The Study of Physical Meaning of Schrödinger Equation Based on de Broglie and Bohm Theory

By

Gong Song[1] & Guan-Long Song[2]

[1]Boeing Commercial Airplanes Company, Seattle, USA

[2]Boeing Commercial Airplanes Company, Seattle, USA

Corresponding Author: Gong Song, Ph.D, Boeing Commercial Company, Seattle, United States.

Email: gongsong991@hotmail.com

Abstract

In this paper, based on the point of view of the pilot-wave theory and Bohmiam mechanics as well as A. Einstein thought, authors analyze the motion of a material point both in the framework of Newtonian Mechanics and in the framework of Classic Quantum Mechanics.

First, in the framework of Newtonian Mechanics, with the help of Hamilton Principal and Hamilton-Jacobi Equation, it is proved that the wave equation from the theory of de Broglie is a displacement equation.

Secondly, in the framework of Classic Quantum Mechanics, with the help of Hamilton Principle and Hamilton-Jacobi Equation, Schrödinger Equation is obtained. So, it is concluded that Schrödinger Equation is a displacement equation and the wave function is a displacement function. Based on these conclusions, the physical meaning of Schrödinger Equation is obtained and then Schrödinger Momentum Equation is derived.

When using Schrödinger Momentum Equation to investigate photon in Double Slit Test, it is first time mathematically found that the collapse of wave function only takes place in the same type of particle. It can be reasonably inferred that the randomness of photon or electron in Double Slit Test is caused by one type or multiple types of particle or quanta, which are mysterious and unknow for human beings up to now. These conclusions are supported by rigorous mathematical deriving and theoretical analysis.

Keywords: Wave-Particle Duality, Schrödinger Equation, Wave Function, Double Slit Test, Randomness of Quantum, Wave Function Collapse, Pilot-Wave Theory, Bohmian Mechanics

1. Introduction

Historically, the pilot-wave approach to quantum theory was initiated by A. Einstein. And then, E. Schrödinger discovered Schrödinger Equation of wave mechanics in 1926. In 1927, de Broglie found an equation of particle motion for a scalar wave function [1]. Later he explained at the 1927 Solvay Congress. However, de Broglie responded very poorly to an objection of W. Pauli [2], de Broglie very quickly abandoned the pilot-wave approach.

In 1952, D.Bohm rediscovered de Broglie's pilot-wave theory in his paper [4], called Bohmian mechanics. In 1960s, 1970, and 1980s, John Bell became the principal proponent of the theory. J. Bell published a lot of paper to investigate this theory, including his famous inequality [5]. After J. Bell, Detlef Dürr, Sheldon Goldstein, Nino Zanghì, and V. Allori published many papers and books to continue to investigate this theory and make great contributions in this area [6,7,8,9].

In Bohmian mechanics, Quantum Potential governs the positions of the particles and these particles are described by their positions with time and the wave function is to obey Schrödinger Equation. This theory over-emphasized the role of particle and paid less attention to the propagation of wave, but the point of view of treating the particle and wave simultaneously is very invaluable insights.

Based on the point of view of the pilot-wave theory and Bohmian mechanics as well as A. Einstein thought [3], authors try to investigate the physical meaning of Schrödinger Equation in this paper. In this sense, this paper can be said to be a continuation and extension of the pilot-wave theory and Bohmian mechanics.

First, in the framework of Newtonian Mechanics, we treat a material point's motion as the motion of particle. And then set up the equation of wave and treat this motion as wave propagation. To solve the wave equation, D'Alembert Method is used to obtain a general solution. With the help of Hamilton Principal and Hamilton-Jacobi Equation, we can prove that no matter how we treat the motion of the material point as particle's motion or wave propagation, the obtained results are the same displacement function. So, the wave equation is a displacement equation.

Secondly, we solve this wave equation in the framework of Classic Quantum Mechanics. Just like what Schrödinger did, Plank Law is used. With the help of the phase velocity of the wave propagation derived from Hamilton Principle and Hamilton-Jacobi Equation, Schrödinger Equation is obtained. Because we have known the wave equation is a displacement equation in the framework of Newtonian Mechanics, it is concluded that Schrödinger Equation is a displacement equation and wave function is a displacement function in the framework of Classic Quantum Mechanics. And then Schrödinger Momentum Equation is derived.

