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Fekete Szego Coefficient Inequality for Certain New Subclass Of Analytic Functions

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ABSTRACT:

We have introduced subclasses of analytic functions and have obtained sharp upper bounds of the Fekete Szego functional $|a_3 - \mu a_2^2|$ for the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ belonging to these classes and subclasses.

KEYWORDS: Univalent functions, Starlike functions, Close to convex functions and bounded functions.

MATHEMATICS SUBJECT CLASSIFICATION: 30C50

1. Introduction : Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

analytic in the unit disc given by $\mathbb{E} = \{z: |z| < 1|\}$. Let $\boldsymbol{S}$ be the class of analytic functions of the form (1.1), which are univalent in $\mathbb{E}$.

In 1916, Bieber Bach ([1], [2]) proved that $|a_2| \leq 2$ for the functions $f(z) \in \boldsymbol{S}$. In 1923, Löwner [10] proved that $|a_3| \leq 3$ for the functions $f(z) \in \boldsymbol{S}$..

With the known estimates $|a_2| \leq 2$and $|a_3| \leq 3$, it was natural to seek some relation between a_3 and ${a_2}^2$ for the class $\boldsymbol{S}$,Fekete and Szegö [4] used Löwner's method to prove the following well known result for the class $\boldsymbol{S}$.

Let $f(z) \in \boldsymbol{S}$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if\ \mu \leq 0; \\ 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right), if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1. \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes $\boldsymbol{S}$([3], [9]).

Let us define some subclasses of $\boldsymbol{S}$.

We denote by S*, the class of univalent starlike functions

$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$ and satisfying the condition

$$Re\left(\frac{zg(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.3}$$

We denote by $\mathcal{K}$, the class of univalent convex functions

$$h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in \mathcal{A}$$

and satisfying the condition

$$Re\frac{((zh'(z))}{h'(z)} > 0, z \in \mathbb{E}. \tag{1.4}$$

A function $f(z) \in \mathcal{A}$ is said to be close to convex if there exists $g(z) \in S^*$ such that

$$Re\left(\frac{zf'(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.5}$$

The class of close to convex functions is denoted by $\mathfrak{C}$ and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

$$S^*(A,B) = \left\{f(z) \in \mathcal{A}; \frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz}, -1 \le B < A \le 1, z \in \mathbb{E}\right\} \tag{1.6}$$

$$\mathcal{K}(A,B) = \left\{f(z) \in \mathcal{A}; \frac{\left(zf'(z)\right)'}{f'(z)} \prec \frac{1+Az}{1+Bz}, -1 \le B < A \le 1, z \in \mathbb{E}\right\} \tag{1.7}$$

It is obvious that $S^*(A,B)$ is a subclass of S^* and $\mathcal{K}(A,B)$ is a subclass of $\mathcal{K}$.

Several authors studied and introduced various classes and subclasses of univalent analytic functions and established Fekete Szego inequality for the same. ([3]-[9], [12]-15], [22]-[62])

N. Kaur [11] introduced a new subclass as

$$S^*(f,f',\alpha,\beta) = \left\{f(z) \in \mathcal{A}; (1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta} \prec \frac{1+z}{1-z}; z \in \mathbb{E}\right\}$$

and have established its coefficient inequality.

We will deal with the subclass of $S^*(f,f',\alpha,\beta)$ defined as follows in the present paper:

$$S^*(f,f',\alpha,\beta,A,B) = \left\{f(z) \in \mathcal{A}; (1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta} \prec \frac{1+Az}{1+Bz}; z \in \mathbb{E}\right\} \tag{1.8}$$

We will deal with the subclass $S^*(f,f',\alpha,\beta,\delta)$ defined as follows in our next paper:

$$S^*(f, f', \alpha, \beta, \delta) = \left\{ f(z) \in \mathcal{A}; (1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta} \prec \left(\frac{1+z}{1-z}\right)^{\delta}; z \in \mathbb{E} \right\} \tag{1.9}$$

Symbol $\prec$ stands for subordination, which we define as follows:

Principle of Subordination: Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to F(z) in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = F(w(z))$; $z \in \mathbb{E}$ and we write $f(z) \prec F(z)$.

