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Generalized Fractional Derivative Operators of the Multi-Index Bessel Functions with Applications

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Abstract

In this paper we use fractional differential operators $D^{n}_{\alpha,\beta,x}$ and D^{η}_{t} to derive a number of key formulas of multivariable H-function. We use the generalized Leibnitz's rule for fractional derivatives in order to obtain one of the aforementioned formulas, which involve a product of generalized multi-index Bessel function. It is further shown that, each of these formulas yield interesting new formulas for generalized multi-index Bessel function.

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INTRODUCTION AND DEFINITIONS

The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[15,18] the derivation of generating function [21,chap.5] and the solution of differential equations motivated by these and many other avenues of applications, the fractional differential operators $D^{n}_{\alpha,\beta,x}$ and ${}_{\alpha}D^{\mu}_{x}$ are much used in the theory of special function of one and more variable

Definitions

1. Generalized Fractional Derivative Operators

We use the fractional derivative operator defined in the following manner [7]

$$D^{n}_{\alpha,\beta,x}(x^{\lambda}) = \prod_{r=0}^{n-1}\left[\frac{\Gamma(\lambda+r\alpha+1)}{\Gamma(\lambda+r\alpha-\beta+1)}\right]x^{\lambda+n\alpha} \qquad (1.1)$$

Where $\beta \neq \lambda + 1$ and α and β are not necessarily integers

We use the binomial expansion in the following manner

$$(ax^{\mu}+b)^{\lambda}=b^{\lambda}\sum_{l=0}^{\infty}\binom{\lambda}{l}\left(\frac{ax^{\mu}}{b}\right)^{l} \qquad where \quad [\frac{ax^{\mu}}{b}]<1 \qquad (1.2)$$

the familiar differential operator ${}_{\alpha}D_{x}^{\mu}$ is defined by [7]

$${}_{\alpha}D_{x}^{\mu}f(\mathrm{x})=\begin{cases}\dfrac{1}{\sqrt{-\mu}}\int_{\alpha}^{x}(x-t)^{-\mu-1}f(t)dt \;, & [\mathrm{Re}(\mu)<0]\\ \dfrac{d^{m}}{dx^{m}}\,{}_{\alpha}D_{x}^{\mu-m}f(x), & [0\le \mathrm{Re}(\mu)<\mathrm{m}]\end{cases} \qquad (1.3)$$

Where m is a positive integer
For $\alpha=0$,(1.3) Defines the classical Riemann-Liouville fractional derivative of order μ (or-μ) when α⟶∞ (1.3) may be identified with the definition of the well known Weyl fraction derivative of order μ (or-μ) [1,chap.13);3] the special case of fractional calculus operator ${}_{\alpha}D_{x}^{\mu}$ when $\alpha=0,\mu=\eta,x=t$ is written as D_{t}^{η} thus we have

$$D_{t}^{\eta}={}_{\alpha}D_{x}^{\mu}$$

$$D_{t}^{\eta}\left(x^{\lambda}\right)=\frac{\Gamma(1+\lambda)}{\Gamma(1+\lambda-\eta)}x^{\lambda-\eta} \qquad \{Re(\lambda)>-1\} \qquad (1.4)$$

2.Generalized Multi-Index Bessel Function

For $A_j, B_j, \lambda, \in \mathbb{C}, \mu>0, \mathcal{R}e(\lambda)>0$ the generalized multi-index Bessel function is defined by Choi and Agarwal [1] in the following summation form:

$$\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x)=\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j+1\right)}\frac{(-x)^{l}}{l!}; \qquad (m\in\mathbb{N}) \qquad (2.1)$$

where $\mathcal{R}e\left(B_j\right)>-1$ and $\sum_{j=1}^{m}\mathcal{R}e\left(A_j\right)>max\,\{0;\,\mathcal{R}e(\mu)\,-1\}$

3. Main Results

Theorem 1. Fractional derivatives operator $D_{t}^{\eta}(x^{\lambda})$ associated with the product of two multi-index Bessel functions

$$D_{t}^{\eta}\left\{t^{\delta-1}\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1t)\times\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2t)\right\}$$

$$= t^{\delta-\eta-1}\left\{ \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t) \times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\} \otimes \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{\Gamma(\delta+k+l)}{\Gamma(\delta+k+l-\eta)} t^{k+l} \quad (3.1)$$

Where $\otimes$ stands for convolution product of two functions.

Proof. We refer to the left hand side of equation (3.1) by the symbol D_1.