In order to verify Schrödinger Momentum Equation, Double Slit Test is investigated. Through using Schrödinger Momentum Equation, theoretically, it is first time found that the collapse of wave function takes only place in the same types of particles. That is to say, the collapse of wave function of photon can only be generated by photon; the collapse of wave function of electron can only be generated by electron.

And also, it can be reasonably and mathematically inferred that the randomness of photon or electron in Double Slit Test is caused by one type or multiple types of particles /quanta, which are mysterious and unknow for human being up to now.

2. Particle Motion of Theory of de Broglie Wave-Particle Duality in Framework of Newtonian Mechanics

Consider a material point with mass, m, is moving forward in a conservative field of force $V = V(x, y, z)$ along axial x toward infinite distance in a Cartesian Coordinates, shown in Figure 1.

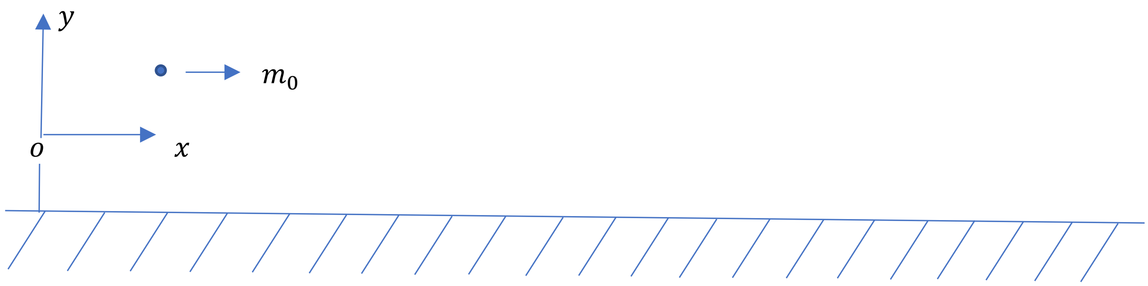

Figure 1 Figure 1 Moving Material Point in Space

Without losing generality, set the initial displacement and velocity of the material point as follows:

Initial displacement: $x(0) = x_0$, at time $t = 0$;

Initial velocity: $\frac{\partial x}{\partial t} = v_0$, at time $t = 0$.

If we do not consider the gravity in y direction and the friction of air, according to Newton Second Law, then the equation of motion of the material point is as follows:

$$m\frac{d^2x}{dt^2} = F_x \quad (1)$$

where F_x is total force applied on the material point in x direction.

After a series of integral operations for time t , then we can solve Equation (1) as follows:

$$x = \frac{F_x}{2m}t^2 + C_0t + C_1 \quad (2)$$

where C_0 and C_1 are integration constants and will be determined by initial conditions.

If we assume F_x is 0, then Equation (2) will become

$$x(t) = C_0t + C_1 \quad (3)$$

Apply the above initial conditions into Equation (3),

$$x(t) = v_0t + x_0 \quad (4)$$

where v_0 is the initial velocity of the material point and x_0 the is the initial displacement of the material point.

Equation (2) and Equation (4) are the equations of which are used to describe the motion of the material point in the framework of Newtonian Mechanics. It is well known to all of us.

Next, we consider the above motion of material point based on the theory of de Broglie wave-particle duality, which was from the fundamental research of L. de Broglie and has been well accepted by all of us.

Apparently, the above equations of motion for material point, m, Equation (2) and (4), are to treat m as a particle, which are equivalent to the particle of which is described in the theory of de Broglie wave-particle duality. So, we can say that Newtonian Mechanics can be used to describe the motion of the particle stated in the theory of de Broglie wave-particle duality.

3. Wave Propagation of Theory of de Broglie Wave-Particle Duality and Its Phase Velocity from Hamilton-Jacobi Equation under Framework of Newton Mechanics

Now we consider the above motion of material point or particle as wave propagation in its "configuration - space"(q-space, not pq-space) [1], which is described in the theory of de Broglie wave-particle duality. It should be noticed that up to now, although we have not known the physical meanings of this kind of wave, we can follow the conventional way to call this wave as "matter wave".

Because the matter wave is a kind of wave, then the propagation of matter wave could be described by an equation, which is called wave equation. Without losing generality, we chose a dimensional wave equation along x direction as follows:

$$\frac{\partial^2\psi}{\partial t^2} = u^2\frac{\partial^2\psi}{\partial x^2} \qquad (5\text{-}1)$$

where ψ is a function which is used to describe the propagation of the matter wave, u is a phase velocity of the matter wave ψ and t is time.

