International Journal of Scientific Research and Engineering Development-– Volume 9 Issue 1, Jan-Feb 2026
Available at www.ijsred.com

RESEARCH ARTICLE OPEN ACCESS

Probabilistic Load Flow Computations in Radial Distribution System Using Monte-Carlo Simulations

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Abstract:

Uncertainty in load demand and operating conditions poses significant challenges to voltage regulation in radial distribution networks. Conventional deterministic load flow methods, which assume fixed system parameters, are unable to adequately capture these uncertainties. This paper presents a Monte Carlo–based probabilistic load flow (PLF) analysis of the 11 kV Ayepe 34-bus radial distribution network operated by the Ibadan Electricity Distribution Company (IBEDC), Nigeria. Load uncertainties were modeled using Gaussian distributions with varying standard deviations of 25 & 5, 45 & 20 and 75 & 50 active and reactive power respectively and repeated load flow solutions are carried out using the backward–forward sweep algorithm. All simulations were performed in the MATLAB R2022a computational environment. The results are analyzed using two different analysis approaches: Deterministic Load Flow & Probabilistic load flow. Statistical voltage indices, including mean voltage, standard deviation, and probability of voltage violation, were evaluated to assess voltage stability and operational risk using voltage profile curves, probability distribution histograms, and bar charts to facilitate comparative and probabilistic analysis. Results show that although probabilistic mean voltage profiles closely follow deterministic results, several high-load and end-of-feeder buses exhibit a high probability of undervoltage violations under increased load variability. The study demonstrates that Monte Carlo PLF provides a more realistic and risk-informed assessment of distribution system performance than deterministic analysis and is suitable for planning voltage support measures in practical radial networks.

Keywords — **MATLAB, Monte-Carlo Simulation, Voltage Stability, Deterministic Load Flow, Algorithm.**

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I. INTRODUCTION

Radial distribution networks form the terminal segment of power delivery, supplying loads with electricity while exhibiting high resistance-to-reactance ratios and weak voltage support under varying operational conditions [3]. Deterministic load flow (DLF) methods such as Newton–Raphson and backward/forward sweep assume fixed loads and generation, providing a single operating solution that fails to capture variability from load dynamics and intermittent distributed energy resources (DERs) in modern systems [3]. As uncertainties from DER penetration and stochastic load behavior increase, deterministic approaches can misrepresent voltage profiles, power losses, and reliability metrics.

Probabilistic load flow (PLF) frameworks address these limitations by modeling uncertain inputs as random variables and quantifying outputs e.g., bus voltages and flows as statistical distributions [8]. Monte Carlo simulation (MCS) is widely used in PLF to generate numerous scenarios of random operating conditions and solve repeated load flow cases, yielding estimates of statistical indices such as means, variances, and violation probabilities ([8],[2]). While MCS is computationally intensive, its ability to integrate with conventional algorithms and handle nonlinearities makes it suitable for radial systems with high DER penetration and complex uncertainty profiles.

PLF with MCS thus supports risk-informed planning, reliability assessment, and voltage stability analysis by providing probabilistic insights unavailable from deterministic studies, enabling more resilient distribution network operation amid growing uncertainty [8].

II. LITERATURE REVIEW

Probabilistic Load Flow (PLF) has emerged as a critical tool for assessing the performance of power systems under uncertainty, especially in radial distribution networks where load and generation variability significantly affect voltage profiles and power losses. Classical deterministic load flow methods such as Newton-Raphson and backward/forward sweep assume fixed system parameters and fail to capture realistic operating uncertainties associated with loads and distributed energy resources (DERs) such as wind and solar PV systems [1].

Monte Carlo Simulation (MCS) is among the most widely adopted stochastic techniques for PLF due to its conceptual simplicity and ability to model arbitrary probability distributions of uncertain inputs. [2] incorporated Monte Carlo methods to assess probabilistic behavior in radial networks with photovoltaic generation, demonstrating improved representation of voltage and power flow variability compared to deterministic methods. Similarly, [7] used data clustering combined with MCS to reduce computation time in PLF analysis of IEEE benchmark radial systems with wind farms, highlighting trade-offs between simulation accuracy and runtime .

The foundational approach to PLF dates back to [8], who first introduced probabilistic methods for load flow problems by modeling uncertain variables as random processes, setting the stage for Monte Carlo-based techniques in later studies [14]. Comparisons between probabilistic and fuzzy approaches within radial systems underscore that while fuzzy methods can address limited uncertainty with lower runtime, Monte Carlo remains more accurate when multiple random variables interact within networks [6].

