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Analyzing Dynamic Systems Using Laplace Transform

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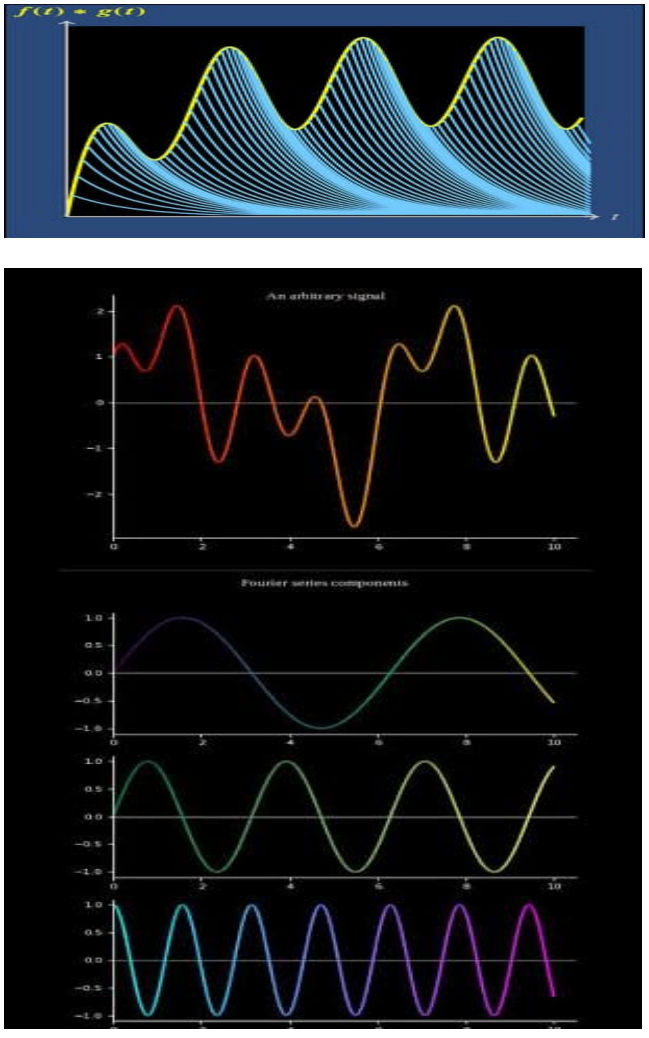
Abstract— In this paper, we are interested to discuss about properties and applications of Laplace transform and Fourier transform in various fields and various areas. Also we discuss Laplace transform has the master techniques used by researchers, scientists and mathematicians and engineers. This paper will discuss the fundamentals of Laplace transform and Fourier transform and basic applications of these fundamentals to electric circuits and signal design and solution to related problems also we will study to solve research problems by using Laplace transform and Fourier transform. The motive of this paper is that a scientific review on properties and applications of Laplace transform and Fourier transform.

Keywords— Laplace transforms, Fourier Transform, Signal Processing, Frequency Domain Analysis, Time-Frequency Analysis, System Analysis, Mathematical Modeling, Linear Systems, Differential Equations, and Applications.

Graphical Abstract



"Time–Frequency Analysis Using Laplace and Fourier Transforms"
Input signal f (t) analyzed in time domain

1. LAPLACE TRANSFORM

Introduction

Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equation, in this paper overview of properties of Laplace transform with definition and its application in engineering and applied science. The Laplace transform is integral transform which is denoted by L [f (t)]. It finds very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics, signal processing. The Laplace transform can be interpreted as a transformation from the time to the frequency domain where inputs and outputs are function of time f(t) to be Laplace transformable, it must satisfy the following conditions;

1) f (t) must be piecewise continuous which means that it must be single valued but can have a finite number of finite isolated discontinuities for t > 0.

2) f (t) must be exponential order which means that f(t) must remain less than $se^{-a_0 t}$ as t approaches ∞ where S is a positive constant and a_0 is a real positive number.