It should be pointed out again that up to now we have not known the physical meaning of matter wave, ψ. Its physical meaning of ψ is what we try to find and will be given in this paper.

First, we consider a general solution of this wave equation, Equation (5-1). Although we do not know the physical meaning of ψ , we can use D'Alembert Method to obtain a general solution for this one-dimensional wave equation as follows [2]:

For wave equation:

$$\frac{\partial^2\psi}{\partial t^2} = u^2\frac{\partial^2\psi}{\partial x^2} \qquad (5\text{-}2)$$

If setting initial conditions as follows:

$\psi(x,0) = f(x)$;

$\frac{\partial\psi}{\partial t}/_{t=0} = g(x)$;

Setting boundary conditions:

$\psi(0,t) = 0$,

$\psi(-\infty,+\infty) = 0$;

Then the general solution is as follows:

$$\psi(x,t) = \frac{1}{2}[f(x-ut)+f(x+ut)] + \frac{1}{2u}\int_{x-ut}^{x+ut} g(\xi)d\xi \tag{6}$$

where u is the phase velocity of wave.

Next we need to obtain the velocity of wave propagation and the phase velocity, u , in Equation (5-2) or Equation (6).

In order to obtain u, we use Hamiltonian Principal and three-dimensional description [1]. Using following conventional notations:

Displacement Vector, $\vec{r} = \vec{r}(x, y. z)$;

Kinetic energy, $T = T(x, y, z)$;

Particle Momentum: p_x, p_y, p_z;

Action Function, $W = W(x, y, z, t) = W(\vec{r}, t)$;

Potential Energy, $V = V(x, y, z)$.

Then, the total kinetic energy of the particle can be written as:

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) \tag{7}$$

Based on Hamiltonian Principal, from time t_0 to time t during the moving of the particle, a Hamiltonian function for action W can be written as follows:

$$W = \int_{t_0}^{t} [(T - V]dt \tag{8}$$

Take Equation (7) into Equation (8), then we have:

$$W = \int_{t_0}^{t} (T - V)dt = \int_{t_0}^{t} [\frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) - V(x, y, z)]dt \tag{9}$$

As all of us are well known that:

$$\begin{aligned} p_x &= m\dot{x} = \frac{\partial W}{\partial x}; \\ p_y &= m\dot{y} = \frac{\partial W}{\partial y}; \\ p_z &= m\dot{z} = \frac{\partial W}{\partial z} \end{aligned} \tag{9-1}$$

In order to obtain Hamilton-Jacobi Equation of the action W, now take derivative for time, t for Equation (9), then we have the following the Hamiltonian partial differential equation, which is a function of the upper limit t and of the final values of the coordinates x, y, z:

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial t}[\int_{t_0}^{t}[\frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) - V(x,y,z)]dt$$

⇒

$$\frac{\partial W}{\partial t} = [-\frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) - V(x,y,z)]$$

⇒

$$\frac{\partial W}{\partial t} = -[\frac{1}{2m}(p_x^2) + \frac{1}{2m}\left(p_y^2\right) + \frac{1}{2m}(p_z^2)] - V(x,y,z)$$

Take Equation (9-1) into the above equation, then

⇒

$$\frac{\partial W}{\partial t} = -\frac{1}{2m}[(\frac{\partial W}{\partial x})^2 + (\frac{\partial W}{\partial y})^2 + (\frac{\partial W}{\partial z})^2] - V(x,y,z)$$

⇒

$$\frac{\partial W}{\partial t} + \frac{1}{2m}[(\frac{\partial W}{\partial x})^2 + (\frac{\partial W}{\partial y})^2 + (\frac{\partial W}{\partial z})^2] + V(x,y,z) = 0 \qquad (10)$$

Equation (10) is Hamilton-Jacobi Equation of the action W.