More advanced Monte Carlo-based studies have expanded beyond basic radial setups to address unbalanced three-phase systems. [4] advanced PLF solutions including unbalanced conditions, validating results against Monte Carlo benchmarks and illustrating applicability in realistic distribution networks. [5] further developed methods for both radial and weakly meshed networks without reliance on standard Y-bus formulations, reinforcing that PLF should handle diverse network configurations.

Researchers have also applied Monte Carlo techniques alongside sampling improvements. For instance, Quasi-Monte Carlo (QMC) and Latin Hypercube Sampling (LHS) have been proposed to reduce variance and computational burden, with comparative analyses showing that LHS can maintain accuracy with fewer samples than simple random sampling in MCS [11]. These enhanced sampling strategies are particularly vital for high-dimensional problems involving correlated uncertainties of load and renewable generation outputs.

Wind and solar power output variability drives significant interest in probabilistic analysis within distribution systems. [1] extended Monte Carlo PLF to incorporate wind and PV uncertainties using multi-linear formulations, offering improved insight into combined generation impacts on voltage and power flows. These approaches consistently highlight that MCS remains the benchmark for validating newer approximation methods due to its ability to converge to true distributional behavior given sufficient samples [9].

Recent advancements also explore machine learning integration with PLF. For example, adaptive kernel density estimation combined with LHS demonstrated improvements in probabilistic representation and computational efficiency when compared against traditional Monte Carlo results on standard systems such as IEEE test networks [10]. These hybrid methods highlight ongoing efforts to balance accuracy and computational efficiency in probabilistic load flow studies [3].

Across the literature, key performance indices such as voltage quality, loss distributions, and the probability of limit violations are derived from probabilistic outputs rather than single-point estimates, offering system planners richer insight into operational risk and robustness compared to deterministic benchmarks [12]. Additionally, probabilistic studies have been used in optimization contexts such as capacitor and DER placement demonstrating that stochastic information can lead to more resilient distribution design decisions [13].

Although probabilistic load flow (PLF) methods particularly those based on Monte Carlo Simulation (MCS) have been widely studied internationally. Most existing PLF studies have focused on standardized IEEE test systems or distribution networks in developed countries with relatively stable demand patterns and extensive data availability. However, there exist notable research gap when considering the unique characteristics and challenges of Nigerian distribution networks. There is a notable lack of comprehensive Monte Carlo PLF studies calibrated to actual Nigerian radial distribution network data such as those operated by the Distribution Companies (DisCos). The topology, loading profiles, and reliability issues in Nigerian networks differ significantly from benchmark systems due to frequent outages, customer behavior variability, and poor data acquisition infrastructure. This study therefore, focuses on Monte-Carlo Probabilistic load flow studies of the AYEPE 34-bus radial distribution network in Ibadan, Nigeria.

III. MATERIALS AND METHOD

The Monte Carlo Probabilistic Load Flow (MCPLF) technique was adopted for analyzing the probabilistic behavior of a radial power distribution network. Unlike conventional deterministic load flow analysis, which assumes fixed or nominal load values, the probabilistic approach accounts for the random and uncertain nature of load demand in distribution systems.

The Monte Carlo simulation method is employed to repeatedly generate random load values based on predefined probability distributions, followed by load flow analysis using the Backward–Forward Sweep (BFS) algorithm. The statistical properties of bus voltages and system losses are then evaluated to quantify voltage stability, uncertainty, and risk of voltage violation.

1) The Radial Distribution Test Network

For the probabilistic load flow analysis, the 11 kV Ayepe 34-bus distribution feeder of the Ibadan Electricity Distribution Company (IBEDC), illustrated in Fig. 2, was adopted in this study. The system comprises 34 buses, with Bus 1 acting as the substation and supplying power to the remaining buses in the network. The aggregate real and reactive power demands of the feeder are 4.12 MW and 2.05 MVAr, respectively.