The solution of linear, ordinary differential equation with constant coefficients such as the third order equation $af'''(t) + bf''(t) + cf'(t) + df(t) = g(t)$ can be solved by first obtaining the general form for the expression $f(t)$. This general form will contain a integration constants whose values can be found by appropriate boundary conditions. A various systematic way of solving these equations is to use the Laplace transform which transform the differential equation into an algebraic equation and has the added incorporate advantage the boundary conditions from the beginning. Furthermore, if f (t) represents function with discontinuities, other methods fail where Laplace transform method can succeed.

Laplace transform techniques also provide powerful in various fields of technology such as control theory, population growth and decay problems where knowledge of the system transfer function is important and at which Laplace transform comes into its own.

Definition of Laplace transform

The Laplace transform of the function (t) for all $t \geq 0$ is defined as [1-5]

$$L[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt = F(s) \ldots\ldots\ldots\ldots\ldots (1)$$

Where L is Laplace transform operator. The Laplace transform of the function (t) for all $t \geq 0$ exist if (t) is exponential order and piecewise continuous. These are only sufficient conditions for the existence of Laplace transform of the function (t).

Features of Laplace transform [2]

Some of the very important properties of Laplace transforms which will be used in its applications to be discussed later on are described as follows:

Linearity

If Laplace transform of function f (t), g (t) and h (t) be $\overline{f(s)}$, $\overline{g(s)}$ and $\bar{h}$ (S) respectively, then

$L\{af(t)+ bg(t)+ h(t)\} = a\bar{f}(s)+ b\bar{g}(s)+ c\bar{h}$ (s) Where a, b, c are constant.

Laplace transform of Derivative

$L[f'(t)] = s\,L[f(t)] - f(0)$,

$L[f''(t)] = s^2 L[f(t)] - s\,f(0) - f'(0)$,

$L[f'''(t)] = s^3 L[f(t)] - s^2 f(0) - sf'(0) - f''(0)$, and so on.

Laplace transform of Integrals

$If\ L[f(t)] = \bar{f}(s), then$

$L\left\{\int_0^t f(u)du\right\} = \frac{1}{s}\bar{f}(s)$ & $\int_0^t f(u)du = L^{-1}\left(\frac{\bar{f}(s)}{s}\right)$

Laplace transform of Periodic function

$L[f(t)] = \frac{1}{1-e^{sT}} \int_0^T e^{-st} f(t)\, dt$

A function f (t) is said to be periodic if and only if there exists a positive real number T such that for all values of t,

$f(t) = f(t+T) = f(t+2t) = --= f(t+nT)$

The positive real number T is called the period of the function f (t) and it is the least value of variable t by which it may be increased or decreased without affecting the value of the function f (t).

Division by t

If $L\{f(t)\} = \bar{f}(s)$, then

$L\left[\frac{f(t)}{T}\right] = \int_S^\infty \bar{f}(s)\, ds$

Unit step function

The unit step function can be said to be the force of magnitude 1 which is applied to a system at time $t \geq 0$ and it is defined as

$$U(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

$$L[U(t)] = \frac{1}{s},\ s > 0$$

Transfer function

Mathematically it is defined as the ratio of Laplace transform of output (response) of the system to the Laplace transform of input (driving function); under the assumption that all initial conditions are zero. [21]

If T(s) is the transfer function of the system then,

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(s)}{R(s)}$$

Time delay

The substitution of $t - \lambda$ for the variable t in the transform Lf (t) corresponds to the multiplication of the function F(s) by $e^{-\lambda s}$ that is

$$L(f(t-\lambda)) = e^{-\lambda s} F(s)$$

Second shifting Theorem

$$If\ L[f(t)] = \bar{f}(s), then$$

$$[f(t-a)u(t-a)] = e^{-as}\bar{f}(s)$$

USABLILITY OF LAPLACE TRANSFORM

1. **Simplifies differential equations**:

It transforms differential equations into algebraic equations, making them easier to solve.

2. **Analyzes systems:**

It's widely used in engineering and physics to analyze the behaviour of linear time-invariant systems, including electrical circuits and mechanical systems.

