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Physical Deriving and Discussion of Heisenberg Uncertain Principal Through Using Schrödinger Equation and Yuji – Masanao Experiment

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Abstract:

In this paper, authors show the process of the physical deriving of the Heisenberg Uncertainty Principal under the assumption of the wave function being a displacement function in Schrödinger Equation. This process indicates the Heisenberg Uncertainty Principal is compatible with the assumption of the wave function being a displacement function.

It is found that, for a single quantum system, there exists a certainty relationship between the position x and the momentum p.

If the wave physical characteristics are not considered, then the product of the position and the momentum satisfies the below inequality:

$$\left(\frac{\bar{h}}{2}\right) \leq xp$$

That is to say, the Heisenberg Uncertainty Principal is true. $\bar{h}$ is the reduced Plank constant.

If the wave physical characteristics are considered, then the product of the position and the momentum satisfies the below inequality:

$$0 \leq xp < \left(\frac{\bar{h}}{2}\right)$$

This equation provides theoretical support to Yuji Hasegawa and Masanao Ozawa Experiment. This experiment does not violate the Heisenberg Uncertainty Principal.

Keywords — **Heisenberg Uncertain Principal, Schrödinger Equation, Schrödinger Momentum Equation, Yuji Hasegawa and Masanao Ozawa Experiment.**

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I. INTRODUCTION

As we known, the Heisenberg Uncertainty Principle is one of the fundamental concepts in quantum mechanics and was first introduced in 1927 by German physicist Werner Heisenberg [1][2]. This principal says that if we measure simultaneously position, x, and momentum, p, then there is a limit to the precision. That is to say, the more the position is accurately measured, the less the momentum can be accurately known.

In 1927 and 1928 later, German theoretical physicist Earle Hesse Kennard and Hermann Weyl gave a rigorous mathematical deriving and description about the Heisenberg Uncertainty Principle of the position and momentum [3][4].

However, the disputes about the physical interpretation of the Heisenberg Uncertainty Principle of quantum mechanics have existed over one hundred of years. Without any questions, all physics community agree to the mathematical derivation of the uncertainty relation. But the disagreements have lasted many years. The reason of causing the disagreements is lack of the physical deriving of the Heisenberg Uncertainty Principal.

Recently, an important dispute happened on January 15, 2012. Yuji Hasegawa and Masanao Ozawa of Nagoya University published their empirical results that refuted Heisenberg's uncertainty principle [5][6], it is called Yuji - Masanao Experiment in this paper. They used two instruments to measure the spin angle of neutrons and calculated the results and obtained a smaller error than that shown by Heisenberg Uncertainty Principle. This proved that the measurement limit advocated by Heisenberg's uncertainty principle may be not correct.

This experiment results remind us of thinking about the Heisenberg's Uncertainty Principle, although a group of researchers from three countries had provided some theoretical analysis to support the uncertainty principal as Heisenberg envisioned it in 2014 [7]. However, they could not clearly explain why this experiment generates the measurement limit less than the prediction from the Heisenberg Uncertainty Principal. The contradiction between the Heisenberg Uncertainty Principal and Yuji - Masanao Experiment makes us feel that it is necessary for a deeper understanding of the Heisenberg Uncertain Principal and providing theoretical explanation to Yuji - Masanao Experiment.

In order to do so, let's recall that E. Schrödinger thought that his wave mechanics was equivalent to Heisenberg's matric mechanics. And E. Heisenberg agreed with E. Schrödinger, too. According to their thoughts and fact of which the Heisenberg Uncertainty Principle is related to the position and the momentum, authors think that Schrödinger Equation should be related to the position and the momentum. So, it is reasonable to assume that the wave function in Schrödinger Equation [8] is a displacement function.

Based on this assumption, authors investigate the Heisenberg Uncertainty Principle through using Schrödinger Equation. It is found that the Heisenberg Uncertainty Principle can be physically derived from Schrödinger Equation. Although there is the certainty relationship between the position and momentum, the Heisenberg Uncertainty Principle is still true. The theoretical analyses indicate that there is existence of the measurement limit for the Heisenberg Uncertainty Principle under some conditions. This conclusion matches the results of Yuji – Masanao experiment.

II. Physical Deriving of Heisenberg's Uncertainty Principle

Without losing generality, we only consider one dimensional case.

We all know that Schrödinger Equation can be write the following form [6]:

$$-\frac{h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+\psi(x,t)V(x)=ih\frac{\partial\psi(x,t)}{\partial t} \tag{1}$$

where

h is Planck's constant;

$\psi(x,t)$ is a wave function;

m is a mass of quantum;

$V(x)$ is a conservative field of force;

x is a displacement;

t is time;

i is an imaginary number, $i=\sqrt{-1}$.

