Realised Returns, Risks and Shortfall Probabilities of Systematic Investment Plans: Evidence from India

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Jayashri vishnu khedikar

Department of Management Studies
Wainganga College of Engineering and Management
Nagpur, Maharashtra, India.
Email : jayashrib0720@gmail.com

Guided by

Dr Ujwala Narkhede

Associate Professor, MBA Department
Wainganga College of Engineering and
Management Nagpur, Maharashtra, India
Email : vaidehi1210@gmail.com

ABSTRACT

Systematic Investment Plans (SIP) play a vital role in accumulating investments over long time horizons in India. We compare returns and volatilities of SIP and Lump Sum Investment (LSI) strategies for selected categories of Indian mutual funds and the S&P BSE 100 TR Index over different tenors. Returns and volatilities between SIP and LSI are not significantly different over long periods. By design, the risks of SIP, as measured by the standard deviation of returns, are lower than equivalent LSI returns over short periods under specific conditions. Under these circumstances, SIP investors enjoy higher risk-adjusted returns for a short period. The SIP frequency plays a role in determining the return outcomes only for short time horizons. Over long periods, the SIP frequency has little impact on returns and risk. Our findings show that SIPs are effective accumulation strategies over long periods. However, SIPs have higher shortfall probabilities and higher average conditional shortfalls than equivalent LSI strategies even at tenors as long as five years. This finding contradicts the widespread perception that “SIPs are safer”. Investors should incorporate the higher shortfall probability and conditional shortfall into their expectations from SIPs.

Keywords— SIP, DCA, Dollar Cost Averaging, Rupee Cost Averaging, India JEL Classification Codes G00, G11, C15

1 ***Introduction***

Systematic investment plans (SIP) have grown significantly amongst retail investors in India. Mutual Fund SIP accounts stood at 4.91 crores, and the total amount collected through SIP during December 2021 was ₹11,305 crores[1] With small investments at regular intervals, these plans have the potential to build a sizeable portfolio over long time horizons. The mutual fund industry argues that SIPs are a disciplined and successful investing process, reducing the 'risk of investment' and diminishing the stress associated with market timing of lumpsum investments (LSI). SIPs have significantly smaller liquidity and budgetary constraints with low minimum investment amount requirements, making them attractive by design. In the Indian context, where the equity participation of households is relatively low, without a doubt, SIPs are effective saving tools.

Adopting a SIP strategy implies a commitment to stay the course over a long period, forcing an investor into a deterministic method irrespective of market conditions. Despite the rapid growth of SIPs in India, little formal analysis of the returns and risks associated with SIPs is available for investors. Most tools available to investors use a straight-line extrapolation of returns to show prospective investors the potential trajectory of SIP values. This naive approach could blind a potential investor to the real risk of SIP investment. As we demonstrate, SIPs are subject to market risks and suffer capital losses, even over extended periods.

Many academic studies show that LSI strategies perform better than SIP strategies. In India, regular savings fund most SIPs. Different from many dollar-cost averaging studies, we do not assume a prior risk-free position for the investor, allowing us to isolate the effect of the unique investment schedule of a SIP. This paper is one of the first to examine the empirical evidence of returns and risks SIPs in India. Our contribution is multi-faceted. First, we outline a formal mathematical formulation to model a SIP strategy. This theoretical formulation gives insight into the expected returns and variance for SIPs. Second, we detail the empirical evidence and report realised returns and risks of SIPs and equivalent LSIs for selected AMFI fund sub-categories. This paper reports the first long time-series analysis on the realised performance of SIPs across several popular mutual fund categories. Finally, we run a bootstrap simulation using the monthly returns of the S&P BSE 100 index as a robustness check of the conclusions arising from the empirical evidence. The bootstrap is a framework for practitioners to estimate a range of SIP terminal values. Amongst other uses, estimates of terminal values are essential in managing sequence risk of long term portfolios. We show that returns of SIP and LSI strategies tend to converge as time increases. The "disciplined" regular investment process without market-timing delivers similar returns as comparable LSI strategies over the long run. Our findings demonstrate that SIP is a very effective strategy for accumulating investment from regular savings. However, while SIP strategies have lower volatility in the short term, they have significantly higher shortfall probabilities and average conditional shortfalls over extended periods than LSI strategies. "Time in the market" does not translate to risk-free return or lower risk. Investors should be aware of the higher shortfall probabilities and average conditional losses even for periods as long as five years and have appropriate expectations from SIPs.