3. **Frequency domain analysis:**

It allows analysis in the frequency domain (s-domain), providing insights into the system's frequency response.

4. **Solves initial value problems**:

The Laplace transform can directly solve initial value problems without first solving the corresponding homogeneous equation.

RESTRICTION:

- The Laplace transform is primarily used for linear, time-invariant systems.
- It may not be suitable for all types of differential equations, especially those with non-constant coefficients.

2. FOURIER TRANSFORM

Introduction

Fourier Transform is useful in the study of solution of partial differential equation to solve initial boundary value problems we use Fourier Transform in signal &image processing. It is also useful in cell phones, LTI system & circuit analysis we obtain Fourier Transform by a limiting process of Fourier series. Since it was first used by French Mathematician Jean Baptist Fourier (1768-1830) in a manuscript submitted to the Institute of France in 1807.He said that Fourier Transform is a mathematical procedure which transforms a function from time domain to frequency domain. Fourier analysis is useful in almost every aspect of the subject ranging from solving LDE to developing computer models, to the processing & analysis of data. The Fourier Transform is a tool that breaks a waveform (a function or signal) into an alternate representation characterized by sine & cosines

Definition of Fourier Transform

The generalisation of the complex Fourier series is known as the Fourier transform. Fourier transform entails representation of a non-periodic function not as a sum but as an integral over a continuous range of frequencies [20][18]. The term "Fourier transform" can be used in the mathematical function, and it is also used in the representation of the frequency domain, The Fourier Transform is a generalization of the Fourier series. Fourier transform basically involve frequency domain representation of non-periodic function, in which such representation is valid over the entire time domain and accomplished by means of Fourier integral[19][18]. It only applies to continuous & a periodic functions. We defined Fourier Transform of a piecewise continuous & absolutely integral function x (t) by

$$f(x) = \int_{-\infty}^{\infty} x(t).e^{-jwt}dt$$

Features of Fourier Transform

Properties of the Fourier transform facilitate the transformation from the time domain to frequency domain & vice versa. [17]

Linearity-

The Fourier Transform satisfies linearity & principle of superposition Consider two functions $x_1(t)$ & $x_2(t)$

If $F[x_1(t)] = X_1(\omega)$, $F[x_2(t)] = X_2(\omega)$

Then $F[a_1 x_1(t) + a_2 x_2(t)] = a_1 X_1(\omega)\, a_2 X_2(\omega)$

Convolution

Fourier transform makes the convolution of 2 signals into the product of their Fourier Transforms. There are two types of convolution properties, one for time domain & other for frequency domain

Scaling

$F[x(t)] = X(\omega)$

If a is real constant then

$F[x(at)] = \frac{1}{|a|} X(\omega)$

Shifting Property

$X(t - t_0) = e^{-jwt_0} x(\omega)$

Symmetry

If $F[x(t)] = X\omega$ then $X(t) = 2\pi X(-\omega)$

Modulation Property

$F\{x(t)\cos at\} = \frac{1}{2}\{X\omega + a) + X(\omega - a)\}$

$F\{x(t)\sin at\} = \frac{1}{2}\{X\omega + a) - X(\omega - a)\}$

Duality

$X(t) = 2\pi x(-\omega)$

Differentiation

$\frac{dx}{dt}(t) = j w X(\omega) \frac{dx}{dt}(t)$

USABLILITY OF FOURIER TRANSFORM

1. **Signal Analysis:**

It allows us to break down complex signals into their constituent frequencies, making it easier to understand and manipulate them**.**

2. **Signal Processing:**

Analyzing and manipulating audio, video, and other signals.

3. **Image Processing**:

Compressing and enhancing images.

4. **Physics:**

Analyzing wave phenomena, heat transfer,and more.

5. **Engineering:**

Analyzing circuits, vibrations, and other systems.

RESTRICTION

- Not all functions can be represented by a Fourier series, and the convergence of the series is not always guaranteed.
- Well-behaved functions, such as smooth functions, have Fourier series that converge to the original function.

