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Dynamic analysis of a Simply Supported Cracked Beam Subjected to a Constant Force by Using Modified Adomian Decomposition Method

Naseradin Abujnah

Faculty of Technical Engineering,Mesallata, Libya

Email:naabujnah@gmail.com

Abstract*:*

Beam models are very common in structural analysis. Obtained equations of motions are complex in generally. Many solution techniques are applied for these equations of motion. Recently, Adomian methods have attracted the attention of researchers. In this study, Modified Adomian Decomposition Method (MADM) is utilized to examine the vibration response of the Euler-Bernoulli beam with a crack subjected to concentrated constant force. The system is modeled as a concentrated constant force acts on the two continuous segments, where the crack modeled as a sectional flexibility rotational spring. The equations of motion of the spans become a fourth order, non-homogenous partial differential equations. The analytical solution of dynamic response is obtained for cracked simply supported beam, it shows the crack effects with different crack location and crack extension for various values of concentrated force. Some numerical results presented by using MATLAB software to compute the vibration analysis and plot the deflection. The solution method was verified with the reference study[18]. The results were obtained as they were expected.

Keywords: Modified Adomian Decomposition Method; Euler-Bernoulli beam; Cracked simply supported beam

1. Introduction

The beam-like bridge structure is extensively used in several aeronautical, civil and mechanical engineering fields. The cracks are the major reason for bridge structure failure and when a bridge structure is subjected to cracks, its stiffness will be reduced, thereby reducing its natural frequencies and reduces the lifetime of the bridge structure. It is probable to presage the depth of a crack and its location based on the changes in vibration parameters. Dynamic, regular or random forces caused by the bridge vibration equations of motion fourth order, non-homogeneous is expressed as a partial differential equation. Various methods have been developed to solve these equations of motion. Adomian decomposition method (ADM) is one of these methods. Hsu, Lai, and Chen have developed a new ADM for the solution of non-uniform beams with several classical boundary conditions and supplying end conditions obtained the beam mods [1]. Lai and Hsu analyzed Euler Bernoulli (EB) beam vibrations for different boundary conditions and calculated some lower natural frequencies and mode shapes using ADM [2]. In addition to, they used the same method for uniform Timoshenko beams [3]. Mao and Pietrzko analyzed the free vibrations of two uniform sections of a stepped EB beam [4]. Mao has benefited from ADM and MADM in the free EB beam with a number of steps [5,6]. Mao analyzed the rotating EB beam under several boundary conditions by using MADM [7]. The ADM method had been developed in the late 1980s, the MADMgeneral equation consists of linear and nonlinear terms and the solution of equation is based on the integration of both sides of the equation according to the highest derivative order [8]. This method has been made more useful and modified due to the increase in the use of ordinary and partial differential equation solutions in the following periods, and it has been facilitated to make the series-opening easier and faster [9-14]. All the previous studies in the literature considered free vibration of a cracked beam by using ADM, Bilik and Karaçay, analyzed the vibration of an intact beam subjected to a constant force that analytically solved by MADM [15]. In their study, a comparatively new computed approach called modified Adomian Decomposition Method (MADM) is used to analyze the vibration of a cracked simply supported beam subjected to a constant force. The location of the applied force expressed in the equation of motion with Dirac Delta function and the fourth order, non-homogeneous, partial differential equation for each section of a cracked beam becomes second order ordinary differential equation. The analytical expression obtained by the solution of these equations by MADM. Some numerical values and the beam displacements in the first mode for the different crack depth ratio, various values of crack position and several applied forces were presented graphically. In the present study, Modified Adomian Decomposition Method (MADM) has been applied to solve a fourth order, non-homogenous partial differential equations which are the equation of motion of the Euler-Bernoulli beam with a crack subjected to concentrated constant force. The analytical solution of dynamic response is obtained.

2.Analysis of MADM

MADM has been used in recent years for solving partial differential equations which are highly complex in mathematics, physics and engineering applications, and modified and rendered more useful [16,17]. The second order non-homogeneous equation in this method and the initial conditions are expressed as follows.

$$w''(t) + P(t)w' + N(t) = \mathrm{g(t)}, w(0) = k_0, w'(0) = k_1 \tag{1}$$

Where N(t) is nonlinear function, P(t) and g(t) are given functions and k_0 and k_1 are constants. According to MAD*M*, the new differential operator*L* is defined as follows:

$$L = e^{-\int P(t)dt} \frac{d}{dt}\left(e^{\int P(t)dt} \frac{d}{dt}\right) \tag{2}$$

If *P(t)*=0, we can rewrite Eq. (1) as

$$Lw = g(t) - N(t) \tag{3}$$

In this method, the inverse of the operator *L*is applied a two-fold integral operator, as follow

$$L^{-1}(\cdot) = \int_0^t e^{-\int P(t)dt} \int_0^t e^{\int P(t)dt} (\cdot)dtdt \tag{4}$$