To solve this equation, we put $W(x,y,z,t)$ in the follow form:

$$W(x,y,z,t) = -Et + S(x,y,z) = -Et + S(\vec{r}) \qquad (11)$$

where $S = S(\vec{r}) = S(x,y,z)$ is Hamiltonian characteristic function;

$\vec{r}$ is displacement vector and $\vec{r} = \vec{r}(x,y.z)$

E is an integration constant, that is, total particle energy;

Notice that:

$$\frac{\partial W}{\partial t} = -E$$

$$\frac{\partial W}{\partial x} = \frac{\partial S}{\partial x};$$

$$\frac{\partial W}{\partial y} = \frac{\partial S}{\partial y};$$

$$\frac{\partial W}{\partial z} = \frac{\partial S}{\partial z};$$

Then we have

$\Rightarrow \quad -E + \frac{1}{2m}[(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + (\frac{\partial S}{\partial z})^2] + V(x, y, z) = 0$

$\Rightarrow \quad \frac{1}{2m}[(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + (\frac{\partial S}{\partial z})^2] = E - V(x, y, z)$

$$\Rightarrow \quad [(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + \left(\frac{\partial S}{\partial z}\right)^2\Big] = 2m[E - V(x, y, z)] \quad (12)$$

Consider that

$$p_x = m\dot{x} = \frac{\partial W}{\partial x} = \frac{\partial S}{\partial x}$$
$$p_y = m\dot{y} = \frac{\partial W}{\partial y} = \frac{\partial S}{\partial y}$$
$$p_z = m\dot{z} = \frac{\partial W}{\partial z} = \frac{\partial S}{\partial z}$$

Then we have

$$\dot{x} = \frac{\partial S}{m\partial x}$$

$$\dot{y} = \frac{\partial S}{m\partial y}$$
$$\dot{z} = \frac{\partial S}{m\partial z}$$

We know the particle velocity, v, is

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} = \{\frac{1}{m}[(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + \left(\frac{\partial S}{\partial z}\right)^2\Big]^{1/2} \quad (13)$$

Take Equation (12) back into Equation (13), then we have

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} = \{\frac{1}{m}[(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + \left(\frac{\partial S}{\partial z}\right)^2\Big]^{1/2} = \frac{1}{m}\{[2m[E - V(x, y, z)]\}^{1/2}$$

$\Rightarrow$

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} = [\frac{2(E - V(x,y,z))}{m}]^{\frac{1}{2}} = [\frac{2(E - V)}{m}]^{\frac{1}{2}} \quad (14)$$

Equation (14) is the velocity of the particle in wave propagation.

Now we get the phase velocity of the wave propagation. Notice according to the definition, the phase velocity is expressed as follows:

$$u = \frac{d\vec{r}}{dt} \quad (15)$$

If we imagine W as a function of the system of the wave front surfaces [1], which represents the propagation of the wave. As the wave surface front is propagating from $(t + dt)$, W will become

$(W + dW_0)$. However, at time $(t + dt_0) = t_0$, $t_0$being a constant, then $(W + dW_0)$ will be a constant along the surface of the wave front. That is to say, $(W + dW_0)$ will not be changed along the surface of the wave front with the change of time t. So, we have the following equation:

$$\frac{dW}{dt} = 0$$

⇒

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + \nabla W \cdot \frac{d\vec{r}}{dt}$$

⇒

$$0 = \frac{\partial W}{\partial t} + \nabla W \cdot \frac{d\vec{r}}{dt} \qquad (16)$$

Using Equation (11), then

$$\frac{\partial W}{\partial t} = -E$$

Take the above equation into Equation (16), then

$$\frac{d\vec{r}}{dt} = \frac{E}{|\nabla W|} \qquad (17)$$

Because we have

$$|\nabla W| = (\frac{\partial W}{\partial x})^2 + (\frac{\partial W}{\partial y})^2 + (\frac{\partial W}{\partial y})^2$$

and

$$(\frac{\partial W}{\partial x})^2 + (\frac{\partial W}{\partial y})^2 + (\frac{\partial W}{\partial y})^2 = [2m[E - V]$$

⇒

$$(\nabla W)^{\frac{1}{2}} = [2m[E - V]^{\frac{1}{2}}$$

⇒

$$|\nabla W| = \{2m[E - V]\}^{\frac{1}{2}} \qquad (18)$$

Taking Equation (18) into Equation (17):

$$0 = -E + \nabla W \cdot \frac{d\vec{r}}{dt}$$

$$u = \frac{d\vec{r}}{dt} = \frac{E}{|\nabla W|} = \frac{E}{[2m(E-V)]^{\frac{1}{2}}} \tag{19}$$

Equation (19) is the phase velocity of the wave propagation.

