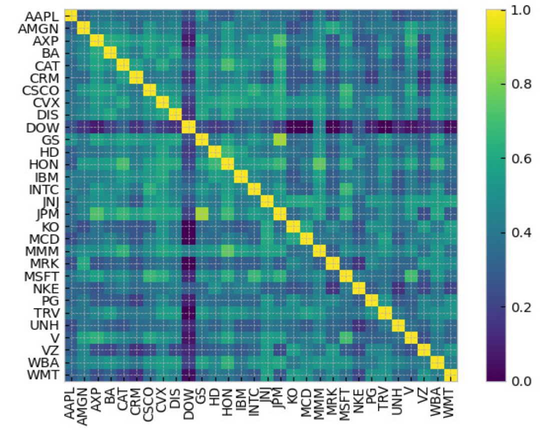

Figure 3a Full Covariance matrix for Dow 30 stocks over 2010-2019 period

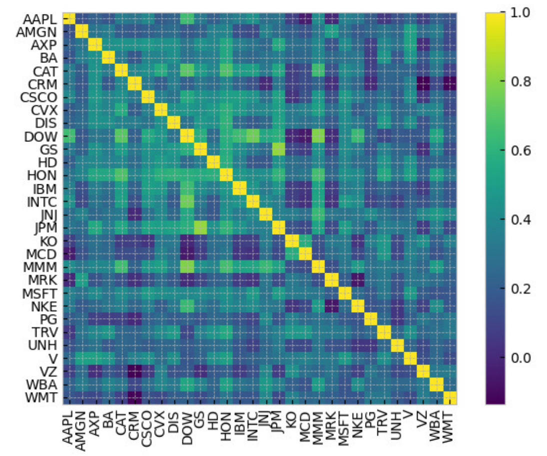

Figure 3b Semi Covariance matrices for Dow 30 stocks over 2010-2019 period

Overall, the variance-covariance matrix confirms that Dow 30 stocks meets diversification objective of portfolio optimization and a good sample of supplies for our analysis. Its utilization assists in constructing efficient portfolios that balance risk and return based on the characteristics and interactions of the underlying assets.

After the covariance matrix generation, we computed the mean returns using the CAPM return method and then calculated the weights using the efficient frontier (EF) module. The EF approach considers the returns and volatility (standard deviation) of each stock in the portfolio. We calculate the Maximum Sharpe ratio and associated weights by balancing higher returns with lower risk.

After finding out the weights, we plotted our portfolio's efficient frontier and various computed shape ratios. Ideally, the portfolio's max Sharpe ratio for a given risk value must fall on the efficient frontier line. We considered a sample of 100,000 different portfolio weights and shape proportions to build an efficient frontier and then computed a risk-return scatter plot.

Each dot on this Figure 4 represents a different possible portfolio, with darker blue corresponding to 'better' portfolios (regarding the Sharpe Ratio). The dotted black line is the efficient frontier itself. The red cross marker represents the portfolio with max Sharpe ratio.

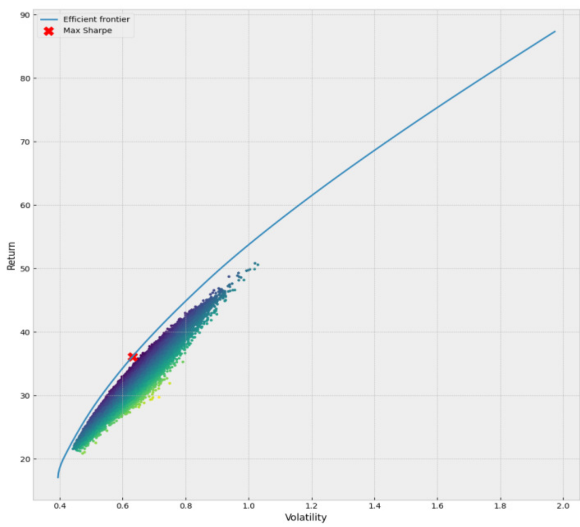

Figure 4 Efficient Frontier Graph for Dow 30 stocks over 2010-2019 period

The Sharpe ratio is the portfolio's return over the risk-free rate, per unit risk (volatility). It is essential because it measures the portfolio returns, adjusted for risk. In practice, rather than trying to minimize volatility for a given target return (as per Markowitz), it often makes more sense to find the portfolio that maximizes the Sharpe ratio.

L. Optimized Portfolio Definitions

Next, we used the Dow 30 stocks universe to create 6 diverse stock portfolios, as listed in Table 3. We utilized the Multi-period Convex Optimization techniques described by Stephen Boyd, Enzo Busseti, and Steven Diamond [7] to determine the optimum weights for these portfolio stocks with maximum annualized returns.