By applying L^{-1}on Eq. (3) becomes as follow:

$$w(t) = \mathrm{k}_0 + \mathrm{k}_1 t + L^{-1}g(t) - L^{-1}N(t) \tag{5}$$

According to MADM, the solution of *w(t)* and *N(t)* introduced by infinite series

$$w(t) = \sum_{n=0}^{\infty} w_n(t) \tag{6}$$

and

$$N(t) = \sum_{n=0}^{\infty} A_n(t) \tag{7}$$

By substituting Eq. (6) and Eq. (7) into Eq. (5)

The general equation of MADM is defined in the serial form as follow:

$$\sum_{n=0}^{\infty} w_n = \mathrm{k}_0 + \mathrm{k}_1 t - L^{-1}\left[\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} g_n\right] \tag{8}$$

Where the components A_n of the solution $N(t)$ and the components $w_n(t)$ of the solution $w(t)$will be resolved recurrently.

$$w_0(t) = \mathrm{k}_0 + \mathrm{k}_1 t \tag{9}$$

$$w_{n+1}(t) = -L^{-1}(A_n - g_n) \qquad ,n \geq 0 \tag{10}$$

2.The mathematical model of the beam subjected to a concentrated force

The equation of motion of the beam under concentrated force *F* is:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = F\delta(x-d) \quad ,0 < x < L \tag{11}$$

For this system, the boundary and initial conditions are:

$$w(0,t) = 0,\quad \frac{\partial^2 w(0,t)}{\partial x^2} = 0\ \ , w(L,t) = 0\ \ , \frac{\partial^2 w(L,t)}{\partial x^2} = 0\,,\ \ \mathrm{w(x,0)} = 0\,,\ \ \frac{\partial \mathrm{w(x,0)}}{\partial \mathrm{t}} = 0 \tag{12}$$

Where *w(x, t), E,I,*ρ, A, F and $\delta$are the vertical displacement , the young's modulus of the material, the moment of inertia of the beam cross-section, the density of the material , the cross-sectional area of the beam, the constant force and is the Dirac delta function, respectively.

A series solution of the Eq. (11) can be written as follow:

$$w(x,t) = \sum_{n=1}^{N} \varphi_n(x)\eta_n(t) \tag{13}$$

Where $\boldsymbol{\varphi_n(x)}$and $\boldsymbol{\eta_n(t)}$ are eigenfunctions and generalized coordinates of the system respectively and *N*is the number of repetitions used to convergent the solution. By substituting Eq. (13) into Eq. (11) and integrating from 0 to *L* after multiplying by $\boldsymbol{\varphi_n(x)}$ lead to

$$EI\sum_{n=1}^{N}\left[\eta_n(t)\int_0^L \frac{d^4\varphi_n(x)}{dx^4}\varphi_n(x)dx\right] + \sum_{n=1}^{N}\rho A\left[\frac{d^2\eta_n(t)}{dt^2}\int_0^L \varphi_n(x)\,\varphi_n(x)dx\right] = \int_0^L F\delta(x-d)\,\varphi_n(x)dx \tag{14}$$

For the simply supported beam, the assumed mode shape may be written as

$$\varphi_n(x) = sin\left(\frac{n\pi x}{L}\right) \tag{15}$$

According to the following condition of orthogonality and properties of Diracdelta functions [6].

$$\int_0^{\mathrm{L}} \varphi_n(\mathrm{x})\,\varphi_n(\mathrm{x})\mathrm{dx} = \begin{cases} 0\,, & \mathrm{n} \neq \mathrm{r} \\ \frac{\mathrm{L}}{2}, & \mathrm{n} = \mathrm{r} \end{cases} \quad and \quad \int \delta(\tau-\tau_0)\,\mathrm{f}(\tau) = \mathrm{f}(\tau_0) \tag{16}$$

Substituting the Eq. (16) into Eq. (14) leads to

$$\frac{d^2\eta_n(t)}{dt^2} + {\omega_n}^2\eta_n(t) = \frac{2F}{L\rho A} sin\left(\frac{n\pi d}{L}\right) \tag{17}$$

where

$$\omega_n = \left(\frac{n\pi x}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \quad (18)$$

2. Analysis of cracked beam eigenfunctions

Consider a cracked simply-supported beam of length L, the crack position at$x = L_1$.A force is applied on the beam at location$d$. Theuniform beam has a cross-section are: width b, crack depth h as shown in Fig. 1

The crack modeled as a rotational spring as shown in Fig. 2

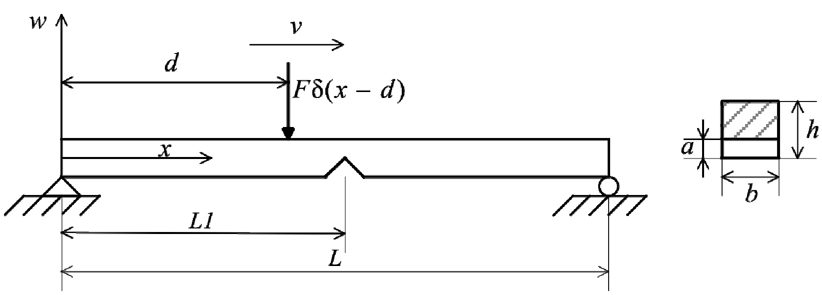

Figure 1. A simply supported cracked beam under concentrated force

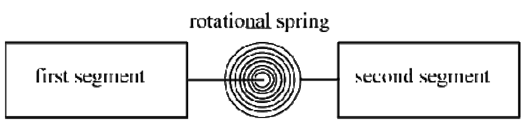

Figure2. Simply supported cracked model

the beam divided into two parts by crack.According to Euler-Bernoulli beam theory theequationof motionthe two parts in the system of free vibration can be written as