The rest of the paper is organized as follows: Section 2 covers the literature, in Section 3 we outline the analytical framework, in Section 4, we discuss our methodology and data; in Section 5, we present our results; and we conclude in Section 6.

2 ***Literature Review***

SIPs are multi-period investment strategies. The introduction of time of entry as a variable into the investment strategy is a growing field of research (see Ling et al. (2020)). Wessmann (1948)) argued for dollar-cost averaging (DCA) plans, calling it "the unbeatable formula". DCA/SIP[2]. The popular press widely endorses SIP/DCA strategies. For instance, in the Forbes India article titled "What Is An SIP And How Does It Work?" published October 27, 2021, the authors conclude "*The simplicity of SIP and ease of understanding has the potential to ride over market cycles for disciplined investors to create long-term wealth. Given the upside to SIP investments, make a start, and invest in mutual funds through SIPs and work towards realising your financial goals*"[3]. Malkiel (2003) writes, "Periodic investments of equal dollar amounts in common stocks can reduce (but not avoid) the risks of equity investment by ensuring that the entire portfolio of stocks will not be purchased at temporarily inflated prices". Most wealth advisers actively promote SIP strategies.

Studies have tried to explain the popularity of SIP using behavioural finance. Statman (1995) attributed the use of DCA to investor irrationality, proposing four behavioural explanations:

- Prospect theory: people tend to give less weight to outcomes that are merely probable than to certain outcomes;
- Aversion to responsibility and regret;
- Cognitive errors caused by recent stock trends; and
- A lack of self-control.

Leggio and Lien (2001) empirically test Statman's conjecture and show that loss-aversion does not explain the "existence of the [DCA] investment strategy". They find DCA is "not an appropriate investment strategy for volatile assets such as small-cap stocks". DCA, they argue, is "best suited for investors interested in a forced savings plan that avoids the consumption of earnings".

Academic studies beginning with Constantinides (1979) have generally concluded that DCA strategies have lower returns than their LSI counterparts (see Knight and Mandell (1993); Abeysekera and Rosenbloom (2000); Leggio and Lien (2003); Kirkby et al. (2020)). When comparing realised returns and shortfall probabilities, Lei and Li (2007) show that returns from DCA and LSI strategies are "statistically identical" across investment horizons. Nevertheless, they show that DCA strategies have "significantly higher shortfall probabilities than lump sum investments for intermediate horizons". Milevsky and Posner (1999) prove if the underlying price ends up exactly where it starts, "the expected payoff from DCA is uniformly better than LSI". Further, the "magnitude of this conditional benefit to DCA, increases with higher volatility of the underlying security". Trainor (2005), using a statistic known as "first-passage time probability", shows that DCA strategies relative to LSI can "significantly reduce the probability, magnitude, and duration of enduring a large loss". He

[2] The DCA in academic literature usually assumes that investors have an initial stock of wealth invested in the risk-free asset from which a fixed dollar amount is invested at equal intervals over a holding period. We assume that regular savings fund a SIP, which differs from many other studies. We use DCA to refer narrowly to the formal academic interpretation.

[3] https://www.forbes.com/advisor/in/investing/what-is-an-sip-and-how-does-it-work/

adds that this is "especially relevant to investors with minimum loss thresholds, possible interim withdrawal needs, changing asset allocations, and/or an uncertain retirement date". Grable and Chatterjee (2015) show that depending on the market conditions and investor circumstances, "DCA can sometimes be a smart investing tactic and occasionally an inefficient allocation approach". Luskin (2017) reported that DCA outperforms LSI strategies when the cyclically-adjusted price-to-earnings (CAPE) ratio is high. Smith and Artigue (2018) argue that DCA is "a valuable way of diversifying investment decisions across time" and this "is particularly appealing when investing in volatile stocks over a substantial horizon". Many of these studies compare DCA and LSI in portfolio terms, which this paper does not. However, we leverage the principles arising from this vast body of work. Research on SIPs in India is more limited in scope and depth (see Jha (2015); Somani and Sharma (2017); Gajera et al. (2020)).