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form $w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1.$ (1.10)

It is known that $|d_1| \leq 1, |d_2| \leq 1 - |d_1|^2.$ (1.11)

2. PRELIMINARY LEMMAS:

For $0 < c < 1$, we write $w(z) = \left(\frac{c+z}{1+cz}\right)$ so that

$$\frac{1+w(z)}{1-w(z)} = 1 + 2cz + 2z^2 + \cdots. \tag{2.1}$$

3. MAIN RESULTS

THEOREM 3.1: Let $f(z) \in S^*(f, f', \alpha, \beta, A, B)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2} - \dfrac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu, \\ if\, \mu \leq \dfrac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}; & (3.1) \\ \dfrac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)} \\ if\, \dfrac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)} \leq \mu \leq \\ \dfrac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}; & (3.2) \\ \dfrac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu - \dfrac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}, \\ if\, \mu \geq \dfrac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} & (3.3) \end{cases}$$

The results are sharp.

Proof: By definition of $S^*(f, f', \alpha, \beta, A, B)$, we have

$$(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \alpha\left(\frac{\left(zf'(z)\right)'}{f'(z)}\right)^{1-\beta} = \frac{1+Aw(z)}{1+Bw(z)}; w(z) \in \mathcal{U}. \tag{3.4}$$

Expanding the series (3.4), we get

$$(1-\alpha)\left\{1+\beta a_2 z+(2\beta a_3+\frac{\beta(\beta-3)}{2}a_2^2)z^2+---\right\}+\alpha\{1+2(1-\beta)a_2 z+2(1-\beta)(3a_3-(\beta+2)a_2^2)z^2+---\}=(1+(A-B)c_1 z+(A-B)(c_2-B{c_1}^2)z^2+---). \quad (3.5)$$

Identifying terms in (3.5), we get

$$a_2=\frac{(A-B)}{(1-\alpha)\beta+2\alpha(1-\beta)}c_1 \quad (3.6)$$

$$a_3=\frac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)}c_2+\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{c_1}^2. \quad (3.7)$$

From (3.6) and (3.7), we obtain

$$a_3-\mu a_2^2=\frac{(A-B)}{2(3\alpha+\beta-4\alpha\beta)}c_2+\left[\frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}-\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu\right]c_1^2. \quad (3.8)$$

Taking absolute value, (3.8) can be rewritten as

$$|a_3-\mu a_2^2|\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}|c_2|+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right||c_1^2|. \quad (3.9)$$

Using (1.9) in (3.9), we get

$$|a_3-\mu a_2^2|\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}(1-|c_1|^2)+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right||c_1^2$$

$$=\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\left|\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}-\mu\right|-\frac{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(A-B)(3\alpha+\beta-4\alpha\beta)}\right]|c_1|^2. \quad (3.10)$$

Case I: $\mu\le\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$.

(3.10) can be rewritten as

$$|a_3-\mu a_2^2\le\frac{(A-B)}{3\alpha+\beta-4\alpha\beta}+\frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}-\mu\right]|c_1|^2.$$

(3.11)

Subcase I (a): $\mu \leq \frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}$.

Using (1.9), (3.11) becomes

$$|a_3-\mu a_2^2| \leq \frac{(A-B)^2(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2} - \frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu \qquad (3.12)$$

Subcase I (b): $\mu \geq \frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)}$.

We obtain from (3.11)

$$|a_3-\mu a_2^2 \leq \frac{(A-B)}{3\alpha+\beta-4\alpha\beta}. \qquad (3.13)$$

Case II: $\mu \geq \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}$

Preceding as in case I, we get

$$|a_3-\mu a_2^2 \leq \frac{1}{3\alpha+\beta-4\alpha\beta} + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\mu - \frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\right]|c_1|^2.$$
(3.14)

Subcase II (a): $\mu \leq \frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}$

(3.14) takes the form

$$|a_3-\mu a_2^2 \leq \frac{(A-B)}{3\alpha+\beta-4\alpha\beta} \qquad (3.15)$$

Combining subcase I (b) and subcase II (a), we obtain

$$|a_3-\mu a_2^2| \leq \frac{(A-B)}{3\alpha+\beta-4\alpha\beta}\, if\, \frac{(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta-4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{(3\alpha+\beta-4\alpha\beta)} \leq \mu \leq \frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} \qquad (3.16)$$

Subcase II (b): $\mu \geq \frac{4\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2-(A-B)8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}$

Preceding as in subcase I (a), we get

$$|a_3-\mu a_2^2| \le \frac{(A-B)^2}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\mu - \frac{(A-B)^2\left(8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta\right)}{4(3\alpha+\beta-4\alpha\beta)\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2} \tag{3.17}$$

Combining (3.12), (3.16) and (3.17), the theorem is proved.