$$EI\frac{\partial^4 w_1}{\partial x^4} + \rho A\frac{\partial^2 w_1}{\partial t^2} = 0,0 < x < L_0 \quad (19)$$

$$EI\frac{\partial^4 w_1}{\partial x^4} + \rho A\frac{\partial^2 w_1}{\partial t^2} = 0\,, L_0 < x < L \quad (20)$$

where $w_1$and $w_2$thevertical displacementbefore and after the crack. The boundary conditions of a cracked simply-supported beam case are

$$w_1(0,t) = w_2(L,t) = 0 \;\;, \qquad w_1{''}(0,t) = w_2{''}(L,t) = 0 \quad (21)$$

The compatibility requirements before and after the crack

$$w_1(L_1{}^-,t) = w_2(L_1{}^+,t)$$

$$w_1''(L_1{}^-,t) = w_2''(L_1{}^+,t) \tag{22}$$

$$w_1'''(L_1{}^-,t) = w_2'''(L_1{}^+,t)$$

where the symbols $L_1{}^-$and$L_1{}^+$indicate the locations promptly before and after the crack position, respectively. Discontinuity point at x= L_1 is considered on the beam dividing the beaminto two parts before and after the crack. The condition discontinuity at the crack for a simply connected beam can be expressed as

$$w_2(L_1{}^+,t) = w_1(L_1{}^-,t) + \left(\frac{EIc}{L_1}\right) L_0 w_2''(L_1{}^+,t) \tag{23}$$

where *c* is the local crack flexibility [18], that can be found from the following function

$$c = \frac{[6\pi(1-\upsilon^2)h\phi(\alpha)]}{EI} \tag{24}$$

where *h* is height of the beam section,υ is Poisson ratio and α is the crack depth ratio

$$\alpha = \frac{a}{h} \tag{25}$$

$$(\alpha) = 0.6272\alpha^2 - 1.04533\alpha^3 + 4.5948\alpha^4 - 9.9736\alpha^5 + 20.2948\alpha^6 - 33.0351\alpha^7 + 47.106\alpha^8 - 40.7556\alpha^9 + 19.6\alpha^{10} \tag{26}$$

The mode shape of vibration of the two parts of the beam, before and after the crack, respectively, are

$$\varphi_{1n}(x) = a_1 \sin(\lambda x) + b_1 \cos(\lambda x) + c_1 \sinh(\lambda x) + d_1 \cosh(\lambda x), \qquad x < L_1 \tag{27}$$

$$\varphi_{2n}(x) = a_2 \sin\lambda(x-L_1) + b_2 \cos\lambda(x-L_1) + c_2 \sinh\lambda(x-L_1) + d_2 \cosh\lambda(x-L_1), \quad L_1 < x < L \tag{28}$$

Where the coefficients a_i , b_i, c_i and $d_i$can be found by substituting previous equations into the boundary condition equations., the two spans boundary conditions are:

$$\varphi_{1n}(0) = 0\,, \quad \varphi_{1n}''(0) = 0, \quad \varphi_{2n}(L) = 0, \quad \varphi_{2n}''(L) = 0 \tag{29}$$

By using the Eq. (29) we get

$$b_1 = d_1 = 0 \tag{30}$$

And

$$a_2 \sin\lambda(L-L_1) + b_2 \cos\lambda(L-L_1) + c_2 \sinh\lambda(L-L_1) + d_2\cosh\lambda(L-L_1) = 0 \quad (31)$$

$$-a_2 \sin\lambda(L-L_1) - b_2 \cos\lambda(L-L_1) + c_2 \sinh\lambda(L-L_1) + d_2\cosh\lambda(L-L_1) = 0 \quad (32)$$

Eqs. (31) and (32) can be expressed in a matrix form as following

$$\begin{bmatrix} \sin\lambda(L-L_1) & \cos\lambda(L-L_1) & \sinh\lambda(L-L_1) & \cosh\lambda(L-L_1) \\ -\sin\lambda(L-L_1) & -\cos\lambda(L-L_1) & \sinh\lambda(L-L_1) & \cosh\lambda(L-L_1) \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (33)$$