The support for DCA in academic literature is mixed at best. Despite this, the popularity of the strategy remains strong. There are some compelling reasons for DCA/SIPs - the discipline of savings and investment, ease of implementation, and intuitive appeal. However, evaluating the SIP investment strategy must use similar standards as any other investment strategy. Investors should estimate expected returns and risks before they make their investment decision. So we begin first by setting out a framework for expectations of returns and variances for a SIP.

3 ***Modelling a SIP Strategy***

To formulate a generalised problem for a SIP, assume that a fixed amount, $|SIP$, is invested into a risky asset every period t over a time horizon $[1,T]$, where 1 is the first investment and T the terminal point of the SIP. To simplify the problem, we assume that the investor has decided their target invested amount, IA, and allocates it through equal amounts of $|SIP$ for T periods such that $SIP = IA/T$. Additionally, we assume that period returns, r_t, of the risky asset, where $r_t = P_{t+1}/P_t$ over the period $[1,T]$ are i.i.d with a mean period return μ and standard deviation σ. Finally, we assume that the investor does not start with an endowment invested in a risk-free asset[4]. The terminal wealth value, W_T, of a SIP strategy is

$$W_T = P_T \cdot \sum_{t=1}^{T} \frac{SIP}{P_t} - \frac{IA}{T} \cdot \sum_{t=1}^{T} \frac{P_T}{P_t} \quad (1)$$

where P_t is the price of the asset at point t, and SIP is the fixed rupee amount for each SIP. At each period, the investor adds $n_t = \frac{SIP}{P_t}$ units of the risky asset. The price determines the number of units purchased for a period at point t as SIP is a fixed amount: if P_t is high, the number of units purchased is low. Since the average cost weights of the purchase prices by the number of units purchased, the average cost of the SIP will be equal to or less than the average of the

[4] In the Indian context, as a majority of SIPs are savings from regular income, our construct reflects the real-world. Many studies model the DCA - where the corpus is moved from a risk-free asset to a risky asset over the SIP duration. See Smith and Artigue (2018); Kirkby et al. (2020) amongst others quoted earlier.

purchase prices. The implication is that SIP will provide a "better" overall purchase price than the average price at entry across the SIP period.

$$\frac{IA}{n_1 + n_2 + ... + n_T} \leq \frac{P_1 + P_2 + ... + P_T}{T} \tag{2}$$

where $n_1,n_2,...,n_T$ are the number of units purchased at t = 1,2,..,T at prices $P_1,P_2,...,P_T$. To demonstrate this, the average cost of the SIP is given by

$$\begin{aligned} &= \frac{IA}{n_1 + n_2 + ... + n_T} \\ &= \frac{IA}{\frac{IA}{T} \cdot \sum_{t=1}^{T} \frac{1}{P_t}} \\ &= \frac{T}{\sum_{t=1}^{T} \frac{1}{P_t}} \\ &= H(P_1, P_2, ..., P_T) \end{aligned} \tag{3}$$

where $H(P_1,P_2,...,P_T)$ are the harmonic means of the entry prices $P_1,P_2,...,P_T$. The "inequality of arithmetic and geometric means" from mathematics states that "the arithmetic mean of a list of non-negative real numbers is equal to or greater than the geometric mean of the same list. Further, the two means are equal if and only if every number in the list is the same". The intuition behind SIP allowing an investor to purchase stock at an average price below the average of the prices prevailing on the purchase dates arises from this mathematical inequality. Equation 2 does not imply that SIPs will necessarily have a positive expected return as implied in the popular messaging promoting SIPs.

We can rewrite Equation 1 as:

$$\begin{aligned} W_T &= \frac{IA}{T} \cdot \sum_{i=1}^{T} \frac{P_T}{P_t} \\ &= \frac{IA}{T} \sum_{k=1}^{T} \prod_{i=k}^{T} r_i \end{aligned} \tag{4}$$

where r_i is the period return for period i. Equation 4 shows that the returns for the different SIP periods are a sum of geometric series. Using the general formula for the sum of a geometric series $\sum_{i=1}^{n} r^k = r\frac{1-r^n}{1-r}$, the expected portfolio value for the SIP is represented by:

!#

$$\begin{aligned} \mathbb{E}[W_T] &= \mathbb{E}\left[\frac{IA}{T} \sum_{k=1}^{T} \prod_{i=k}^{T} r_i\right] \\ &= \mathbb{E}\left[\frac{IA}{T} \left(\sum_{k=1}^{T} \prod_{i=k}^{T} r_i \right.\right. \\ &= \frac{IA}{T} \sum_{k=1}^{T} \prod_{i=k}^{T} \mathbb{E}[r_i] \\ &= \frac{IA}{T} \cdot \mu \cdot \left(\frac{1-\mu^T}{1-\mu}\right) \end{aligned} \tag{5}$$

The expected value of SIP is a function of T, the SIP term and μ the expected return[5]. Where T = 1, the buy-and-hold scenario, the expected value of the SIP portfolio is the same as a lump-sum investment ($\frac{IA}{1} \cdot \mu$).