TABLE 3 COMPOSITION OF PORTFOLIOS

Portfolio No.	**Portfolio Name**	**Public Company Stock Ticker Symbols**
1	Dow 30 Optimized	'MMM, ' 'AXP', 'AMGN', 'AAPL', 'BA', 'CAT', 'CVX', 'CSCO', 'KO', 'DIS','DOW', 'GS', 'HD', 'HON', 'IBM','INTC', 'JNJ', 'JPM', 'MCD', 'MRK', 'MSFT', 'NKE', 'PG', 'CRM', 'TRV', 'UNH', 'VZ', 'V', 'WBA', 'WMT'
2	Dow 30 Optimized ESG Leaders Only - 15 stocks	'AMGN', 'AXP', 'CRM', 'HD', 'HON', 'DOW', 'IBM', 'INTC', 'KO', 'MMM', 'MRK', 'MSFT', 'UNH', 'VZ', 'CSCO'
3	Dow 30 Optimized ESG Average Only - 15 stocks	'AAPL', 'BA', 'CAT', 'CVX', 'DIS', 'GS', 'JNJ', 'JPM', 'MCD', 'NKE', 'PG', 'TRV', 'V', 'WBA', 'WMT'
4	Dow 30 Optimized ESG Leaders with ESG Risk Rating Factors - 15 stocks	'AMGN', 'AXP', 'CRM', 'HD', 'HON', 'DOW', 'IBM', 'INTC', 'KO', 'MMM', 'MRK', 'MSFT', 'UNH', 'VZ', 'CSCO'
5	Dow 30 Optimized ESG Leaders with ESG Risk Rating Factors - 15 stocks	'AAPL', 'BA', 'CAT', 'CVX', 'DIS', 'GS', 'JNJ', 'JPM', 'MCD', 'NKE', 'PG', 'TRV', 'V', 'WBA','WMT'
6	Dow 30 Optimized ESG Average Per Employee Revenue > USD 500K - 5 stocks only	'AAPL', 'CVX', 'GS','TRV','V'

Using *CVXPORTFOLIO* Python library and building multi period portfolio model with hyperparameters as specified in Section 6, we generated and compared the Annualized returns for each portfolio.

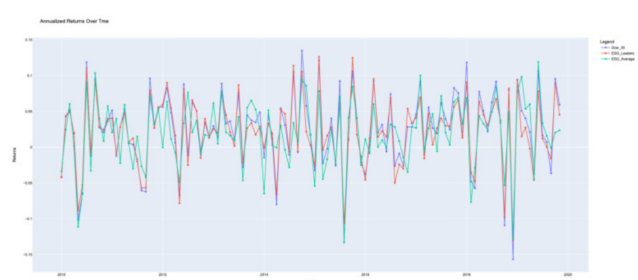
Figure 4 : Annualized Returns over time for Portfolio 1,2,3

Figure 5 : Annualized Returns over time for Portfolio 4,5,6

In Figure 6,displays the distribution of various stocks of Portfolio 1 in terms of weight allocation over a time period in consideration. MPO model distributes the weight allocation based on the model and some of the stocks do not get any allocation to reduce the risks.

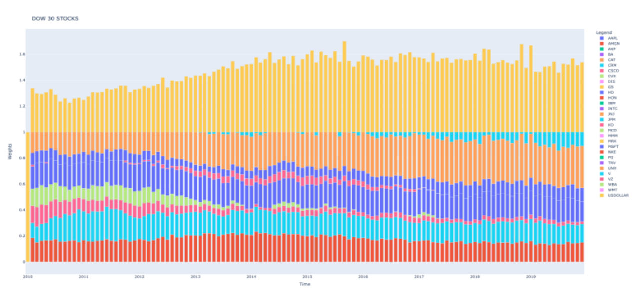

Figure 6 Weight distribution ofDow 30 stocks over time 2010-2019 withmonthly trade frequency for Portfolio 1

VII. Results

Table 4 shows Multi-period Portfolio Optimization using Convex Optimization technique-basedresults: Portfolio 4 shows improvements in annualized returnand a comparable Sharpe Ratio with Portfolio 1, utilizing ESG Risk ratings as afactor.