Fourier Transform in Two Dimensions

Fourier transform in two-dimensions is given as follows:

$$F(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(k_x,k_y) e^{-2\pi i(k_x x + k_y y)} dk_x dk_y$$

$$f(k_x,k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{2\pi i(k_x x + k_y y)} dx dy$$

Relationship between LT and FT chart

	Laplace transform	Fourier transform	Relationship
1.	Transforms a function of time (t) into a function of a complex variable (s).	Transforms a function of time (t) into a function of frequency (ω).	The Laplace transform is a more general transform than the Fourier transform.
2.	The complex variable s has a real part (σ) and an imaginary part (j ω).	The Fourier transform is a special case of the Laplace transform where the complex variable s is restricted to the imaginary axis (s = j ω).	The Laplace transform can be used to analyze both transient and steady-state behavior of systems, while the Fourier transform is primarily used for steady-state analysis
3.	The Laplace transform can handle signals that decay or grow exponentially, which the Fourier transform cannot.	The Fourier transform is suitable for analyzing periodic or sinusoidal signals	If a signal is absolutely integral and one-sided (meaning it is zero for t < 0), then the Laplace transform evaluated on the imaginary axis (s = j ω) is equal to the Fourier transform.
4.	Lt is used only for transient signal analysis	Ft is used only for steady state signal analysis	
5.	Lt can be functioned for time varying signals	Ft can be functioned for periodic signals	

3. Applications of Laplace Transform

A. Population Growth problem

The growth of population (growth of a species, an organ, or a plant, or a cell) is can be written as first order linear ordinary Differential equation [6-15]

$$\frac{dP}{dt} = \alpha P \ldots\ldots\ldots\ldots (2)$$

With the initial condition as

$$P(t_0) = P_0 \ldots\ldots\ldots (3)$$

Where α is a positive real number, P is the amount of population at time t and $P_0$is the initial population at time t_0 Equation (2) is also known as Malthusian law of population growth.

The decay problem of the substance is defined mathematically by the first order linear ordinary differential equation [12, 14-15]

$$\frac{dP}{dt} = -\alpha P \ldots\ldots\ldots\ldots (4)$$

With the initial condition as

$$P(t_0) = P_0 \ldots\ldots\ldots (5)$$

Where P is the amount of substance at time t, α is a positive real number and $P_0$is the initial population at time t_0.

In equation (4), the negative sign in the right side is taken as mass of the substance is decreasing with time and so derivative $\frac{dP}{dt}$must be negative.

In this, we present Laplace transform for population growth problem given by (2) and (3)

By applying Laplace transform on both sides of (2),

$$L[\frac{dP}{dt}] = \alpha L[P(t)] \ldots\ldots (6)$$

Now applying the property, Laplace transform of derivative of function, on (6), we have

$$sL[P(t)] - P(0) = [P(t)] \ldots\ldots\ldots (7)$$

Using (3) in (7) and on simplification, we have

$$(s - \alpha)[P(t)] = P_0$$

$$\Rightarrow L[P(t)] = \frac{P_0}{(s-\alpha)} \ldots\ldots\ldots\ldots\ldots (8)$$

By operating inverse Laplace transform on both sides of (8), we have

$$P(t) = L^{-1}\left[\frac{P_0}{(s-\alpha)}\right]$$

$$\Rightarrow P(t) = P_0 e^{\alpha t} \ldots\ldots\ldots\ldots (9)$$

This is required amount of population at time t.

Similarly by applying Laplace transform for decay problem given in (4) and (5)

$$\Rightarrow P(t) = P_0 e^{-\alpha t} \ldots\ldots\ldots\ldots (10)$$

This is required amount of substance at time t.

Example.01: The population of town grow at a at a tide proportional to number of people presently living in the town if after two years the population has doubled and after 4 year the population in 300000 estimated the number of people initially living in the town.