Kirkby et al. (2020) analytically show that "*expected terminal wealth always declines as we increase the number of investments between t0 and T*". Smith and Artigue (2018) argue that expected returns of SIP investments are determined by the underlying asset's expected return as equation 5 shows.

The variance of the SIP portfolio value is represented as:

$$\begin{aligned}
\mathrm{Var}\left(W_T\right) &= \mathrm{Var}\left(\frac{IA}{T}\sum_{k=1}^{T}\prod_{i=k}^{T} r_i\right) \\
&= \left(\frac{IA}{T}\right)^2\left(\sum_{k=1}^{T}\mathrm{Var}\left(\prod_{i=k}^{T} r_i\right)\right) \\
&= \left(\frac{IA}{T}\right)^2\sum_{k=1}^{T}\left(\mathbb{E}\left[\left(\prod_{i=k}^{T} r_i\right)^2\right] - \left(\mathbb{E}\left[\prod_{i=k}^{T} r_i\right]\right)^2\right) \\
&= \left(\frac{IA}{T}\right)^2\sum_{k=1}^{T}\left(\left(\prod_{i=k}^{T}\left(\left(\mathbb{E}\left[r_i\right]\right)^2 + \mathrm{Var}(r_i)\right)\right) - \left(\mathbb{E}\left[\prod_{i=k}^{T} r_i\right]\right)^2\right) \\
&= \left(\frac{IA}{T}\right)^2\sum_{k=1}^{T}\left(\left(\prod_{i=k}^{T}(\mu^2+\sigma^2)\right) - \left(\mathbb{E}\left[\prod_{i=k}^{T} r_i\right]\right)^2\right) \\
&= \left(\frac{IA}{T}\right)^2 \cdot \left((\mu^2+\sigma^2)\cdot\frac{1-(\mu^2+\sigma^2)^T}{1-(\mu^2+\sigma^2)} - \mu^2\cdot\frac{1-\mu^{2T}}{1-\mu^2}\right)
\end{aligned} \tag{6}$$

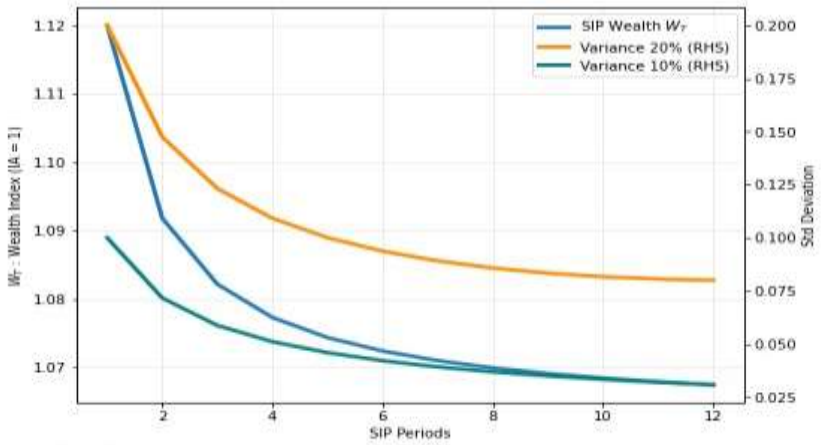
When the SIP term T = 1, equation 6 reduces to $\left(\frac{IA}{T}\right)^2 \cdot \sigma^2$ as the expression in the brackets simplifies to $(\mu^2 + \sigma^2) \cdot 1 - \mu^2 \cdot 1 = \sigma^2$. We plot the results of equations 5 and 6 to show the relationship between E[W_T], Var(W_T) in Figure 1 using annualised μ = 12% and annualised σ = 10% and 20% while varying T = 1,2,...,12. These results are consistent with academic studies showing SIPs have lower expected returns compared to LSI.