TABLE 4 COMPARISON OF PORTFOLIO RESULTS

Period: Jan 2010 - Dec 2019						
Port. No.	**Portfolio Name**	**Annualized Excess Returns (%)**	**Annualized Excess Risk (%)**	**Sharpe Ratio**	**Annualized Turnover (%)**	**Average of Stocks Holding Cost per period (bp)**
1	Dow 30 Optimized Portfolio Stocks: 30	29.73	18.633	**1.570**	45.091	-2.062
2	Dow 30 Optimized ESG Leaders Only Stocks: 15	26.46	16.911	1.536	**29.014**	**-1.443**
3	Dow 30 Optimized ESG Average Only Stocks: 15	23.18	16.235	1.397	46.514	-1.847
4	Dow 30 Optimized ESG Leaders with ESG Risk Factors Stocks: 15	**30.24**	19.333	1.539	36.837	-2.393
5	Dow 30 Optimized ESG Average with ESG Risk Factors Stocks: 15	26.08	18.476	1.385	191.910	-2.689
6	Select Dow Index Optimized Portfolio with ESG Average + High Average Per Employee Revenue: > USD 500K Stocks: 6	20.17	**14.841**	1.324	122.944	**-0.407**

From the above Table 4, based on our experiments, we observed the following:

- Portfolio 1 with all Dow 30 stocks performed well with ~29% with higher stocks holding cost of ~2%. Sharpe Ratio of this portfolio is reasonably high, in the range of 1.57.
- Portfolio 2 with 15 ESG Rating of "Leaders" returned less than Portfolio 1. However, the annualized turnover of the stocks is significantly reduced with a sound reduction of the Average of Stocks Holding Cost as well.
- Portfolio 3 with15 ESG Rating of "Average" returned less than Portfolio 2. The annualized turnover of the stocks is not significantly reduced, with a slight reduction of the Average of Stocks Holding Cost when compared to Portfolio 1.
- Portfolio 4 with 15 stocks filtered with ESG "Leaders" rank plus ESG Risk Rating value used as factors in stock portfolio weights produced a relatively higher return with somewhat higher holding cost of -2.39%.
- Portfolio 5 with 15 stocks filtered with ESG "Average" rank plus ESG Risk Rating value used as factors in stock portfolio weights produced a relatively comparable return with Portfolio 2. This portfolio has the highest holding cost of -2.69%.
- Portfolio 6 with select 5 Dow Index stocks with ESG "Average" Rating across 3 different sectors and "higher average revenue per employee" yielded relatively less annualized returns. However, Annualized Risk and Holdings costs

were lower than other portfolios and shows a new dimension for optimization.

All 6 portfolio models beat the popular US indices annualized returns we quoted in Section 7 and two out of six portfolios resulted in ~30% Annualized returns with an excellent *Sharpe Ratio* of 1.5 and above.

Given the experiments were conducted using synthetic ESG Risk Ratings scores and assumed constant ratings for the entire period as one of the factor constraints, these results are encouraging the use more factors in MPO Multi-period portfolio optimization models.

As evident in Figure 5, the swings in the AVERAGE-rated ESG stock portfolio's monthly returns are more significant than ESG LEADER-rated stock portfolio. More swings would mean increased risk and reduced returns as implied in Table 3.

VIII. FUTURE ENHANCEMENTS

This research can be further continued by extending the stocks universe to NASDAQ,and S&P 500 universe with a focus on small and mid-caps and various industry segments. Similarly, the research can be extended to non-US financial markets.

For our experiments, considering the typical investor profile, we used trading frequency as “Monthly” and validated with “Quarterly” and “Annual” frequency-based portfolio returns also. In the real world, considering an investment manager as a persona for these portfolio optimizations, researchers can use Daily as trading frequency and compare the portfolio returns and risk results.

Similarly, for this research assumed starting cash of USD 1 Million and no new money was added to the portfolio. Instead, we could add external cash periodically to the portfolio and compare the returns. Another option is to take additional risk by shorting stocks and going negative on US Cash equivalents.

Researchers used Embedded Conic Solver (ECOS) to find an efficient frontier. Other popular solvers include GLPK MI, CBC, SCIP as supported by CVXPY[16], and these can be used to run the same portfolio optimizations to see the returns gain or loss.Researchers used synthetic ESG Risk Ratings in a data frame. We kept it as constant over the time horizon, primarily to simulate an investor interest to use a company's current ESG Risk rating. This can be easily replaced with real historical ESG Risk Ratings and see the weight change and impact on portfolio returns.

Lastly, we can create a Machine learning model out of this and apply Deep learning techniques for forecasting future portfolio weights [4].