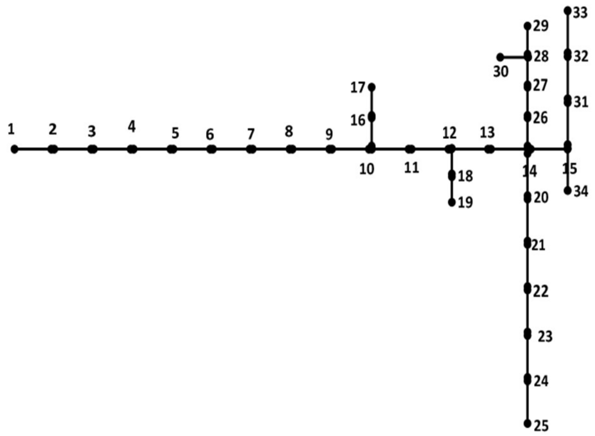

Fig. 2. Ayepe 34-bus radial distribution network

2) Deterministic Load Flow Using Backward–Forward Sweep Method

The Backward–Forward Sweep (BFS) method is adopted for load flow analysis due to its fast convergence, numerical stability, and suitability for radial networks. The BFS method involves two iterative steps:

1. **Backward Sweep** – Calculation of branch currents from the terminal buses to the source
2. **Forward Sweep** – Updating of bus voltages from the source to the terminal buses

1. Backward Sweep: Branch Current Calculation

For each load bus i, the injected current is calculated as:

$I_i = \frac{P_i - j\ \ _i}{V_i^*}$

where:

P_i and Q_i are the active and reactive power demands

V_i is the bus voltage

$(.)^*$ denotes the complex conjugate

Branch currents are then obtained by summing the load currents of downstream buses, starting from the terminal buses and moving toward the source

2. Forward Sweep: Bus Voltage Update

During the forward sweep, voltages are updated sequentially from the source bus to the terminal buses using:

$V_j = V_i - I_{ij}(R_{ij} + jX_{ij})$

$\boldsymbol{V_i}$ *and* $\boldsymbol{V_j}$ *are sending and receiving end voltages*

$\boldsymbol{I_{ij}}$ *is branch current*

$\boldsymbol{R_{ij}}$ *and* $\boldsymbol{X_{ij}}$ *are line resistance and reactance*

The source bus voltage is fixed at 1.0 p.u., and the process continues until the voltage mismatch is below a predefined tolerance.

3) Monte Carlo Probabilistic Load Flow Framework

The Monte Carlo simulation involves repeated random sampling of load values and execution of load flow analysis. For each simulation iteration:

1. Random load values are generated for all buses
2. Backward–Forward Sweep load flow is executed
3. Bus voltages and system losses are recorded

This process is repeated for N Monte Carlo iterations**,** typically ranging from 1,000 to 10,000, to ensure statistical convergence.

4)Modeling of Load Uncertainty

1) Probability Distribution of Loads

In this study, the active and reactive loads are assumed to follow a normal (Gaussian) distribution**:**

$P_i = \mu_{Pi} + \sigma_{Pi}.N(0,1)$

$Q_i = \mu_{Qi} + \sigma_{Qi}.N(0,1)$

where:

μ_{Pi}, μ_{Qi} are the nominal load values

σ_{Pi}, σ_{Qi} are the standard deviations

$N(0,1)$ is a standard normal random variable

A standard deviation of the nominal load is used to reflect realistic operating conditions.

2) Mean Voltage

$V_{mean,i} = \frac{1}{N_s}\sum_{k=1}^{N_s} V_i^{(k)}$

where:

$V_i^{(k)}$ is the voltage magnitude at bus i for the $k-th$ Monte Carlo sample

N_s is the total number of Monte Carlo samples

The mean voltage represents the expected or average voltage at each bus under load uncertainties. Buses near the substation have higher V_{mean} showing a better voltage support. Remote buses have Lower V_{mean}, showing higher risk of undervoltage. The mean voltage is used to assess overall voltage profile under stochastic conditions.

3) Voltage Standard Deviation $(\boldsymbol{\sigma_V})$

$$\sigma_{V,i} = \sqrt{\frac{1}{N_s - 1}\sum_{k=1}^{N_s}\left(V_i^{(k)} - V_{mean,i}\right)^2}$$

The Standard deviation quantifies voltage variability or uncertainty at each bus. It is used to identify buses with high voltage dispersion and highlights the need for voltage regulation devices (DG, capacitor banks, STATCOM).