This higher expected return comes with the higher expected variance (see Kirkby et al. (2020)).

The mathematical formulation shows the importance of the underlying distribution of prices in determining

Figure 1: Terminal Wealth and Std Deviation for a SIP with annualise μ=12%, annualised σ = 10%, 20% and T = 1,2,...,12

[5] It is important to adjust μ by the frequency of SIP, *t*. When T is large relative to *t*, then a continuous compounding method will serve as a good approximation.



SIP terminal wealth computed with annualised return, μ, = 12% and std deviation, σ, = 10%, 20% for T = 1,2,...12.

Source: Author's calculations

the terminal value of a SIP, W_T. Two price distributions with the same μ but different σ will result in different E[W_T] and Var(W_T). As we show, E[W_T] is path-dependent, making estimating it through a closed-form extremely challenging. Two pragmatic solutions to estimate terminal values are using the historical prices directly or running simulations from historical returns. In the former, one assumes that historical trends persist, while the latter assumes that the historical distribution of returns remains the same in the future.

4 ***Methodology and Data***

4.1 ***Methodology***

4.1.1 ***Empirical Analysis***

Table 1: XIRR and Annualised Value Return

SIP Date	Entry Price (Rs)	Num Units	Cumm Units	Cash Flow (Rs)	SIP Value (Rs)
15/1/16	1.15	17,391.30	17,391.30	(20,000.00)	20,000.00
16/1/17	1.65	12,121.21	29,512.52	(20,000.00)	48,695.65
15/1/18	1.34	14,925.37	44,437.89	(20,000.00)	59,546.77
15/1/19	1.54	12,987.01	57,424.90	(20,000.00)	88,434.35
15/1/20	1.10	18,181.82	75,606.72	(20,000.00)	83,167.39
15/1/21	1.68	11,904.76	87,511.48	127,019.29	147,019.29
Annualised Terminal Value/Invested Amount					**8.01%**
XIRR				**8.07%**	

SIPs and LSIs are very different approaches: investing a lump sum implies the funds are working over more extended periods, while a SIP drip-feeds investments over time. To calculate comparable returns for the two styles, we use Extended Internal Rate of Return (XIRR) to calculate rates of return using cash flows and their corresponding dates. The XIRR is the Internal Rate of Return (IRR) for a series of cash flows assigning specific dates to each cash flow. Table 1 shows a hypothetical example where the investor invests |20,000 on 15 January annually from 2016. Where the 15th falls on a holiday, the investment occurs on the next business day (in the case of 2017). We assume that the SIP terminates on 15 January 2021. The table shows the entry price, the number of units bought annually, the cumulative number of units,

the cash flow and the SIP value every period. As expected, the annualised LSI return based on the terminal value of the SIP, W_T, and the invested amount, IA[6], is different to the XIRR based on the cash flows and the specific cash flow dates.

It is immediately intuitive that SIP returns are dependent on the price path. To illustrate the path dependence, table 2 summarises five hypothetical scenarios of different price paths and compares the XIRR of a SIP versus the CAGR of an LSI strategy. In all cases, the price on the first day is 1, and if we assume a fixed SIP investment of |10,000 for each period, the LSI strategy would have bought 13,000 units. Price A is the scenario of monotonically increasing prices. As the SIP purchases fewer units every period, it has 11,654 units[6] at the end. This price path results in the XIRR being less than the LSI CAGR. In scenario Price B, the price is flat. Both the XIRR and the CAGR will be the same[7]. In a scenario when the price is monotonically declining, Price C, the SIP Strategy's XIRR is lower than the CAGR of the LSI strategy. As SIP is a deterministic strategy that commits the investor to purchase a fixed Rupee amount every period, the hapless investor is buying more and more units as the price falls, increasing the cumulative loss at the end of the period. Price scenarios D and E are the same except for the final price. With the lower terminal price in D, the SIP performs worse than the LSI. Recall that XIRR uses both the cash flows and dates. With the final price being higher in scenario E, the XIRR for this scenario is better than the CAGR of the LSI strategy. Note how sensitive the returns are to the final terminal price. SIP returns are affected by a. the entry prices, P_i; b. the frequency of the SIP, t; c. the SIPs term/tenor, T; and d. the terminal price, P_T. SIPs suffer from exposure to sequence risk similar to an LSI strategy.