Now we assume that this wave function, $\boldsymbol{\psi(x,t)}$, is a displacement function. Based on this assumption, we have

$$\frac{\partial\psi(x,t)}{\partial t}=v \tag{2}$$

where v is a velocity of quantum.

Take Equation (2) into Equation (1), then we have that

$$-\frac{h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+m\psi(x,t)V(x)=ihv$$

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+\frac{m\psi(x,t)V(x)}{h}]=ip \tag{3}$$

where

$p=vm$ is momentum of quantum . $p=\frac{h}{\lambda}=hk$, λ is wave's length and k is the number of wave. This is well-known de Broglie formula. We call Equation (3) as Schrödinger Momentum Equation.

Without losing generality, we assume that $V(x)$ =0. That is, the motion is not affected by the conservative field of force. Then we have that

$$[-\frac{h}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}] = ip \qquad (4)$$

If integrate x for the two sides of Equation (4), then we have that

$$\Rightarrow \quad \left[-\frac{h}{2}\int_0^x \frac{\partial^2\psi(x,t)}{\partial x^2}\,dx\right] = i\int_0^x (p)\,dx$$

If we notice p is not a function of position according to de Broglie formula, then we have that

$$\Rightarrow [-\frac{h}{2}\left(\frac{\partial\psi(x,t)}{\partial x} - \frac{\partial\psi(0,t)}{\partial x}\right) = i\,p(x-0) = ixp$$

Assume $\frac{\partial\psi(0,t)}{\partial x} = 0$, then we have that

$$[-\frac{h}{2}\frac{\partial\psi(x,t)}{\partial x}] = i\,xp \qquad (5)$$

Rewrite Equation (5) as follows:

$$[-\frac{h}{2}\frac{\partial\psi(x,t)}{\partial t}\frac{\partial t}{\partial x}] = i\,xp \qquad (6)$$

Now we assume $\psi(x,t) = Ae^{i(kx-\omega t)}$, here k is wave number, ω is angular frequency, A is a wave amplitude. Take derivate for t, then we have that

$$\frac{\partial\psi(x,t)}{\partial t} = -\omega Aie^{i(kx-\omega t)} \qquad (7)$$

Take Equation (7) into the above equation (6), then we have that

$$\Rightarrow \quad [-\frac{h}{2}(-\omega Aie^{i(kx-\omega t)})\frac{\partial t}{\partial x}] = i\,xp$$

$$\Rightarrow \quad [\frac{h}{2}\omega Ae^{i(kx-\omega t)}\frac{\partial t}{\partial x}] = xp \qquad (8)$$

If notice Euler formula,

$$e^{i(kx-\omega t)} = \cos(kx-\omega t) + i\sin(kx-\omega t)$$

Take Euler formula into Equation (8), then we have that

$$\{\frac{h}{2}\omega A[\cos(kx-\omega t) + i\sin(kx-\omega t)]\frac{\partial t}{\partial x}\} = xp$$

Compare the two sides of the above equation, we have that

$$\{\frac{h}{2}\omega A[\cos(kx-\omega t)]\frac{\partial t}{\partial x}\} = xp \qquad (9)$$

We rewrite $\frac{\partial t}{\partial x} = \frac{1}{\frac{\partial x}{\partial t}}$ and take it into Equation (9), then we have that

$$\{\frac{h}{2}\omega A[\cos(kx-\omega t)]\frac{1}{\frac{\partial x}{\partial t}}\} = xp \qquad (10)$$

Now we refer to the circle in Figure 1.

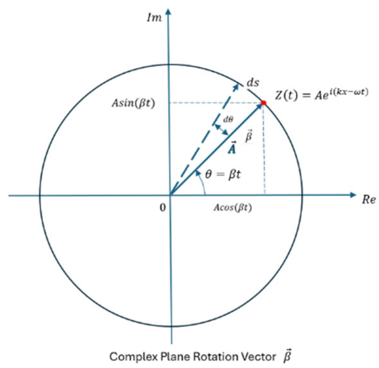

Figure 1

Notice $\frac{\partial x}{\partial t}$ is a linear velocity of the rotation vector tip, we have that

$$\Rightarrow \quad \frac{\partial x}{\partial t} = \frac{dS}{dt} = \frac{d(A\theta)}{dt} = A\frac{d\theta}{dt} = A\beta = 2\pi\omega A$$

where
S is an arc length;
θ is a rotation angle;