This problem mathematically written as

$$\frac{dp}{dt}(t) = \alpha P(t) \ldots\ldots\ldots\ldots\ldots$$

Where P Denote the number of people living in the town at any time t and a is the constant of proportionality

Consider P_o is the number of people initially living in the town at t=0

So,

$$\frac{dp}{dt}(t) = a\,P(t) \qquad \ldots\ldots\ldots\ldots\ldots 1$$

Applying Laplace transform

$$L\left[\frac{dp}{dt}\right] = a\,L[P(t)] \qquad \ldots\ldots\ldots\ldots\ldots 2$$

Now,

Applying the property, Laplace transform of derivative of function on eq. 2

$$L[P(t)] = \frac{P0}{(s-k)}$$

$$P(t) = P_0 e^{kt}$$

$$P(2) = 2P_0 = P_0 e^{2k}$$

Divide both sides by P_0

$$2 = e^{2k} = \log(2) = 2k$$

$$K = \frac{log(2)}{2}$$

The population after 4 years

We now plug into

$$P(4) = 300000 = P_0\, e^{4k}$$

We know from above

$$K = \frac{ln(2)}{2}$$

So,

$$P(4) = P_0\, e^{\frac{4In(2)}{(2)}} = P_0\, e^{2\log(2)}$$

$$P_0\, e^{\ln(4)} = P_0\, 4$$

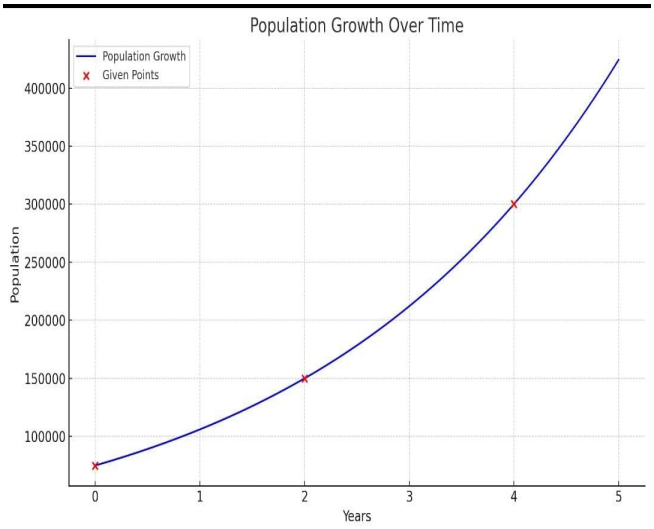
So,

$$300000 = 4P_0$$

$$P_0 = \frac{300000}{4}$$

$$= 75{,}000$$

This is the initially population in the town

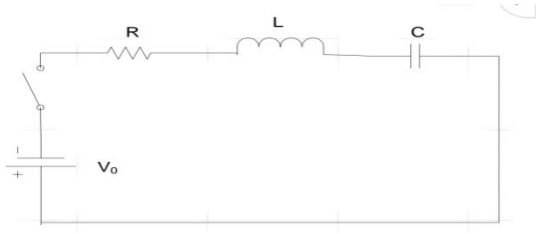


Graph for population growth

B. Application in electric circuit theory

The Laplace transform can be applied to solve the switching transient phenomenon in the series or parallel RL, RC or RLC circuits [16]. A simple example of showing this application follows next. Let us consider a series RLC circuit

R L C Consider simple electric circuit where R-resistance, L-inductance, C-capacity and E-electromotive power of voltage in a series. A switch is connected in the circuit.



Circuit diagram for electric circuit theory

Then by Kirchhoff law

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E \ldots\ldots\ldots$$

Now, applying Kirchhoff's Voltage Law (KVL) to the circuit,

We have,

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_0$$

Differentiating both sides,

$$L\frac{d^2i}{dt^2} + \frac{1}{C}i + R\frac{di}{dt} = 0$$

$$L\frac{d^2i}{dt^2} + \left(\frac{R}{L}\right)\frac{di}{dt} + \left(\frac{1}{LC}\right)i = 0. \text{ ---------- 4}$$

Now, applying Laplace transform to this equation let us

Assume that the solution of this equation is

$(t) = Ke^{st}$ where K and s are constants which may be real , Imaginary or complex.