Then making the use of equation (2.1) in equation (3.1), we have

$D_1 \equiv$

$$D_t^{\eta}\left\{ t^{\delta-1} \sum_{k=0}^{\infty} \frac{(\lambda)_{\mu k}}{\prod_{j=1}^{m} \Gamma\left(A_j\, k + B_j + 1\right)} \frac{(-x_1 t)^k}{k!} \times \sum_{l=0}^{\infty} \frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m} \Gamma\left(A_j\, l + B_j + 1\right)} \frac{(-x_2 t)^l}{l!} \right\}$$

After changing the order of summations and derivative operator under the conditions of theorem, we obtain the above as

$$= \sum_{k=0}^{\infty} \frac{(\lambda)_{\mu k}}{\prod_{j=1}^{m} \Gamma\left(A_j\, k + B_j + 1\right)} \frac{(-x_1 t)^k}{k!} \times \sum_{l=0}^{\infty} \frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m} \Gamma\left(A_j\, l + B_j + 1\right)} \frac{(-x_2 t)^l}{l!}$$

$$\times D_t^{\eta}(t^{\delta+k+l-1})$$

We use the fractional derivative operator $D_t^{\eta}(x^{\lambda})$ after simplification we get

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m} \Gamma\left(A_j\, k + B_j + 1\right) \prod_{j=1}^{m} \Gamma\left(A_j\, l + B_j + 1\right)} \times \frac{(-x_1)^k\,(-x_2)^l}{k!\; l\,!}$$

$$\times \frac{\Gamma(\delta+k+l)}{\Gamma(\delta+k+l-\eta)} t^{\delta+k+l-\eta-1}$$

Further, applying the definition (2.1) and convolution product on two series, we obtain.

$$D_1 \equiv t^{\delta-\eta-1}\left\{ \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t) \times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\} \otimes \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{\Gamma(\delta+k+l)}{\Gamma(\delta+k+l-\eta)} t^{k+l}$$

Where $\otimes$ stands for convolution product of two functions.

Theorem 2. Fractional derivatives operator $D^{n}_{\alpha,\beta,x}(x^{\lambda})$ associated with the product of two multi-index Bessel functions.

$$D^{n}_{\alpha,\beta,t}\left\{t^{\delta-1}\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t)\times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$= t^{\delta+n\alpha-1}\left\{\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t)\times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$\otimes\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta+k+l+r\alpha)}{\Gamma(\delta+k+l+r\alpha-\beta)}\right\}t^{k+l}\right] \qquad (3.2)$$

Where $\otimes$ stands for convolution product of two functions.

Proof. We refer to the left hand side of equation (3.2) by the symbol D_2.

Then making the use of equation (2.1) in equation (3.2) , we have

$$D_2 \equiv D^{n}_{\alpha,\beta,t}\left\{t^{\delta-1}\sum_{k=0}^{\infty}\frac{(\lambda)_{\mu k}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j+1\right)}\frac{(-x_1 t)^k}{k!}\times\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j+1\right)}\frac{(-x_2 t)^l}{l!}\right\}$$

After changing the order of summations and derivative operator under the conditions of theorem, we obtain the above as

$$=\sum_{k=0}^{\infty}\frac{(\lambda)_{\mu k}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j+1\right)}\frac{(-x_1 t)^k}{k!}\times\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j+1\right)}\frac{(-x_2 t)^l}{l!}$$

$$\times D^{n}_{\alpha,\beta,t}(t^{\delta+k+l-1})$$

We use the fractional derivative operator $D^{n}_{\alpha,\beta,x}(x^{\lambda})$ after simplification we get

$$=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j+1\right)\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j+1\right)}\frac{(-x_1)^k}{k!}\frac{(-x_2)^l}{l!}$$

$$\times\prod_{r=0}^{n-1}\left[\frac{\Gamma(\delta+k+l+r\alpha)}{\Gamma(\delta+k+l+r\alpha-\beta)}\right]t^{\delta+k+l-1+n\alpha}$$

Further, applying the definition (2.1) and convolution product on two series, we obtain.

$$D_2 \equiv t^{\delta+n\alpha-1}\left\{\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t)\times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$\otimes\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta+k+l+r\alpha)}{\Gamma(\delta+k+l+r\alpha-\beta)}\right\}t^{k+l}\right]$$

Where $\otimes$ stands for convolution product of two functions.

Theorem 3. Double fractional derivatives operators $D_t^{\eta}(x^{\lambda})$ and $D^n_{\alpha,\beta,x}(x^{\lambda})$ associated with the product of two multi-index Bessel functions.

$$D^n_{\alpha,\beta,t}\left[D_t^{\eta}\left\{t^{\delta-1}\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t)\times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}\right]$$

$$= t^{\delta-\eta+n\alpha-1}\left\{\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t)\times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$\otimes\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta+k+l-\eta+r\alpha)}{\Gamma(\delta+k+l-\eta+r\alpha-\beta)}\right\}t^{k+l}\right] \qquad (3.3)$$

Where $\otimes$ stands for convolution product of two functions.

Proof. We refer to the left hand side of equation (3.3) by the symbol D_3.