From Equation (14), because the velocity of the particle does not contain time t, this means the motion of the particle moves in a constant velocity during the wave propagation. This should include its initial velocity state, $\frac{\partial\psi}{\partial t}$. That is, the initial velocity should be v_0. Because if the initial velocity is v_1, then there exists a state, say State 1, where v_1 is changed to v_0. That is, there is an external force which changes the velocity of particle from v_1 to v_0. However, in the wave propagation, this external force does not exist according to our assumption. Realizing this point and without losing generality, we can write initial condition as follows:

Initial velocity: $\frac{\partial\psi}{\partial t} = v_0 = [\frac{2(E-V)}{m}]^{\frac{1}{2}}$, at time $t = 0$.

Notice that up to now, we have not known the physical meaning of $\boldsymbol{\psi(x,t)}$, that is to say, we do not know the physical meaning of $\boldsymbol{f(x)}$. We only know $\boldsymbol{\frac{\partial\psi}{\partial t} = v_0}$ is a particle motion velocity in the wave propagation.

Take the initial conditions $\psi(x,0)$ and $u = \frac{E}{[2m(E-V)]^{1/2}}$ into Equation (6):

$$\psi(x,t) = \frac{1}{2}[f(x-ut) + f(x+ut)] + \frac{1}{2u}\int_{x-ut}^{x+ut} g(\xi)d\xi$$

$$= \frac{1}{2}[f(x-ut) + f(x+ut)] + \frac{1}{2u}\int_{x-ut}^{x+ut}[\frac{2(E-V)}{m}]^{\frac{1}{2}}d\xi$$

$$= \frac{1}{2}[f(x-ut) + f(x+ut)] + \frac{1}{2u}\left[[\frac{2(E-V)}{m}]^{\frac{1}{2}}\right][(x+ut)-(x-ut)]$$

$$= \frac{1}{2}[f(x-ut) + f(x+ut)] + \frac{1}{2u}[\frac{2(E-V)}{m}]^{\frac{1}{2}}[(x+ut)-(x-ut)]$$

$$= \frac{1}{2}[f(x-ut) + f(x+ut)] + \frac{1}{2u}[\frac{2(E-V)}{m}]^{\frac{1}{2}}(2ut)$$

$$= \frac{1}{2}[f(x-ut) + f(x+ut)] + v_0 t$$

$\Rightarrow$

$$\psi(x,t) = \frac{1}{2}[f(x-ut)+f(x+ut)] + \frac{1}{2u}\int_{x-ut}^{x+ut} g(\xi)d\xi = \frac{1}{2}[f(x-ut)+f(x+ut)] + v_0 t \tag{20}$$

We have

$$\psi(x,t) = \frac{1}{2}[f(x-ut)+f(x+ut)] + v_0 t$$

$$\{\psi(x,t) - \frac{1}{2}[f(x-ut)+f(x+ut)]\} = v_0 t \tag{21}$$

In this equation, because the physical meaning or dimension of $v_0 t$ is a distance, then

$\{\psi(x,t) - \frac{1}{2}[f(x-ut)+f(x+ut)]\}$ physically or dimensionally must be a displacement, too. In the big parentheses, we have $\psi(x,t)$ " minus " $\frac{1}{2}[f(x-ut)+f(x+ut)]$, it indicates that these two items must have the same physical meaning or dimension. So, $\psi(x,t)$ is a displacement function. That is, Matter Wave Equation (5-1), which is based on the theory of de Broglie wave-particle duality, is a displacement wave equation.

4. Wave Propagation of Particle in Framework of Classic Quantum Mechanics and Plank Law and Physical Meaning of Schrödinger Equation

In the previous section, D'Alembert Method is used to prove the wave equation, matter wave equation, is a displacement equation from the theory of de Broglie wave-particle duality. It should be noticed that the proof of the method is in the framework of Newtonian mechanics. In order to view the wave propagation as a moving particle, now we view it in the framework of classic quantum mechanics with the help of Plank Law. This is because the theory of de Broglie wave-particle duality is not only suitable to the macro-world but also is suitable to the micro world, that is, the Equation (5-2) is not only suitable for Newtonian mechanics, and also is suitable for Classic Quantum Mechanics:

$$\frac{\partial^2\psi}{\partial t^2} = u^2\frac{\partial^2\psi}{\partial x^2} \tag{5-2}$$

In classic quantum mechanics, Plank discovered his famous law, Plank Law. According to this law, if a moving particle possesses a frequency, then it must satisfy:

$$E = h\nu \tag{22}$$

or

$$\nu = \frac{E}{h} \tag{22-1}$$

where E is energy of particle, h is Planck's constant and ν is frequency.