4) Probability of Voltage Violation$(\boldsymbol{P(P < V_{min}}))$

$P_{viol,i} = \frac{Number\ of\ samples\ where\ \ _i^{(k)} < V_{min}}{N_s}$

where:

$N_{Vi<0.95}$ is the number of occurrences of voltage violation

N is the total Monte Carlo simulation

V_{min} is usually 0.95 pu (acceptable voltage limit in distribution systems).
The probability of voltage violation provides a risk-based measure of voltage violations at each bus. It highlights buses that occasionally violate voltage limits, even if $V_{mean} > V_{min}$ and allows quantitative ranking of critical buses. It identifies buses for targeted voltage support.

5) Performance Evaluation Criteria

The effectiveness of the probabilistic analysis is evaluated based on:

- Mean bus voltage profile
- Voltage standard deviation
- Probability of voltage violation
- Expected system power losses

These indices provide a comprehensive understanding of system behavior under uncertainty.

IV. RESULTS AND DISCUSSION

The Monte Carlo Probabilistic Power Flow (MCPPF) was performed on the AYEPE 34-bus radial distribution system in Ibadan, Nigeria, to evaluate the impact of stochastic load variations on bus voltages. A total of 2000 random samples of active and reactive loads were generated using a normal distribution of 25 & 5, 45 & 20 and 75 & 50, standard deviation of active and reactive power respectively. The results are analyzed using two different analysis approaches: Deterministic Load Flow & Probabilistic load flow. The Deterministic load flow of backward-forward sweep load flow algorithm uses fixed (nominal) load values at each bus and produces a single voltage profile curve**.** While the probabilistic Load Flow assumes load uncertainty (typically modeled as a normal or uniform distribution) and uses Monte Carlo simulation to repeatedly run load flow with random load variations. It produces: the Mean voltage profile (average of all simulations), ±1σ band, representing one standard deviation around the mean, and probability of voltage violation (P (V<0.90 pu)) i.e P>5%. These indicators provide a comprehensive view of voltage stability and critical buses under realistic operating conditions.

CASE 1: Monte Carlo Probabilistic Power Flow Results at 25&5 standard deviation of real and reactive power

The deterministic load flow represents the nominal operating point using base load values and it was carried out using the Backward-Forward Sweep load flow analysis. Figure 2 compares the deterministic voltage profile with the probabilistic mean voltage profile and ±1σ confidence band. Fig. 2 shows that the mean probabilistic voltage profile closely follows the deterministic voltage profile, confirming consistency of the BFS algorithm. Figure 3 shows that buses 1–8 and buses 19-27 maintain voltages above the minimum voltage limit of 0.9pu showing strong voltage support since they are closer to the substation and their voltages remain stable even when loads fluctuate. Figure 4 shows that high load buses 8-18 and end-of-feeder buses 28–34 are critical or weak buses. They show higher probability bars which indicates that under load uncertainty, their voltages frequently drop below the acceptable limit of 0.9pu. They show lower mean voltages and wider ±1σ bands, indicating higher sensitivity to load variations. The mean voltage provides a first-order assessment of voltage adequacy, but does not capture the full risk of violation. It's the probabilistic load flow that show the full risk of voltage violation.

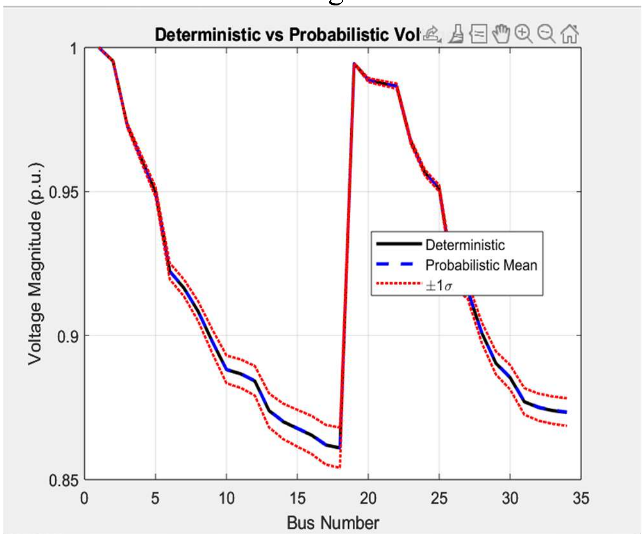

Fig. 2: Deterministic vs Probabilistic Voltage Profile (AYEPE 34-Bus)