Using Eqs. (22) and (23) and matching the boundary conditions at the position of crack

$$a_1 \sin(\lambda L_1) + b_1 \cos(\lambda L_1) + c_1 \sinh(\lambda L_1) + d_1\cosh(\lambda L_1) = d_2 + b_2$$

$$\sin(\lambda L_1) - b_1 \cos(\lambda L_1) + c_1 \sinh(\lambda L_1) + d_1\cosh(\lambda L_1) = d_2 - b_2 \quad (34)$$

$$-a_1 \cos(\lambda L_1) + b_1 \sin(\lambda L_1) + c_1 \cosh(\lambda L_1) + d_1\sinh(\lambda L_1) = c_2 - a_2$$

$$a_1 \cos(\lambda L_1) - b_1 \sin(\lambda L_1) + c_1 \cosh(\lambda L_1) + d_1\sinh(\lambda L_1) = c_2 + a_{2+}EIc\lambda(b_2 - d_2) \quad (35)$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} \quad (36)$$

Where

$$s_{11} = \cos\lambda L_1 - \delta\sin\lambda L_1 \quad, s_{12} = -\sin\lambda L_1 - \delta\cos\lambda L_1 \quad, s_{13} = \delta\sinh\lambda L_1 \quad, s_{14} = \cosh\lambda L_1$$

$$s_{11} = \cos\lambda L_1 - \delta\sin\lambda L_1 \quad, s_{12} = -\sin\lambda L_1 - \delta\cos\lambda L_1 \quad, s_{13} = \delta\sinh\lambda L_1 \quad, s_{14} = \cosh\lambda L_1$$

$$s_{21} = \sin\lambda L_1 \quad, s_{22} = \cos\lambda L_1 \quad, s_{13} = 0\,, s_{14} = 0 \quad (37)$$

$$s_{31} = -\delta\sin\lambda L_1 \quad, s_{32} = -\delta\cos\lambda L_1 \quad, s_{13} = \cosh\lambda L_1 + \delta\sinh\lambda L_1\,, s_{14} = \sinh\lambda L_1 + \delta\cosh\lambda L_1$$

and

$$\delta = \frac{\lambda EIc}{2} \quad (38)$$

SubstitutingEq. (36) into Eq. (33) yields

$$\begin{bmatrix} \sin\lambda(L-L_1) & \cos\lambda(L-L_1) & \sinh\lambda(L-L_1) & \cosh\lambda(L-L_1) \\ -\sin\lambda(L-L_1) & -\cos\lambda(L-L_1) & \sinh\lambda(L-L_1) & \cosh\lambda(L-L_1) \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{39}$$

by solving matrix equation (39) the coefficients relations yields

$$\begin{cases} c_1 = \dfrac{a_1\left(\delta\sin(\lambda L_1)\sinh\left(\lambda(L-L_1)\right)\right)}{\sinh\left(\lambda(L-L_1)\right)(\cosh(\lambda L_1)+\delta\sinh(\lambda L_1))+\cosh(\lambda(L-L_1))\sinh(\lambda L_1)} \\ a_2 = a_1\,(\cos(\lambda L_1)-\delta\sin(\lambda L_1))+\cosh\left(\lambda(L-L_1)\right)\sinh(\lambda L_1) \\ b_2 = a_1 \sin(\lambda L_1) \\ c_2 = c_1\left(\cosh(\lambda L_1)+\delta\sinh(\lambda L_1)\right)-a_1\delta\sin(\lambda L_1) \\ d_2 = c_1\sinh(\lambda L_1) \end{cases} \tag{40}$$

all values of coefficients depend on value of a_1

$$\varphi_1(x) = a_1(\sin(\lambda x)+z\sinh(\lambda x)), \qquad 0 < x < L_1 \tag{41}$$

$$\begin{aligned} \varphi_2(x) = a_1 \Big(& (\cos(\lambda L_1)-\delta\sin(\lambda L_1)) + z\delta\sinh(\lambda L_1)\sin\lambda(x-L_1) \\ & + \sin(\lambda L_1)\cos\lambda(x-L_1)\ z\left(\cosh(\lambda L_1)+\delta\sinh(\lambda L_1)\right) - \delta\sin(\lambda L_1)\sinh\lambda(x-L_1) \\ & + z\sinh(\lambda L_1)\cosh\lambda(x-L_1)\Big), L_1 < x < L \end{aligned} \tag{42}$$

where

$$z = \frac{(\delta\sin(\lambda L_1)\sinh(\lambda(L-L_1))}{\sinh\left(\lambda(L-L_1)\right)(\cosh(\lambda L_1)+\delta\sinh(\lambda L_1))+\cosh(\lambda(L-L_1))\sinh(\lambda L_1)} \tag{43}$$

there is only one unknown coefficient a_1 in the Eigen functions, can be found it by using orthonormality condition

$$\|\varphi_n(x)\| = \sqrt{\int_0^L \varphi_n^2(x)dx} = \left(\int_0^{L_1} \varphi_{n1}^2(x)dx + \int_{L_1}^{L} \varphi_{n2}^2(x)dx\right)^{1/2} = 1 \tag{44}$$

a_1 can be obtained by using Eq. (41), (42) and (43) as

$$a_1 = \left(\frac{1}{\int_0^{L_1}\varphi_1^2(x)dx + \int_{L_1}^{L}\varphi_2^2(x)dx}\right)^{1/2} \tag{45}$$