Equation (22-1) tells us that the energy of a moving particle is very small, if equal to h, then the motion of the particle will be the propagation of wave, since this moving particle possesses the frequency of motion. Mathematically, this wave-function, say $\psi(x,t)$, will be of the form of sinusoidal waves.

$$\psi(x,t) = A(x)\sin(\nu t + B)$$

where A is an "amplitude" function and B is an initial phase of the wave.

In order to obtain the equation of wave for Equation (5-2) in the framework of Classic Quantum Mechanics, we assume the following form [11]:

$$\psi(x,t) = A(x)e^{i\frac{W(x,t)}{h}} \tag{23}$$

That is, the complex plane wave is assumed. According to the previous discussion, Equation (11), we have:

$$W(x,t) = -Et + S(x) \tag{24}$$

Take Equation (23) into Equation (24), then

$$\psi(x,t) = A(x)e^{i\frac{W(x,t)}{h}} = A(x)e^{i\frac{[S(x)-Et]}{h}} = A(x)e^{i\left[\frac{S(x)}{h}-\frac{Et}{h}\right]} = A(x)e^{i\left[\frac{S(x)}{h}-\frac{Et}{h}\right]} = A(x)e^{i\left[\frac{S(x)}{h}\right]}e^{i\left(-\frac{Et}{h}\right)}$$

$$= \psi_0(x)e^{i(-\frac{Et}{h})}$$

where $\psi_0(x) = A(x)e^{i[\frac{S(x)}{h}]}$, it is an integration constant.

Notice that:

$$\frac{\partial\psi}{\partial t} = \psi_0(x)e^{i\left(-\frac{Et}{h}\right)}\left(-i\frac{E}{h}\right) = -i\psi_0(x)(\frac{E}{h})e^{i\left(-\frac{Et}{h}\right)}$$

$$\frac{\partial^2\psi}{\partial t^2} = (-i))\psi_0(x)(\frac{E}{h})e^{i\left(\frac{Et}{h}\right)}\left(-i\frac{E}{h}\right) = -\psi_0(x)\left(\frac{E}{h}\right)^2 e^{i\left(-\frac{Et}{h}\right)} = -\psi(x,t)\left(\frac{E}{h}\right)^2 \tag{25}$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi_0(x)}{\partial x}e^{i\left(-\frac{Et}{h}\right)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi_0(x)}{\partial x^2}e^{i\left(-\frac{Et}{h}\right)} = \frac{\partial^2\psi(x,t)}{\partial x^2} \tag{26}$$

Now take Equation (25) and Equation (26) into Equation (5-2), then

$$\frac{\partial^2\psi}{\partial t^2} = u^2\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{\partial^2\psi}{\partial x^2} - \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{1}{u^2}[\psi(x,t)\left(\frac{E}{h}\right)^2] = 0 \qquad (27)$$

Because we said that the theory of de Broglie wave-particle duality is not only suitable to the macro-world, and also is suitable for the micro world, that is, the phase velocity of wave, u, which was obtained from Hamilton-Jacobi Equation in the framework of Newtonian Mechanics, can be used in the propagation of wave in the framework of Classic Quantum Mechanics:

$$u = \frac{E}{[2m(E-V(x))]^{1/2}} \qquad (6)$$

Take Equation (6) into Equation (26), then we have

$$\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{1}{[\frac{E}{[2m(E-V(x))]^{\frac{1}{2}}}]^2}[\psi(x,t)\left(\frac{E}{h}\right)^2] = 0$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} + [\frac{2m(E-V(x))}{E^2}]\psi(x,t)\left(\frac{E}{h}\right)^2 = 0$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{2m[E-V(x)]}{E^2}\psi(x,t)\left(\frac{E}{h}\right)^2 = 0$$

$$\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{2m[E-V(x)]}{h^2}\psi(x,t) = 0$$

$$(\frac{h^2}{2m})\frac{\partial^2\psi(x,t)}{\partial x^2} + E\psi(x,t) - \psi(x,t)V(x) = 0$$

$$\frac{h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} - \psi(x,t)V(x) = -E\psi(x,t)$$