IX. CONCLUSION

As described in [14], investors' financial and investment gain motivations motivate companies to take their Environmental, Social, and Governance objectives more seriouslyto improve or maintain the ESG Risk ratings. At the same time, Socially Responsible Investing is a growing area in the investment management industry. In this research paper, we applied a multi-period portfolio optimization model with ESG-rated stocks, where we enhanced the classical single-period Mean-Variance optimization with a third non-financial goal represented by the ESG score and Risk ratings. The resulting tri-objective optimization problem was formulated as a convex Quadratic Programming (QP). It consisted of minimizing the portfolio variance with parametric lower bounds on the portfolio's expected return levels and the ESG portfolio. We then provided an extensive empirical analysis from 2010–2019 using real-world datasets from the US stock market. To better examine the ESG impact on portfolio performance and to capture possible effects of SRI regulatory developments over the past 10+ years, we also take a select diversified portfolio of technology and financial services Dow 30 stocks to compare with the returns of Dow 30 portfolio stocks.

Our study shows empirical evidence of slightly higher returns and reduced risk on ESG Leader-rated stock portfolios. All our portfolio models beat the popular US indices annualized returns, and two portfolios resulted in ~30% Annualized returns with an excellent Sharpe Ratio of 1.5 and above.

With more experiments and back testing, more empirical evidence can bring further focus on the ESG Risk rating framework methodology and improve the confidence issues concerning ESG rating and SRI investing.

REFERENCES

[1] Markowitz Harry. Portfolio Selection. *The Journal of Finance*, 1952,paper

[2] Zhou. The stock portfolio analysis, based on the mean-variance model. *Economic Research Tribune,* 2022 paper,p 85-88

[3] Multi-Period Portfolio Optimization, by Edmond Lezmi and Thierry Roncalli, Jiali Xu[Online]. Available: http://www.thierryroncalli.com/download/Multi_Period_Portfolio_Optimization.pdf
[4] Perrin, S., and Roncalli, T., Machine Learning for Asset Management: New Developments and Financial Applications, Chapter 8, Wiley, p 261-328, 2020
[5] Multi-Period Portfolio Optimization with Constraints and Transaction Costs, Joelle Skaf and Stephen Boyd, [Online]. Available: https://web.stanford.edu/~boyd/papers/pdf/dyn_port_opt.pdf
Convex Optimization - Stephen Boyd and Lieven Vandenberghe, [Online]. Available: https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf
[6] Multi-Period Trading via Convex Optimization (Foundations and Trends in Optimization), by Stephen Boyd, Enzo Busseti, Steven Diamond, [Online]. Available: https://stanford.edu/~boyd/papers/pdf/cvx_portfolio.pdf
[7] Multi-Period Portfolio Optimization with Investor Views under Regime Switching [Online]. Available: https://www.mdpi.com/1911-8074/14/1/3
[8] Software: Stanford University Convex Optimization Group - Python libraries on Convex Portfolio Optimization[Online]. Available: https://www.cvxportfolio.com/en/latest/
[9] CVXPY: A Python-Embedded Modelling Language for Convex Optimization by Steven Diamond and Stephen Boyd, Dept of CS, Stanford University[Online]. Available: https://web.stanford.edu/~boyd/papers/pdf/cvxpy_paper.pdf
[10] Tutorial on Python-embedded modelling library for convex optimization problems[Online]. Available: https://arxiv.org/abs/1705.00109
[11] Does ESG Impact Really Enhance Portfolio Profitability? Francesco Cesarone, Manuel Luis Martino and Alessandra Carleo[Online]. Available:https://www.mdpi.com/2071-1050/14/4/2050
[12] A new method for mean-variance portfolio optimization with cardinality constraints,FrancescoCesarone, AndreaScozzari, FabioTardella, Ann Oper Res 205, 213–234 (2013). [Online]. Available:
[13] https://doi.org/10.1007/s10479-012-1165-7
[14] Is the ESG portfolio less turbulent than a market benchmark portfolio, Abdessamad Ouchen, 2021, Springer[Online]. Available: https://link.springer.com/article/10.1057/s41283-021-00077-4
[15] The impact of environmental, social and governance score on shareholder wealth: A new dimension in investment philosophy, Abhishek Parikh, Divya Kumari, Maria Johann, Dušan Mladenovic,2023 [Online]. Available: https://doi.org/10.1016/j.clrc.2023.100101
[16] Advanced CVXPY Solvers: GLPK MI,CBC, SCIP,[Online]. Available: https://www.cvxpy.org/tutorial/advanced/index.html
[17] Software: PyPortfolioOpt – A Python library implementing Mean-Variance Optimization[Online]. Available: https://pyportfolioopt.readthedocs.io/en/latest/
[18] Software: CVXPY: A Python Library for Convex Optimization https://www.cvxpy.org/tutorial/index.html