Now, from eq^{n} (4),

$$LKs^2e^{st} + RKe^{st} + \frac{1}{C}Ke^{st} = 0$$

This on simplification gives

$$\text{Or } s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

The roots of the equation would be

$$s_1, s_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R^2}{4L^2}\right)} - \sqrt{\left(\frac{1}{LC}\right)}$$

The general solution of the differential equation is thus,

$$i\,(t) = K_1e^{s1t} + K_2e^{s^2\,t}$$

Where K_1 and K_2 are determined from the initial conditions.

Now, if we define,

$$\alpha = Damping\ Coefficient = \frac{R}{2L}$$

$$\text{And natural frequency, } \omega_n = \frac{1}{\sqrt{(LC)}}$$

This is also

Known as undamped natural frequency or resonant frequency.

Thus roots are: $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega n^2}$

The final form of solution depends on whether

$$\left(\frac{R^2}{4L^2}\right) > \frac{1}{LC}; \left(\frac{R^2}{4L^2}\right) = \frac{1}{LC} \text{ and } \left(\frac{R^2}{4L^2}\right) < \frac{1}{LC}$$

Example.02: Consider resistor of 20 ohms inductance of 5 Henry, capacitor of 0.05 farad at t=0, the change on the current and capacitor in the circuit is zero. What is current and charge at any time t>0

R = 20 ohms

L = 5, H

C = 0.05 F

Consider $I\ and\ Q$ are instantaneous current and charge respectively at time t,

$$5\frac{dI}{dt} + 20I + 20Q = 0$$

$$\frac{d^2Q}{dt^2} + 4\frac{dQ}{dt} + 4Q = 0$$

By applying Laplace transform we have

$$\left[\frac{d^2Q}{dt^2}\right] + \left[4\ \frac{dQ}{dt}\right] + 4[Q] = 20[1]$$

$$(s^2 + 4s + 4)L(Q) = \frac{20}{s}$$

$$L(Q) = \frac{20}{s(s^2 + 4s + 4)}$$

$$Q = L^{-1}\left[\frac{20}{s(s^2 + 4s + 4)}\right]$$

$$Q = L^{-1}\left[\frac{5}{s} + \frac{-5s - 20}{(s^2 + 4s + 4)}\right]$$

For t > 0 required the current and charge expression is

$$= \left[\frac{5}{s} - \frac{5}{(s+2)} - \frac{10}{s+2^2}\right]$$

$$= 5 - 5e^{-2t} - 10\,t\,e^{-2t}$$

C. Problem in Mechanical Engineering

Vibrating Mechanical systems: If we discuss the suspension system of the car the mass is an important, damper and springs used to join the body of the car. Mechanical systems may be used to model many situations, and involve three basic elements: masses (mass M measured in kg), dampers (damping coefficient B measured in Nsm^{-1}.The associated variables are force F (t) (measured in N) and displacement Y (t) (measured in M).

Example.03: Consider Mass-damper- spring system using Newton's Hooke's Law therefore differential equation given by If M=2, B=6, K=20 & $(t) = 5sinwt$ By Laplace transform

To solve the mass-damper-spring system using Laplace transform, we start from the second-order differential equation derived from Newton's second law and Hooke's law

For a mass spring damper system

Where:

M=2

B = 6

K = 20

$f(t) = 5\sin(\omega t)$

So the differential equation becomes

$$2\ddot{x}(t) + 6\ddot{x}(t) + 20x(t) = 5\sin(\omega t)$$

Assume initial conditions;

$$X(0) = 0 \dot{x}(0) = 0$$

Taking Laplace transform on both sides

$$2[s^2X(s)] = 6[s\,X(s)\} = 20\,X(s) = 5.\frac{\omega}{(S^2+\omega^2)}$$

Factor X (s);

$$X(s)\,[2s^2+6s+20] = \frac{5\omega}{(S^2+\omega^2)}$$

$$X(s) = \frac{5\omega}{(S^2+\omega^2)(2s^2+6s+20)}$$

This is the Laplace transform of the solution. You can do partial fraction decomposition to find the inverse Laplace transform and get in time domain.