Notice $E\psi(x,t) = ih\frac{\partial\psi(x,t)}{\partial t}$

$$-\frac{h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + \psi(x,t)V(x) = ih\frac{\partial\psi(x,t)}{\partial t} \quad (28)$$

We can see that Equation (28) is Schrödinger Equation and it is a displacement equation. In this equation, wave function, $\psi(x,t)$, is a displacement function. The physical meaning of imaginary number i in Equation (28) tells us that the motion of the particle is the propagation of wave.

Notice that $\psi(x,t)$ is a displacement function, then $\frac{\partial\psi(x,t)}{\partial t} = v$ is velocity of the quantum. Rewrite the above equation, then we have

$$-\frac{h^2}{2}\frac{\partial^2\psi(x,t)}{\partial x^2} + m\psi(x,t)V(x) = ihmv$$

Let $p = mv$ be the momentum of the quantum, then we have

$$-\frac{h^2}{2}\frac{\partial^2\psi(x,t)}{\partial x^2} + m\psi(x,t)V(x) = ihp$$

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{m\psi(x,t)V(x)}{h}] = ip \quad (29)$$

Equation (29) is called **Schrödinger Momentum Equation**. It is a dynamic wave equation and used to describe the motion of quantum.

In this equation, the left side is about the particle's characteristic and the right side is about the characteristic of wave. This is because the right side includes the imaginary number i. The physical meaning of i is to indicate the characteristic of wave, this is, the particle moves in space in the way of wave and only can be interfered by another particle with the characteristic of wave.

In addition to this, this equation possesses one important physical characteristic: any one-time momentum input or "measurement" with the characteristic of wave for the quantum can change the original state of the quantum. In other words, in order to change the state of the quantum, it is enough to input or "measurement" only one-time. This is explanation for Uncertainty Principal based on Schrödinger Momentum Equation.

Next Schrödinger Momentum Equation will be used to investigate the collapse of wave function and the randomness of photon or electron in Double Slit Test.

5. Verification of Schrödinger Momentum Equation Through Double Slit Test

As we know that there are two very mysterious physical phenomena in Double Slit Test. First one is when the quantum, for example, photon, is viewed by a watcher, the characteristic of wave of the photon will disappear and only show the characteristic of the particle, that is, the wave function collapse takes place. The second one is the randomness of the quantum, that is, people could not predict the quantum, for example, photon, will pass through which slot of two slots. These two mysterious phenomena cause many guesses and explanations. Now we can try to use Schrödinger Momentum Equation to investigate them.

Historically, because photon had been tested early than electron in Double Slit Test, without the loss of generality, we only consider the photon as an example.

5.1 Discussion of Collapse of Wave Function

If assume one photon with momentum p is emitted and is moving, then according to Schrödinger Momentum Equation (29), we have

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}]=ip \qquad (29\text{-}1)$$

Let us imagine another photon is moving with momentum p in opposite direction, then its Schrödinger Momentum Equation can be written as

$$\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]=-ip \qquad (29\text{-}2)$$

Aften these two photons interfere each other, we can add Equation (29-1) to Equation (29-2). Then we have that

$$\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]+\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]=ip-ip$$

$$\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]=0$$

If we assume $V(x)=V(0)$ is a constant, then we have

$$\frac{\partial^2\psi(x,t)}{\partial x^2}-\frac{2mV(0)}{h^2}\psi(x,t)=0$$

The above equation is an ordinary different equation without the characteristic of wave, that is, the characteristic of the wave has disappeared. This is the so-called the collapse of wave function. The above derivation is the mathematic explanation of wave function collapse.

If we further assume $V(x)=V(0)=0$, then we have

$$\frac{\partial^2\psi(x,t)}{\partial x^2}=0$$

Then

$$\psi(x,t)=Ax+B$$

where A and B are integral constants.

That is to say, the motion of the quantum is a straight line after the wave function collapses. This conclusion matches the test result.