Corollary 3.2: Putting $\alpha = 1, \beta = 0, A = 1, B = -1$ in the theorem, we get

$$|a_3-\mu a_2^2| \le \begin{cases} 1-\mu, if \mu \le 1; \\ \frac{1}{3} if 1 \le \mu \le \frac{4}{3}; \\ \mu - 1, if\ \mu \ge \frac{4}{3} \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent convex functions.

Corollary 3.3: Putting $\alpha = 0, \beta = 1, A = 1, B = -1$ in the theorem, we get

$$|a_3-\mu a_2^2| \le \begin{cases} 3-4\mu, if \mu \le \frac{1}{2}; \\ 1 if \frac{1}{2} \le \mu \le 1; \\ 4\mu - 3, if\ \mu \ge 1 \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

Corollary 3.4: Putting $A = 1, B = -1$ in the theorem, we get

$$|a_3-\mu a_2^2| \le \begin{cases} \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[\frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)} - 4\mu\right], \\ if \mu \le \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}; \\ \frac{1}{3\alpha+\beta-4\alpha\beta} \\ if \frac{8\alpha+3\beta+4\alpha^2-\beta^2-3\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)} \le \mu \le \\ \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)}; \\ \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[4\mu - \frac{8\alpha+3\beta+4\alpha^2-12\alpha^2\beta-9\alpha\beta^2-7\alpha\beta}{(3\alpha+\beta-4\alpha\beta)}\right], \\ if\ \mu \ge \frac{8\alpha+3\beta+8\alpha^2+\beta^2-24\alpha^2\beta-6\alpha\beta^2-7\alpha\beta}{4(3\alpha+\beta-4\alpha\beta)} \end{cases}$$

These estimates were derived by N. Kaur [11] and are results for the subclass $S^*(f, f', \alpha, \beta)$ of univalent starlike functions.

References:

[1] Alexander, J.W *Function which map the interior of unitcircle upon simple regions*, Ann. Of Math., **17** (1995), 12-22.

[2] Aoufet. al.,*Fekete – Szego Inequalities for p – valentstarlike and convex functions of complex order*, Journal of the Egyptian Mathematical Society,**22** (2014), 190 – 196.

[3] Bieberbach, L. Uber die KoeffizientemderjenigenPotenzreihen, welcheeineschlichteAbbildung des Einheitskrsisesvermitteln,*S. – B. Preuss. Akad. Wiss.* **38** (1916), 940-955.

[4] De Branges L., A proof of Bieberbach Conjecture, *Acta. Math*., **154** (1985), 137-152.

[5] Duren, P.L., Coefficient of univalent functions, Bull. *Amer. Math. Soc.*, **83** (1977), 891-911.

[6] Fekete, M. and Szegö, G, EineBemerkunguberungeradeschlichtefunktionen, *J.London Math. Soc*., 8 (1933), 85-89.

[7] Garabedian, P.R., Schiffer, M., AProof for the Bieberbach Conjecture forthe fourth coefficient, *Arch. Rational Mech. Anal*., 4 (1955), 427-465.

[8] Kaur, C. and Singh, G., Approach To Coefficient Inequality For A New Subclass Of Starlike Functions With Extremals, *International Journal Of Research In Advent Technology,* **5**(2017)

[9] Kaur, C. and Singh, G., Coefficient Problem For A New Subclass Of Analytic Functions Using Subordination, *International Journal Of Research In Advent Technology,* **5**(2017)

[10] Kaur. G, Singh. G, Arif. M, Chinram. R, Iqbal. J, A study of third and fourth Hankel determinant problem for a particular class of bounded turning functions, *Mathematical Problems in Engineering*, 22, 511-526, 2021

[11] Kaur N., Fekete Szego Inequality Alongwith Their Extremal Functions Making Results Sharp For Certain Subclasses Of Analytic Functions, *Webology*, 18(4), 2021, 3075-3083

[12] Keogh, F.R., Merkes, E.P., A coefficient inequality for certain classes of analytic functions,*Proc. Of Amer. Math. Soc.*, **20**, 8-12, 1989.

[13] Koebe, P., Uber Die uniformisiesrungbeliebigeranalyischeerKurven, *Nach. Ges. Wiss.Gottingen* (1907), 633-669.

[14] Lindelof, E., Memoire surcertainesinegalitiesdans la theorie des functions monogenes et surquelques proprieties nouvellles de cesfontionsdans la voisinage d′un point singulier essential, *Acta Soc. Sci. Fenn*., **23** (1909) , 481-519.