Then making the use of equation (2.1) in equation (3.3), we have

$$D_3 \equiv$$

$$D^n_{\alpha,\beta,t}\left[D_t^{\eta}\left\{t^{\delta-1}\sum_{k=0}^{\infty}\frac{(\lambda)_{\mu k}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j+1\right)}\frac{(-x_1 t)^k}{k!}\times\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j+1\right)}\frac{(-x_2 t)^l}{l!}\right\}\right]$$

After changing the order of summations and derivative operator under the conditions of theorem, we obtain the above as.

$$D^n_{\alpha,\beta,t}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j\right)\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j\right)}\frac{(-x_1)^k}{k!}\frac{(-x_2)^l}{l!}\times\left\{D_t^{\eta}\left(t^{\delta+k+l-1}\right)\right\}\right]$$

We use the fractional derivative operator $D_t^{\eta}(x^{\lambda})$ after simplification we get

$$D^n_{\alpha,\beta,t}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma\left(A_j\,k+B_j\right)\prod_{j=1}^{m}\Gamma\left(A_j\,l+B_j\right)}\frac{(-x_1)^k}{k!}\frac{(-x_2)^l}{l!}\times\left\{\frac{\Gamma(\delta+k+l)}{\Gamma(\delta+k+l-\eta)}\,t^{\delta+k+l-\eta-1}\right\}\right]$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma(A_j\, k + B_j)\prod_{j=1}^{m}\Gamma(A_j\, l + B_j)} \frac{(-x_1)^k}{k!}\frac{(-x_2)^l}{l!} \times \frac{\Gamma(\delta + k + l)}{\Gamma(\delta + k + l - \eta)}$$

$$\times \left\{ D^{n}_{\alpha,\beta,t}\left(t^{\delta+k+l-\eta-1}\right)\right\}$$

We use the fractional derivative operator $D^{n}_{\alpha,\beta,x}(x^{\lambda})$ after simplification we get

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(\lambda)_{\mu k}(\lambda)_{\mu l}}{\prod_{j=1}^{m}\Gamma(A_j\, k + B_j)\prod_{j=1}^{m}\Gamma(A_j\, l + B_j)} \frac{(-x_1)^k}{k!}\frac{(-x_2)^l}{l!} \times \frac{\Gamma(\delta + k + l)}{\Gamma(\delta + k + l - \eta)}$$

$$\times \prod_{r=0}^{n-1}\left[\frac{\Gamma(\delta + k + l - \eta + r\alpha)}{\Gamma(\delta + k + l - \eta + r\alpha - \beta)}\right] t^{\delta+k+l-\eta-1+n\alpha}$$

Further, applying the definition (2.1) and convolution product on two series,

we obtain.

$$D_3 \equiv t^{\delta-\eta+n\alpha-1}\left\{ \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t) \times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$\otimes \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta + k + l - \eta + r\alpha)}{\Gamma(\delta + k + l - \eta + r\alpha - \beta)}\right\} t^{k+l}\right]$$

Where $\otimes$ stands for convolution product of two functions.

4. Special cases

Our main provides unifications and extensions of various (known or new) results fractional differential operators. For the sake of illustration, we mentior the following special cases

Corollary 1. Let the conditions of Theorem 1 be satisfied and $\eta = 1$, then

the theorem 1 reduced in the following form:

$$D^{1}_{t}\left\{ t^{\delta-1}\mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t) \times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\}$$

$$= t^{\delta-2}\left\{ \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_1 t) \times \mathrm{J}^{(A_j)_{m,\lambda}}_{(B_j)_{m,\mu}}(x_2 t)\right\} \otimes \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\Gamma(\delta + k + l)}{\Gamma(\delta + k + l - 1)} t^{k+l} \qquad (4.1)$$

Where $\otimes$ stands for convolution product of two functions.

Corollary 2. Let the conditions of Theorem 2 be satisfied and $\alpha = 0,\ \beta = 1$ then

the theorem 2 reduced in the following form:

$$D_{0,1,t}^{n}\left\{t^{\delta-1}\mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_1 t)\times \mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_2 t)\right\}$$

$$= t^{\delta-1}\left\{\mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_1 t)\times \mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_2 t)\right\}$$

$$\otimes\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta+k+l)}{\Gamma(\delta+k+l-1)}\right\}t^{k+l}\right] \qquad (4.2)$$

Where $\otimes$ stands for convolution product of two functions.

Corollary 3. Let the conditions of Theorem 3 be satisfied and $\alpha = 1,\ \beta = 0,$

$\eta = 1$then the theorem 3 reduced in the following form:

$$D_{1,0,t}^{n}\left[D_{t}^{1}\left\{t^{\delta-1}\mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_1 t)\times \mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_2 t)\right\}\right]$$

$$= t^{\delta+n-2}\left\{\mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_1 t)\times \mathrm{J}_{(B_j)_{m,\mu}}^{(A_j)_{m,\lambda}}(x_2 t)\right\}$$

$$\otimes\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[\prod_{r=0}^{n-1}\left\{\frac{\Gamma(\delta+k+l+r-1)}{\Gamma(\delta+k+l+r-1)}\right\}t^{k+l}\right] \qquad (4.3)$$

Where $\otimes$ stands for convolution product of two functions.

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