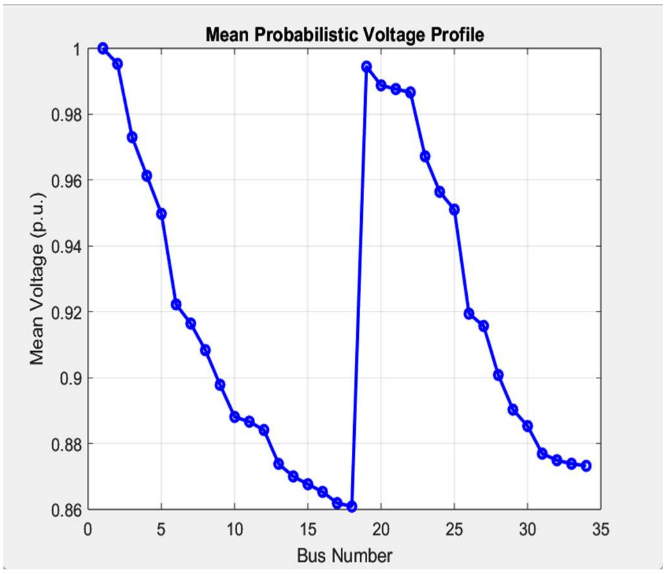

Fig 3: Mean Probabilistic Voltage Profile (25&5 standard deviation of real and reactive power)

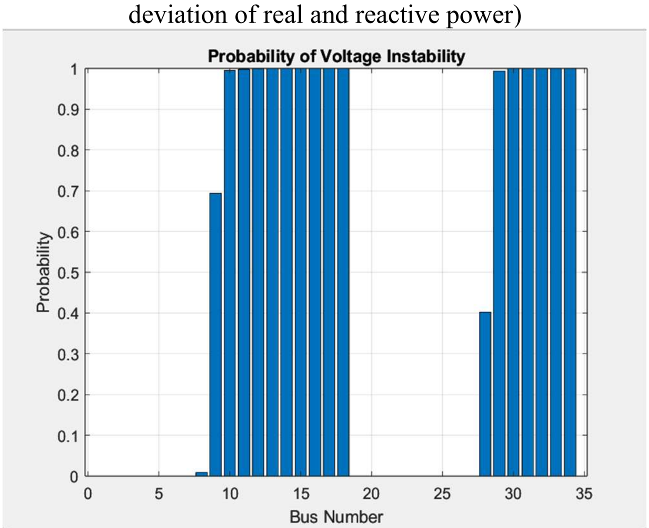

Fig. 4: Probability of voltage violation per bus at 25&5 % standard deviation of real and reactive power

Using the criterion P(V<0.95)>5%, the following buses are critical:
In figure 4, high load buses 9-18 and end of feeder buses 28-34 where voltage drop accumulates are critical buses under probabilistic load flow using a bench mark of Probability of voltage violation greater than 5%. Buses closer to the source show low or zero voltage probability violation and their probability of voltage violation was less than 5%. The critical buses shown in figure 4 are high-priority targets for voltage support interventions such as DG integration, capacitor banks, or STATCOM integration. The probabilistic approach provides a more realistic and robust assessment than deterministic analysis, which might classify certain buses with high mean voltage, as safe.

CASE 2: Monte Carlo Probabilistic Power Flow Results at 45&25 standard deviation of real and reactive power

Figure 5 shows that at 45 & 25 standard deviation of the real and reactive power, the probabilistic load flow plot follows the deterministic load flow plot closely showing the effectiveness of the backward-forward sweep load flow algorithm. Figure 6 shows the mean voltage profile when there was 45&25 standard deviation of the real and reactive power. Figure 7 shows that the near substation buses which are buses 1 to 7, have their voltages remain stable even when loads fluctuate and they maintained the minimum voltage of 0.9pu due to the strong voltage support they got from the substation. Low load buses 9-19 also have their voltages remain stable even when loads fluctuate and they maintain the minimum bus voltage of 0.9pu. While, high load buses 8-18 and end of feeder buses 8-18, 28-34 are critical buses due to their high level of sensitivity to load variations. Increase in the standard deviation of real and reactive power to 45 & 25, forced buses 8 and 28 to become critical buses when the loads were varied.