5. Equation of motion of cracked beam subjected to concentrated force

According to the MADM,equation (17) can be written as followsfor the first span of beam

$$\eta_{1n}(t) = \mathrm{k}_0 + \mathrm{k}_1 t - L^{-1}[\omega_n{}^2\,\eta_n(t) - F_s(a_1\,sin(\lambda d) + c_1\,sinh(\lambda d))] \tag{46}$$

$$\sum_{r=0}^{\infty}\eta_r(t) = \mathrm{k}_0 + \mathrm{k}_1 t - L^{-1}\left[\omega_n{}^2\sum_{r=0}^{\infty}\eta_r(t) - F_s(a_1\,sin(\lambda d) + c_1\,sinh(\lambda d))\right] \tag{47}$$

Where $r$is the number of terms

$$\begin{cases}\eta_0(t) = k_0 + k_1 t - L^{-1}[F_s(a_1\sin(\lambda t) + c_1\sinh(\lambda t))] \\ \eta_{r+1}(t) = -L^{-1}[\omega_n{}^2\,\eta_r(t)]\end{cases} \tag{48}$$

the initial conditions are written in the zero-term, the non-homogeneous two-time integral is taken, and in the other terms for r= 0,1,2,3... for first segment of a cracked beam $k_0 = 0$ and $k_1 = 0$

$$\eta_0(t) = -L^{-1}[F_s(a_1\sin(\lambda d) + c_1\sinh(\lambda d))]$$
$$= \iint_0^t F_s(a_1\sin(\lambda d) + c_1\sinh(\lambda d))dtdt = F_s[(a_1\sin(\lambda d) + c_1\sinh(\lambda d))]\frac{t^2}{2!}$$
$$\eta_1(t) = -L^{-1}[\omega_n{}^2\,\eta_0(t)]$$
$$= -\iint_0^t \omega_n{}^2 F_s(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^2}{2!}dtdt = -\omega_n{}^2 F_s\left[(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^4}{4!}\right]$$
$$\eta_2(t) = -L^{-1}[\omega_n{}^2\,\eta_1(t)]$$
$$= \iint_0^t \omega_n{}^4 F_s(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^4}{4!}dtdt = \omega_n{}^4 F_s\left[(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^6}{6!}\right] \tag{49}$$
$$\eta_3(t) = -L^{-1}[\omega_n{}^2\,\eta_2(t)]$$
$$= -\iint_0^t \omega_n{}^6 F_s(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^6}{6!}dtdt = -\omega_n{}^6 F_s\left[(a_1\sin(\lambda d) + c_1\sinh(\lambda d))\frac{t^8}{8!}\right]$$

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From Eq. (49) we can get an approximate amount of $\eta_{1\mathrm{n}}(t)$ for first segment of cracked beam

$$\begin{cases} \eta_{1\mathrm{n}}(t)=\sum_{r=1}^{N}\eta_{\mathrm{r}}(t)=\eta_0+\eta_1+\eta_2+\eta_3+\cdots \\ \eta_{1\mathrm{n}}(t)=\frac{F_s}{{\omega_n}^2}[a_1\sin(\lambda d)+c_1\sinh(\lambda d)]\left[\frac{(\omega_n t)^2}{2!}-\frac{(\omega_n t)^4}{4!}+\frac{(\omega_n t)^6}{6!}-\cdots\right] \end{cases} \quad (50)$$

$$\eta_{1\mathrm{n}}(t)=\frac{F_s}{{\omega_n}^2}[a_1\sin(\lambda d)+c_1\sinh(\lambda d)][1-cos(\omega_n t)] \quad (51)$$

For the second segment of cracked beamof the beam

$$\begin{cases} \eta_{2n}(t)=k_0+k_1 t-L^{-1}\left[{\omega_n}^2\,\eta_n(t)-F_s\begin{pmatrix} a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1) \\ +c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)\end{pmatrix}\right] \\ \sum_{r=0}^{\infty}\eta_r(t)=k_0+k_1 t-L^{-1}\left[{\omega_n}^2\sum_{r=0}^{\infty}\eta_r(t)-F_s\begin{pmatrix} a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1) \\ +c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)\end{pmatrix}\right] \\ \eta_0(t)=\ k_0+k_1 t-L^{-1}\left[F_s\begin{pmatrix} a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1) \\ +c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)\end{pmatrix}\right] \\ \eta_{r+1}(t)=-L^{-1}[{\omega_n}^2\,\eta_r(t)] \end{cases} \quad (52)$$

for r= 0,1,2,3 we obtained

$$\begin{cases} \eta_0(t)=k_0+k_1 t-L^{-1}\left[F_s\big(a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1)+c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)\big)\right] \\ =k_0+k_1 t-\iint_0^t F_s\big(a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1)+c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)\big)dtdt \\ \eta_0(t)=k_0+k_1 t-F_s[a_2\,sin\,\lambda(d-L_1)+b_2 cos(d-L_1)+c_2\,sinh\,\lambda(d-L_1)+d_2 cosh(d-L_1)]\frac{t^2}{2!} \end{cases} \quad (53)$$