Since we know that the photon and electron have different momentums. Based on the above discussion, if a photon or electronic, is interfered by another photon or electronic, the collapse of wave function takes only place in the same particles. That is to say, the collapse of wave function of photon can only be generated by photon; the collapse of wave function of electron can only be generated by electron. The photon cannot cause the collapse of wave function of electron. And also, the electron cannot cause the collapse of the photon.

5.2. Discussion of Phenomenon of Randomness

Now we investigate the phenomenon of randomness from both physical aspect and mathematical aspect.

Without the loss of generality, we only consider photon.

a. Physically

About the phenomenon of randomness of photon, since the wave function of the quantum is a displacement function, this indicates the quantum-self does not have characteristic of randomness. However, the characteristic of randomness of photon really exists in Double Slit Tests. Based on this fact, we can infer that the motion of the photon is randomly interfered by one or two kinds of quanta, which are mysterious and unknown by human beings up to now. These random interferences from the quanta cause the randomness of the photon in the Double Slit Test.

It should be pointed out that so far we do not know anything about these mysterious quanta and only can infer that the mass of the quanta is smaller than the photon or electron. We expect more investigation and tests to discover these mysterious quanta.

b. Mathematically

Now we assume that one photon, A, with momentum p is emitted and is moving, then its Schrödinger Momentum Equation is as follows:

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}]=ip \qquad \text{(A)}$$

We imagine another particle, B, is moving in space with momentum p_1, its Schrödinger Momentum Equation can be written as

$$\left[-\frac{h}{2}\frac{\partial^2\psi_1(x,t)}{\partial x^2}+\frac{m_1\psi_1(x,t)V_1(x)}{h}\right]=ip_1 \qquad \text{(B)}$$

After particle B interferes with this photon A, their Schrödinger Momentum Equation can be written as follows:

$$\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]+\left[-\frac{h}{2}\frac{\partial^2\psi_1(x,t)}{\partial x^2}+\frac{m_1\psi_1(x,t)V_1(x)}{h}\right]=ip+ip_1 \qquad \text{(C)}$$

In order to consider the motion of photon, we assume the mass of particle B is very small, compared with the mass of photon. Then we can re-write Equation (C) as follows:

$$\left[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]+\left[-\frac{h}{2}\frac{\partial^2\psi_1(x,t)}{\partial x^2}+\right]=ip+ip_1$$

$$\left[-\frac{h}{2}\left(\frac{\partial^2[\psi(x,t)+\psi_1(x,t)]}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}\right]=i(p+p_1) \qquad \text{(D)}$$

If comparing Equation (D) with Equation (A), then it can be found that the difference of the motion of photon between two equations is from particle B. If particle B disappears, then the motion of the photon is totally the same. So, we can say if the motion of photon A is changed, then the photon must be inferred by particle B. If photon A is interfered by numeric and random particles, B_1, B_2, B_3 ... $B_{n,} C_1, C_2, C_3,$ ... C_n and so on, then the motion of photon A will certainly show the characteristics of randomness. However, In Double Slit Test, the motion of the photon shows indeed the randomness. Based on this fact and Equation (D), **we can infer reasonably that there does exist one type or two types of mysterious and unknown particles for us, which cause the randomness of the photon in Double Slit Test.**

Summarize the above discussion, we can say that Schrödinger Momentum Equation can not only reasonably explain the collapse of wave function, and also it predicts the existence of mysterious and unknown particles in Double Slit Test. So, Schrödinger Momentum Equation has been verified through the test.

6. Conclusions

In this paper, we use theory of de Broglie and Bohm as well as Einstein's thought as a foundation to analyze the motion of a particle both in the frame of Newtonian Mechanics and in the frame of Classic Quantum Mechanics. The results indicate that

a. Schrödinger Equation is a displacement equation and the wave function is a displacement function;

b. Schrödinger Momentum Equation is as follows:

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}]=ip$$

c. According to Schrödinger Momentum Equation, the collapse of wave function of photon can only be generated by photon; the collapse of wave function of electron can only be generated by electron. The photon cannot cause the collapse of wave function of electron and the electron cannot cause the collapse of the photon.

d. It is concluded that the randomness of photon or electron in Double Slot Test is caused by mysterious and unknow particles for human beings up to now.

Declarations

• Ethical Approval and Consent to participate

No Applicable

• Consent for publication

No Applicable

• Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

• Competing interests

The authors declare that they have no competing interests

• Funding

No Applicable

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