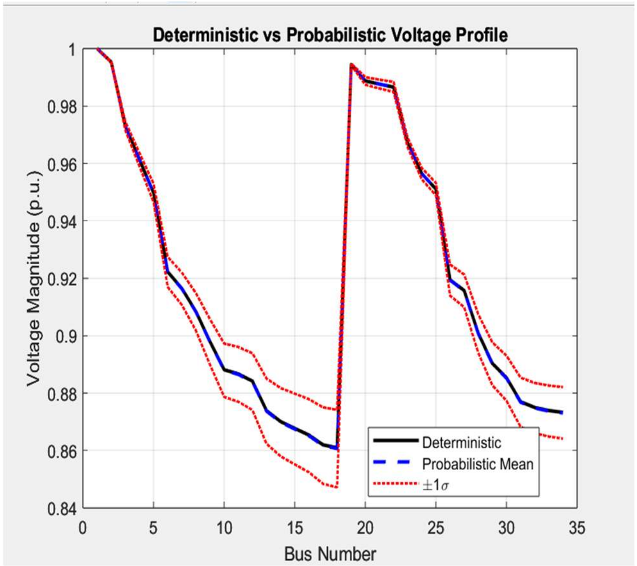

Fig. 5: Deterministic vs Probabilistic Voltage Profile (AYEPE 34-Bus)

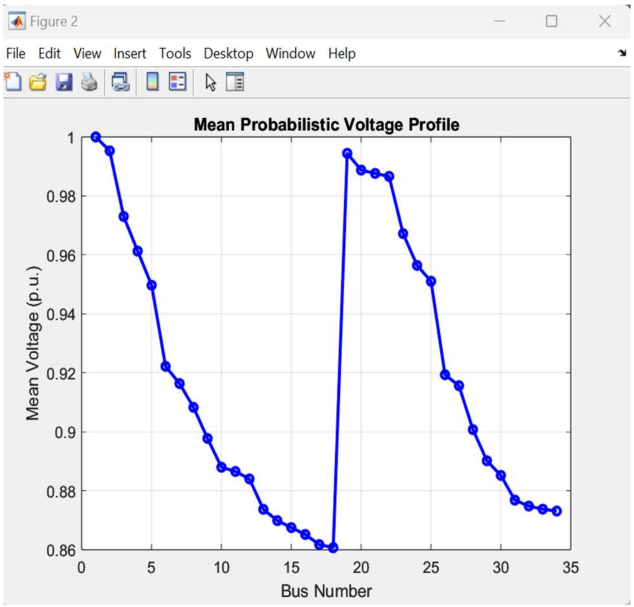

Fig 6: Mean Probabilistic Voltage Profile (45&25 standard deviation of real and reactive power)

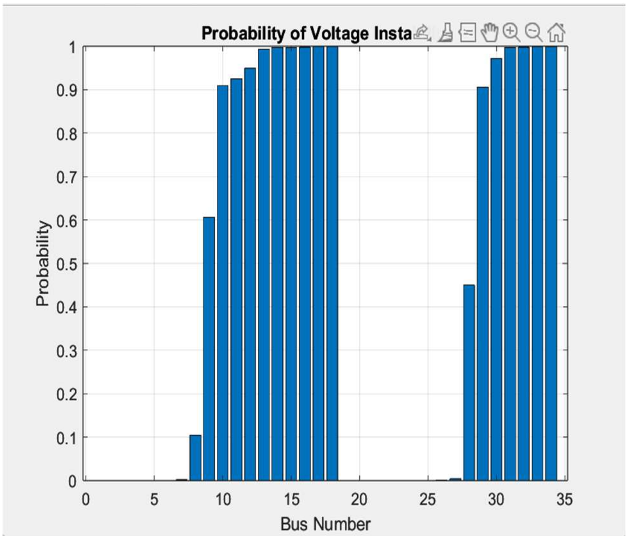

Fig. 7: Probability of voltage violation per bus at 45&25 % standard deviation of real and reactive power

CASE 3: Monte Carlo Probabilistic Power Flow Results at 75&55 standard deviation of real and reactive power

Figure 8 shows that at 75 & 55 standard deviation of the real and reactive power, the probabilistic load flow plot follows the deterministic load flow plot closely showing the effectiveness of the backward-forward sweep load flow algorithm. Figure 9 shows the mean voltage profile when there was 75&55 standard deviation of the real and reactive power. Figure 10 shows that the near substation buses which are buses 1 to 5 and 19-26 maintained the minimum voltage of 0.9pu due to the strong voltage support they got from the substation while high load buses 6-16 and end of feeder buses 27-34 are critical buses due to their high level of sensitivity to load variations. Increase in the standard deviation of real and reactive power to 75 & 55, caused buses 6, 7 and 27 to become critical buses which were not critical buses in case 2, when the loads were varied.