$$\begin{cases} \eta_1(t) = -L^{-1}[\omega_n{}^2\, \eta_0(t)] \\ = -\omega_n{}^2 \iint_0^t k_0 + k_1 t - F_s[a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) + c_2 \sinh\lambda(d-L_1) + d_2 cosh(d-L_1)]\dfrac{t^2}{2!} \\ \eta_1(t) = -\omega_n{}^2\left[\dfrac{k_0 t^2}{2!} + \dfrac{k_1 t^3}{3!} - F_s\begin{pmatrix} a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) \\ +c_2 \sinh\lambda(d-L_1) + d_2 cosh(d-L_1)\end{pmatrix}\dfrac{t^4}{4!}\right] \end{cases} \quad (54)$$

$$\begin{cases} \eta_2(t) = -L^{-1}[\omega_n{}^2\, \eta_1(t)] \\ = \omega_n{}^4 \iint_0^t \dfrac{k_0 t^2}{2!} + \dfrac{k_1 t^3}{3!} - F_s\begin{bmatrix} a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) + c_2 \sinh\lambda(d-L_1) \\ +d_2 cosh(d-L_1)\end{bmatrix}\dfrac{t^4}{4!}dtdt \\ \eta_2(t) = \omega_n{}^4\left[\dfrac{k_0 t^4}{4!} + \dfrac{k_1 t^5}{5!} - F_s\begin{bmatrix} a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) + c_2 \sinh\lambda(d-L_1) \\ +d_2 cosh(d-L_1)\end{bmatrix}\dfrac{t^6}{6!}\right] \end{cases} \quad (55)$$

$$\begin{cases} \eta_3(t) = -L^{-1}[\omega_n{}^2\, \eta_2(t)] \\ -\omega_n{}^6 \iint_0^t \dfrac{k_0 t^4}{4!} + \dfrac{k_1 t^5}{5!} - F_s\,[a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) + c_2 \sinh\lambda(d-L_1) \\ \qquad +d_2 cosh(d-L_1)]\dfrac{t^6}{6!}dtdt \\ \eta_3(t) = -\omega_n{}^6\left[\begin{matrix}\dfrac{k_0 t^6}{6!} + \dfrac{k_1 t^7}{7!} - F_s[a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) + c_2 \sinh\lambda(d-L_1) \\ +d_2 cosh(d-L_1)]\dfrac{t^8}{8!}\end{matrix}\right] \end{cases} \quad (56)$$

$$\begin{cases} \eta_{2\mathrm{n}}(t) = \sum_{r=1}^{N} \eta_{\mathrm{r}}(t) = \eta_0 + \eta_1 + \eta_2 + \eta_3 + \cdots \\ \eta_{2\mathrm{n}}(t) = k_0\left[1 - \dfrac{(\omega_n t)^2}{2!} + \dfrac{(\omega_n t)^4}{4!} - \dfrac{(\omega_n t)^6}{6!} + \cdots\right] + \dfrac{k_1}{\omega_n}\left[(\omega_n t - \dfrac{(\omega_n t)^3}{3!} + \dfrac{(\omega_n t)^5}{5!} - \dfrac{(\omega_n t)^7}{7!} + \cdots\right] \\ + \dfrac{F_s}{\omega_n{}^2}\begin{bmatrix} a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) \\ +c_2 \sinh\lambda(d-L_1) + d_2 cosh(d-L_1)\end{bmatrix}\left[\dfrac{(\omega_n t)^2}{2!} - \dfrac{(\omega_n t)^4}{4!} + \dfrac{(\omega_n t)^6}{6!} - \cdots\right] \end{cases} \quad (57)$$

Finally, $\eta_{2\mathrm{n}}(t)$ is obtained as follow

$$\eta_{2\mathrm{n}}(t) = k_0 cos(\omega_n t) + \frac{k_1}{\omega_n} sin(\omega_n t) + \frac{F_s}{\omega_n{}^2}\begin{bmatrix} a_2 \sin\lambda(d-L_1) + b_2 cos(d-L_1) \\ +c_2 \sinh\lambda(d-L_1) + d_2 cosh(d-L_1)\end{bmatrix}[1 - cos(\omega_n t)] \quad (58)$$

To determine k_0 and k_1 in Eq. (59), the initial conditions are used

$$\eta_{n1}\left(\frac{L_0}{v}\right) = \eta_{n2}\left(\frac{L_0}{v}\right) = H\left(\frac{L_0}{v}\right) + k_0 sin\left(\frac{\omega_n L_0}{v}\right) + \frac{k_1}{\omega_n} cos\left(\frac{\omega_n L_0}{v}\right) \quad (59)$$

and

$$\dot{\eta}_{n1}\left(\frac{L_0}{v}\right) = \dot{\eta}_{n2}\left(\frac{L_0}{v}\right) = \dot{H}\left(\frac{L_0}{v}\right) + k_0\omega_n\, cos\left(\frac{\omega_n L_0}{v}\right) - k_1 sin\left(\frac{\omega_n L_0}{v}\right) \quad (60)$$