A is a length of the rotation complex vector;
ω is angular frequency;
β is angular velocity, $\beta = 2\pi\,\omega$.
Take the above formula, $\beta = 2\pi\,\omega$, into Equation (10), then we have that

$$\{\frac{h}{2}\omega A[\cos(kx-\omega t)]\frac{1}{\frac{\partial x}{\partial t}}\} = xp$$

$$\Rightarrow \quad \{\frac{h}{2}\omega A[\cos(kx-\omega t)]\frac{1}{2\pi\omega A}\} = xp$$

$$\Rightarrow \quad \{\frac{h}{4\pi}[\cos(kx-\omega t)]\} = xp$$

Let $\bar{h} = \frac{h}{2\pi}$ is the reduced Plank constant, we have that

$$\{\frac{\bar{h}}{2}[\cos(kx-\omega t)]\} = xp \qquad (11)$$

Since $0 < x < +\infty$, $0 < t < +\infty$, then we have that $0 < xp < +\infty$.

Equation (11) indicates that there exists a certainty relationship between the position x and the momentum, p. If x or p are known, then p or x can be known by using Equation (11). However, since the existence of $[\cos(kx-\omega t)]$, it will directly affect the scope of values of xp.

Now we discuss these two cases: consider the wave physical characteristics and do not consider the wave physical characteristics.

Case 1 $\quad |\cos(kx-\omega t)| < 1$

For this case, it means that we consider the wave physical characteristics, k, t and ω are not zero. Since $|\cos(kx-\omega t)| < 1$, Equation (11) will become that

$$0 \le xp < \left(\frac{\bar{h}}{2}\right) \qquad (a)$$

This inequality indicates that the value of xp could be less than $\frac{\bar{h}}{2}$.

This conclusion provides theoretical support to the results of Yuji – Masanao Experiment. In the experiment of Yuji Hasegawa and Masanao Ozawa, they indeed considered the wave physical characteristics.

Case 2 $\quad |\cos(kx-\omega t)| = 1$

For this case, it means that we do not consider the wave physical characteristics, k, t and ω are zero. Since $|\cos(kx-\omega t)| = 1$. Then we have

$$\left(\frac{\bar{h}}{2}\right) = xp \qquad (b)$$

However, if consider the domain of xp, $0 < xp < +\infty$, this equation is equivalent to the following one:

$$\left(\frac{\bar{h}}{2}\right) \le xp \qquad (c)$$

The above equation is the Heisenberg Uncertainty Principal.

If consider p is the change of momentum for Δx, we can rewrite $p = \Delta p$. So, we have the convention expression of the Heisenberg Uncertain Principle as follows:

$$\left(\frac{\bar{h}}{2}\right) \le \Delta x \Delta p \qquad (12)$$

This indicates that there exists the smallest limit for Heisenberg Uncertainty Principal. This smallest limit is $\frac{\bar{h}}{2}$.

Discussions:

From the above discussions, for a single quantum system, if we do not consider the wave physical characteristics and time, the Heisenberg Uncertainty Principle is true and the smallest limit is $\frac{\bar{h}}{2}$. If we consider the wave physical characterist-ics and time, the Heisenberg Uncertainty Principle could not be used. The scope of the product of the position and the momentum is between $0 \le xp < \left(\frac{\bar{h}}{2}\right)$.

We can see that, from the above process of the physical deriving, the Heisenberg Uncertainty

Principal is compatible with the assumption of the wave function being a displacement function in Schrödinger Equation. That is, if the Heisenberg Uncertainty Principal is true, then the assumption of the wave function being a displacement function in Schrödinger Equation is true. Vice versa.

If the wave physical characteristics are not considered, then the product of the position and the momentum satisfies the Heisenberg Uncertainty Principal.:

$$\left(\frac{\hbar}{2}\right) \leq xp$$

III. CONCLUSIONS

In this paper, inspired by the thoughts of E. Schrödinger and W. Heisenberg, we make the assumption of the wave function being a displacement function. Under the assumption, we show the process of the physical deriving of the Heisenberg Uncertainty Principal from Schrödinger Equation. This process indicates the Heisenberg Uncertainty Principal is compatible with the assumption of the wave function being a displacement function.

It is found that there exists a certainty relationship between the position x and the momentum, p. For a single quantum system, if the wave physical characteristics are considered, then the product of the position and the momentum satisfies the below inequality:

$$0 \leq xp < \left(\frac{\hbar}{2}\right)$$

This inequality explains Yuji – Masanao experiment. This experiment does not contradict the Heisenberg Uncertainty Principal.

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