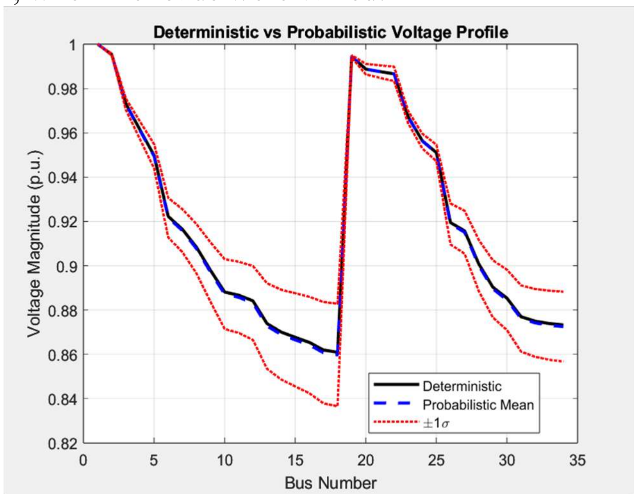

Fig. 8: Deterministic vs Probabilistic Voltage Profile (AYEPE 34-Bus)

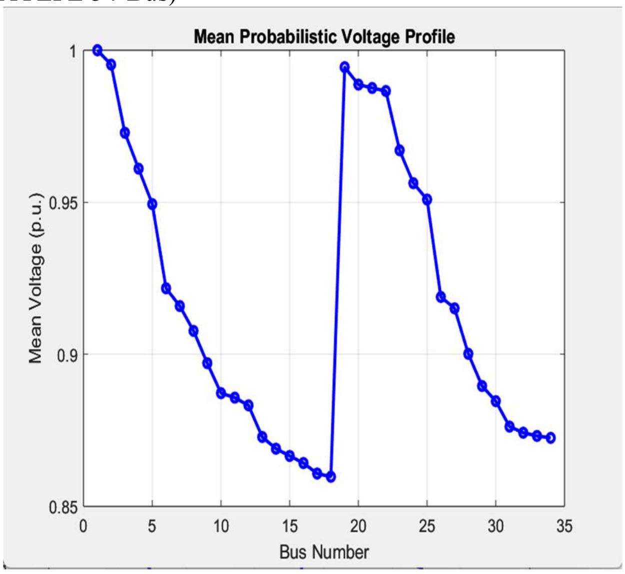

Fig 9: Mean Probabilistic Voltage Profile (75&55 standard deviation of real and reactive power)

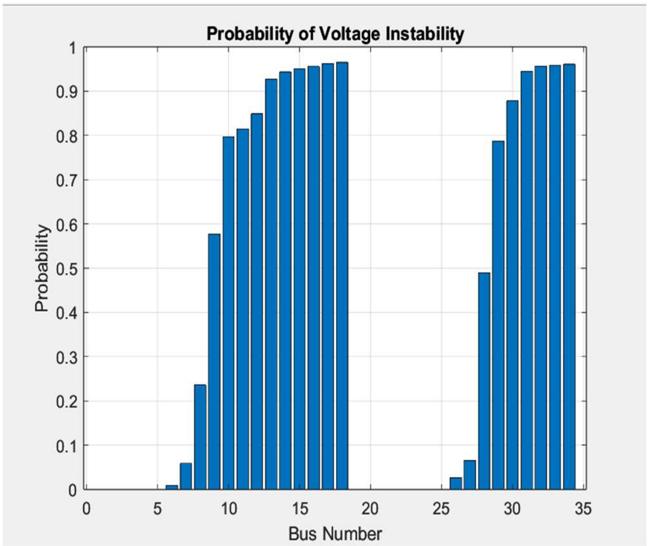

Fig. 10: Probability of voltage violation per bus at 75&55 % standard deviation of real and reactive power

CONCLUSION

This paper applied a Monte Carlo–based probabilistic load flow approach to assess voltage performance in the 11 kV Ayepe 34-bus radial distribution network under load uncertainty. While probabilistic mean voltages closely matched deterministic results, high-load and end-of-feeder buses exhibited significant probabilities of undervoltage, which increased with load variability. These results confirm the limitations of deterministic load flow for voltage risk assessment in radial systems. The proposed framework enables risk-informed identification of weak buses for planning purposes. Future work will incorporate stochastic distributed generation, and voltage support devices such as capacitor banks and STATCOMs.

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