Where

$$H(t) = \frac{F_s}{{\omega_n}^2}\begin{bmatrix} a_2\, sin\,\lambda(d - L_1) + b_2 cos(d - L_1) \\ +c_2\, sinh\,\lambda(d - L_1) + d_2 cosh(d - L_1)\end{bmatrix}[1 - cos(\omega_n t)] \quad (61)$$

By using the modal expansion theorem, we get

$$w(x,t) = \sum_{n=1}^{N} \varphi_n(x)\eta_n(t) \quad (62)$$

$$w_1(x,t) = \sum_{n=1}^{N} \varphi_{n1}(x)\frac{F_s}{{\omega_n}^2}[a_1 \sin(\lambda d) + c_1 \sinh(\lambda d)][1 - cos(\omega_n t)] \quad (63)$$

$$w_2(x,t) = \sum_{n=1}^{N} \varphi_{n2}(x)\left[\frac{F_s}{{\omega_n}^2}\begin{bmatrix} a_2\, sin\,\lambda(d - L_1) + b_2 cos(d - L_1) \\ +c_2\, sinh\,\lambda(d - L_1) + d_2 cosh(d - L_1)\end{bmatrix}[1 - cos(\omega_n t)] + k_0 cos(\omega_n t) \quad + \frac{k_1}{\omega_n} sin(\omega_n t)\right] \quad (64)$$

Numerical results

A simply supported cracked beam with$L = 30m,\ L_0 = 15m, h = 3m, b = 8\ m, \rho = 3824kg/m$ and$E = 28\ GPa, I = 18m^4$ is considered. In order to verify the accuracy of AMDM for a cracked beam subjected to a 60 kN concentrated force, firstly we consider an intact beam, by this assumption (α) in Eq.26 becomes equal to zero is equivalent the crack fully closed, or beam is intact. Comparison between results obtained by deflection of a cracked beam subjected to concentrated force when α=0 and intact beam subjected to a concentrated force that has been investigated by Bilik and Karaçay [18] is shown in Fig (3). Having an excellent agreement between the results, we can use the AMDM to analyze the vibration of the cracked beam subjected to concentrated force.

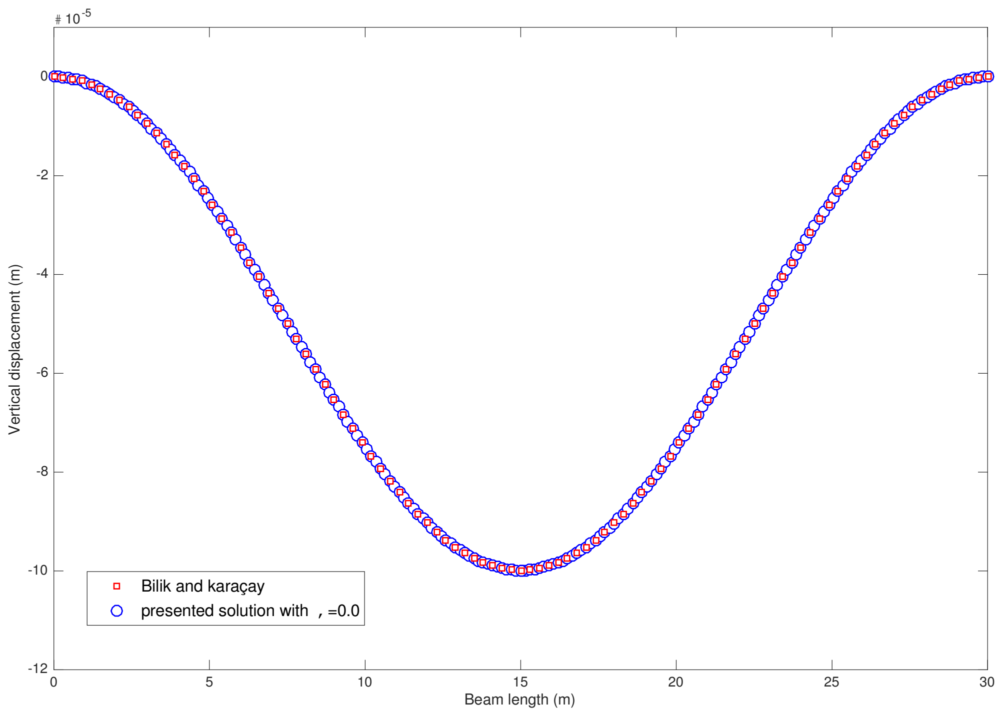

Figure 3. Comparison of the results of the presented study withBilik and Karaçay for intact and cracked simply-supported beam subjected to concentrated force when $\alpha = 0.0$

In Fig.4, the vertical axis shows the vertical displacement and horizontal axis representsthe length of the cracked simply supported beam under concentrated force. The vertical displacement has been plotted for several of crack depth ratios (α), as expectant the deflection of cracked beam increases as the crack depth ratio (α) increases