We need to simplify;

$$2s^2+6s+20$$

Factor out 2

$$2(s^2+3s+10)$$

Now complete the square for

$$s^2+3s+10$$

$$s^2+3s+10 = \left(s+\frac{3}{2}\right)^2 + \left(10-\frac{9}{4}\right)^2 = \left(s+\frac{3}{2}\right)^2 + \frac{31}{4}$$

$$2s^2+6s+20 = [2\left(s+\frac{3}{2}\right)^2 + \frac{31}{4}]$$

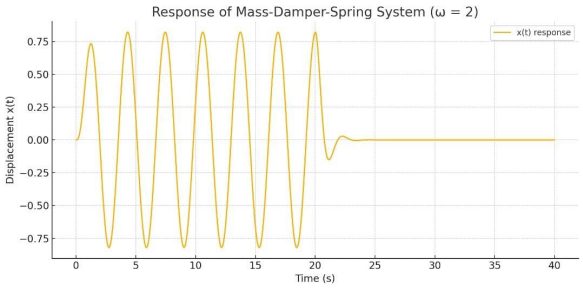
$$X(s) = \frac{5\omega}{[(s^2+\omega^2).2[\left(s+\frac{3}{2}\right)^2 + \frac{31}{4}]}$$

$$X(s) = \frac{5\omega}{[2(s^2+\omega^2)[\left(s+\frac{3}{2}\right)^2 + \left(\frac{\sqrt{31}}{4}\right)^2]}$$

This is a bit complex to invert directly, so we use convolution in time domain, because:

This integral is the convolution of the two functions

$$X(t) = \frac{5\omega}{2}.\,[\mathrm{Sin}\,(\omega t) * (e^{-\frac{3}{2}t}\sin\left(\frac{\sqrt{31}}{4}\,t\right))]$$



Graph for mass- damper spring system

Here's the graph of the system's displacement response for an input. Let me know if you'd like to change the frequency ω or see other system variations (e.g., different damping or stiffness values).

4. Conclusion

The Laplace Transform and Fourier Transform are powerful mathematical tools widely used in engineering, physics, and signal processing. While both are integral transforms that convert time-domain functions into a frequency-domain representation, they serve different purposes.

The Laplace Transform is especially useful in solving differential equations and analyzing linear time-invariant systems, providing insights into system stability and control. It is suited for both transient and steady-state analysis due to its ability to handle initial conditions and exponential growth/decay.

The Fourier Transform, on the other hand, is primarily used to analyze the frequency content of signals, especially in steady-state and periodic systems. It breaks down complex signals into their sinusoidal components, making it essential for signal processing, audio analysis, and communication systems. In conclusion the Laplace transform is a powerful mathematical tool that simplifies the analysis of linear, time-invariant systems described by differential equations by mapping the time domain to the complex s-domain, converting differential equations into algebraic equations ,and Fourier series provide a powerful tool for analyzing and representing periodic functions by decomposing them into a sum of sins and cosines, enabling efficient analysis and manipulation of signals in various applications

In summary, both transforms offer unique advantages, and choosing between them depends on the nature of the problem—transient behavior and control systems often call for Laplace, while frequency analysis and periodic signals are better served by Fourier.

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Authors' Contributions

Shivani Sharma: Conceptualization, methodology, data Collection, formal analysis, and writing – original draft

Preparation.
Garima singh: Data curation, validation, visualization, and writing – review and editing.
Nidhi Gargav: Supervision, project administration, and Funding acquisition.
All authors reviewed and approved the manuscript and approved the final version of the manuscript.

Conflict of Interest

Professor Dr. Nidhi Gargav and Dr. Pooja Sharma disclose a potential conflict of interest in the form of consulting roles, advisory positions, equity holdings, or affiliations.

Data Availability

The data supporting the findings of this review article are Available from various publicly accessible sources, including Peer-reviewed research papers, books and academic Publications. Specific references are cited throughout the Article.

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