[15] Löewner, C. (1917), *Untersuchungenuber die VerzerrungbeiKonformenAbbildungendes Einheitskreises* |z| < *1, diedurchFunktionenmitnichtver-schwindenderAbleitunggeliefertwardin*, Ber. Verh. Sachi. Ges. Wigg. Leipzig, Vol. 69, pp.89-106

[16] Ma, W. and Minda, D. unified treatment of some special classes of univalent functions, *In Proceedings of the Conference on Complex Analysis* , Int. Press Tianjin (1994), 157-169.

[17] Miller, S.S., Mocanu, P.T. And Reade, M.O., All convex functions are univalent and starlike, *Proc. of Amer. Math. Soc.*, 37 (1973), 553-554.

[18] Nehari, Z. (1952), *Conformal Mappings*, McGraw- Hill, New York.

[19] Nevanlinna, R., Uber die Eigenshafteneineranalytischenfunkion in der umgebungeinersingularen stele order Linte*, Acta Soc. Sci. Fenn*., 50 (1922), 1-46.

[20] Pederson, R., A proof for the Bieberbach conjecture for the sixth coefficient,*Arch. Rational Mech. Anal*., 31 (1968-69), 331-351.

[21] Pederson, R. and Schiffer, M., A proof for the Bieberbach conjecture for the fifth coefficient, *Arch. Rational Mech. Anal.*, 45 (1972), 161-193.

[22] Rani, M., Singh, G., Some Classes Of Schwarzian Functions And Its Coefficient Inequality That Is Sharp, *Turk. Jour. Of Computer and Mathematics Education*, **11** (2020), 1366-1372.

[23] Rathore, G. S., Singh, G. and Kumawat, L. et.al., Some Subclaases Of A New Class Of Analytic Functions under Fekete-Szego Inequality, *Int. J. of Res. In Adv. Technology,* **7**(2019)

[24] Rathore. G. S., Singh, G., Fekete – Szego Inequality for certain subclasses of analytic functions , *Journal Of Chemical , Biological And Physical Sciences,* **5**(2015) *,*

[25] Singh. G, Fekete – Szego Inequality for a new class and its certain subclasses of analytic functions , *General Mathematical Notes*, **21** (2014),

[26] Singh. G, Fekete – Szego Inequality for a new class of analytic functions and its subclass, *Mathematical Sciences: International Research Journal*, 3 (2014),

[27] Singh. G., Construction of Coefficient Inequality For a new Subclass of Class of Starlike Analytic Functions, *Russian Journal of Mathematical Research Series*, **1** (2015), 9-13.

[28] Singh, G., Introduction of a new class of analytic functions with its Fekete–Szegö Inequality, *International Journal of Mathematical Archive,* **5** (2014), 30-35.

[29] Singh, G, An Inequality of second and third Coefficients for a Subclass of Starlike Functions Constructed Using nth Derivative, *Kaav Int.J. Of Sci. Eng. And Tech.*, **4** (2017), 206-210.

[30] Singh, G, Fekete–Szego Inequality for asymptotic subclasses of family of analytic functions, *Stochastic Modelling And Applications*, 26 (2022),

[31] Singh, G, Coefficient Inequality for Close to Starlike Functions Constructed Using Inverse Starlike Classes, *Kaav Int. J. Of Sci. Eng. And Tech.*, **4** (2017), 177-182.

[32] Singh, G, Coeff. Inequality for a subclass of Starlike functions that is constructed using nth derivative of the functions in the class, *Kaav Int. J. Of Sci. Eng. And Tech.*, **4** (2017), 199-202.

[33] Singh G., Fekete Szego Inequality For A Complicated Class Of Analytic Functions Approaching To A Class In The Limit Form And Other Class Directly, *IJRES*, 10 (9), 619-624, 2022

[34] Singh, G., Fekete–Szegö Inequality for functions approaching to a class in the limit form and another class directly, *Journal Of Information And Computational Sciences*, 12 (4), 2022, 181-186

[35] Singh, G., Garg, J., Coefficient Inequality For A New Subclass Of Analytic Functions, *Mathematical Sciences: International Research Journal,* **4**(2015)

[36] Singh G, Sharma N., Two new subclasses of already defined class of Analytic functions and establishment of their coefficient inequality, NeuroQuantology, 20 (10), 4967-4976, 2022

[37] Singh, G, Singh, Gagan, Fekete–Szegö Inequality For Subclasses Of A New Class Of Analytic Functions , *Proceedings Of The World Congress On Engineering* , (2014) , .