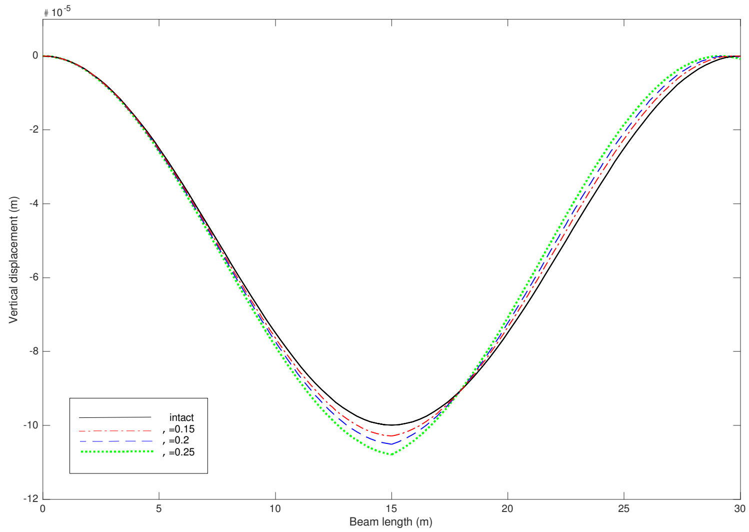

Figure 4. Vertical displacement with respect beam length with various crack depth ratio

$$L_1 = 15\ m\ ; F = 60\ kN$$

In Fig.5, the vertical axis shows the vertical displacement and horizontal axis represents the length of cracked simply supported beam under concentrated force. The vertical displacement has been plotted for several values of crack position, $L_1 = (0.3L, 0.5L$ and $0.7L)$ and $\alpha = 0.2$. As the crack position shift away from the midpoint of the beam, the deflections decrease.

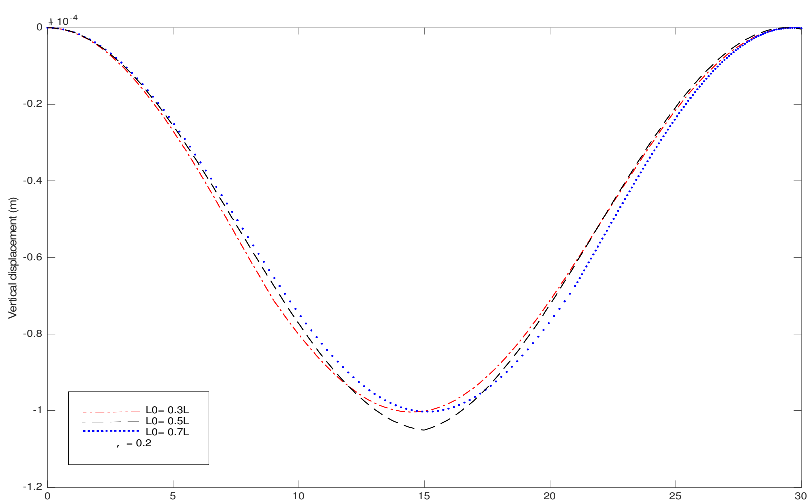

Figure 5. Vertical displacement with respect beam length with various crack position

In Fig.6, the vertical axis shows the vertical displacement and horizontal axis represents the length of cracked simply supported beam under concentrated force. The vertical displacement has been plotted for several values of applied forces F= (60 kN, 70 kN, 80 kN) and $\alpha = 0.2$. As the force increases on the cracked beam, the deflections also increase.

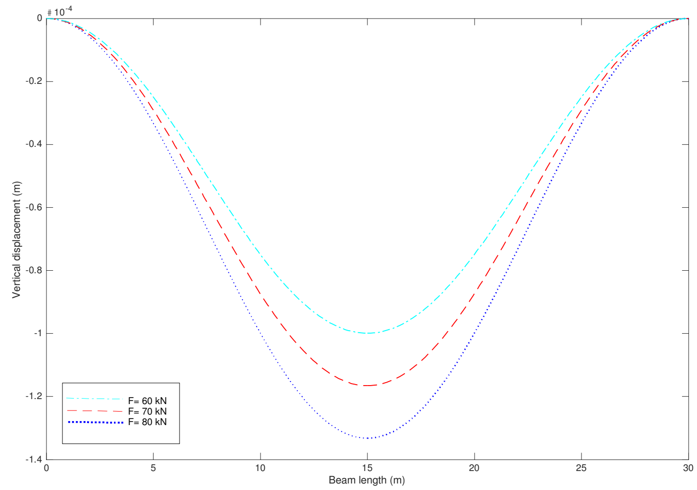

Figure 6. Vertical displacement with respect beam length with various forces($\alpha = 0.2$)

3. **Conclusions**

In this paper, the vibration of a cracked beam subjected to concentrated force was analyzed using Modified Adomian Decomposition Method (MADM). The theory of Euler-Bernoulli beam has been used. The formulation of the vertical displacement of the cracked beam was presented. The presented method was validated by comparing its result with the published result, it was found that the results of the MADM have good agreement with the published result. The effect of crack depth ratio(α) on the beam deflection was also investigated. It was found that by increasing the crack depth ratio, the deflection of a cracked beam also increased. The effect of crack location on the beam deflection was also investigated. It was seen that as the crack location shift away from the midpoint of the beam, the deflections decrease. future works are planned to analyze the vibration of the cracked beam under the moving force by usingMADM.

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