[38] Singh G, Houque M. A., A specially designed class of univalent functions and establishment of its coefficient inequality, *NeuroQuantology*, 20 (10), 2855-2859, 2022

[39] Singh G, Kaur C, Analytic functions subordinate to leaf-like domain, *Advances in Mechanics*, 10 (1), 1444-1448, 2022

[40] Singh, G, Sarao, M. S., and Mehrok, B. S., Fekete – Szegö Inequality For A New Class Of Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013).

[41] Singh. G, Singh. Gagan, Sarao. M. S., Fekete – Szegö Inequality for a New Class of Convex Starlike Analytic Functions, *Conf. Of Information and Mathematical Sciences*, (2013).

[42] Singh, G., Kaur, G., Coefficient Inequality for a Subclass of Starlike Function generated by symmetric points, *Ganita*, **70** (2020), 17-24.

[43] Singh ,G., Kaur, G., Coefficient Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) *,*

[44] Singh, G., Kaur, G., Fekete-Szegö Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) *,*

[45] Singh, G., Kaur, G., Fekete-Szegö Inequality For Subclass Of Analytic Function Based On Generalized Derivative, *Aryabhatta Journal Of Mathematics And Informatics,* **9**(2017) ,

[46] Singh, G., Kaur, G., Coefficient Inequality For a subclass of analytic function using subordination method with extremal function, *Int. J. Of Advance Res. In Sci&Engg* , **7** (2018)

[47] Singh, G., Kaur, G., Arif, M., Chinram R, Iqbal J, A study of third and fourth Hankel determinant problem for a particular class of bounded turning functions, *Mathematical Problems in Engineering*, 2021

[48] Singh, G. and Kaur, G., 4th Hankel determinant for α bounded turning function, *Advances in Mathematics: Scientific Journal*, 9 (12), 10563-10567

[49] Singh, G., Kaur, N., Fekete-Szegö Inequality For Certain Subclasses Of Analytic Functions, *Mathematical Sciences: International Research Journal,* **4**(2015)

[50] Singh G, Patil A. S., An extraordinary class of asymptotic analytical functions with coefficient inequality, *NeuroQuantology*, 20 (10), 4960-4966, 2022

[51] Singh, G, Singh, B, Fekete Szego Coefficient Inequality of Regular Functions for A Special Class, *International Journal of Research in Engineering and Science*, 10(8), 2022, 556-560

[52] Singh, G, Singh, P., Fekete – Szegö Inequality For Functions Belonging To A Certain Class Of Analytic Functions Introduced Using Linear Combination Of Variational Powers Of Starlike And Convex Functions, *Journal Of Positive School Psychology*, **6** (2022), 8387-8391.

[53] Singh. G, Rani M, An advance subclass of Analytic Functions having a unique coefficient inequality, *Int. J. of Research in Engineering and Science*, 10 (8), 2022, 474-476

[54] Singh, G., Singh, G., Singh, G., A subclass of bi-univalent functions defined by generalized Sãlãgean operator related to shell-like curves connected with Fibonacci numbers, *International Journal of Mathematics and Mathematical Sciences*, 2019

[55] Singh, G., Singh, G., Singh, G., A generalized subclass of alpha convex biunivalent functions of complex order, *Jnanabha*, 50 (1), 65-71, 2020

[56] Singh, G., Singh, G., Singh, G., Upper bound on fourth Hankel determinant for certain subclass of multivalent functions, *Jnanabha*, 50 (2), 122-127, 2020

[57] Singh, G., Singh, G., Singh, G., Certain subclasses of univalent and biunivalent functions related to shell-like curves connected with Fibonacci numbers, *General Mathematics*, 28 (1), 125-140, 2020

[58] Singh, G., Singh, G., Singh, G., Certain subclasses of Sakaguchitype bi-univalent functions, *Ganita*, 69 (2), 45-55, 2019

[59] Singh, G., Singh, G., Singh, G., Certain Subclasses of Bi-Close-to-Convex Functions Associated with Quasi-Subordination, *Abstract and Applied Analysis*, 1, 1-6, 2019

[60] Singh, G., Singh, G., Singh, G., Fourth Hankel determinant for a subclass of analytic functions defined by generalized S˜al˜agean operator, *Creat. Math. Inform.*, 31(2), 229-240, 2022

[61] Singh, G., Singh, G., Singh, G., Certain subclasses of multivalent functions defined with generalized Salagean operator and related to sigmoid function and lemniscate of Bernoulli, *J. Frac. Calc. Appl*, 13 (1), 65-81, 2022

[62] Srivastava H. M., G. Kaur, Singh. G, Estimates of fourth Hankel determinant for a class of analytic functions with bounded turnings involving cardioid domains, *Journal of Nonlinear and Convex Analysis*, 22 (3), 511-526, 2021