Table 2: Path dependence of SIP Returns

Date	***Price A***	***Cumm Unite***	***Price B***	***Cumm Units***	***Price C***	***Cumm Unit***	***Price D***	***Cumm Units***	***Price E***	***Cumm Units***
1/12/18	1.00	1,000.00	1.00	1,000.00	1.00	1,000.00	1.00	1,000.00	1.00	1,000.00
1/1/19	1.02	1,980.00	1.00	2,000.00	0.98	2,020.00	1.02	1,980.00	1.02	1,980.00
1/2/19	1.04	2,941.00	1.00	3,000.00	0.96	3,061.00	0.98	3,000.00	0.98	3,000.00
1/3/19	1.06	3,884.00	1.00	4,000.00	0.94	4,124.00	1.03	3,970.00	1.03	3,970.00
1/4/19	1.08	4,809.00	1.00	5,000.00	0.92	5,210.00	0.99	4,980.00	0.99	4,980.00
1/5/19	1.10	5,718.00	1.00	6,000.00	0.90	6,321.00	1.04	5,941.00	1.04	5,941.00
1/6/19	1.12	6,610.00	1.00	7,000.00	0.88	7,457.00	1.00	6,941.00	1.00	6,941.00
1/7/19	1.14	7,487.00	1.00	8,000.00	0.86	8,619.00	1.05	7,893.00	1.05	7,893.00
1/8/19	1.16	8,349.00	1.00	9,000.00	0.84	9,809.00	1.01	8,883.00	1.01	8,883.00
1/9/19	1.18	9,196.00	1.00	10,000.00	0.82	11,028.00	1.06	9,826.00	1.06	9,826.00
1/10/19	1.20	10,029.00	1.00	11,000.00	0.80	12,278.00	1.02	10,806.00	1.02	10,806.00
1/11/19	1.22	10,848.00	1.00	12,000.00	0.78	13,560.00	1.07	11,740.00	1.07	11,740.00
1/12/19	1.24	11,654.00	1.00	13,000.00	0.76	14,875.00	1.03	12,710.00	1.08	12,665.00
SIP XIRR		***23.03%***		***0.00%***		***-24.96%***		***1.41%***		***10.59%***
LSI CAGR	***24.00%***		***0.00%***		***-24.00%***		***3.00%***		***8.00%***	

For our empirical study, we compare three strategies: a monthly SIP, a quarterly SIP and a simple LSI Strategy (Buy and Hold). Starting from the first date of prices for each security in our sample, we generate the cash flows for each strategy. The amount invested, IA, is the same for all three. For the monthly SIP, IA is divided equally by the number of months; for the quarterly SIP, the number of quarters and a single lump sum at LSI start. We compute XIRR for the three strategies from the price series of the underlying security, accounting for the dates of investment and terminal value, W_T, when the strategy is complete. During the SIP's term, we invest the fixed sum on the same date every

[6] We ignore the decimals in the calculation.

[7] Following the endowment model, where the investor has the original amount IA in a risk-free asset which is reduced by the SIP amount periodically, the SIP would perform better than the LSI strategy. But that is not our approach.

period[8]. If the SIP starts on the 1st of the month, subsequent investments occur on the 1st of the remaining months. If the 1st of any month is a holiday, the investment is on the next trading day.

Similarly, for a quarterly SIP, the investment is made on the same date of the month quarterly, starting from the month of the first investment. If the investment begins in January, the subsequent investments will be in April, August, and November. If the first investment starts in December, the subsequent investments will be in March, July and October.

For each strategy, we vary the term of the SIP, T = 1, 3, 5, 7 years[9]. We buy the units with the final investment amount and simultaneously calculate the terminal value on the final day. We use actual dates of the investments every month for the fixed investment amount and the terminal date and terminal value of the SIP[10] to calculate the XIRR. The annualised standard deviation is computed from the time-weighted monthly returns throughout the term of the SIP[11].

To measure risk-adjusted performance, we compute the raw Sharpe ratios. A second risk measure is the chances that the terminal value of the investment falls below a pre-specified level: the probability of shortfall. For the paper, we use a zero risk-free rate[12], consequently, the pre-specified level is the total nominal investment, IA. The probability of shortfall is:

$$p = \mathrm{P}[W_T < IA] \tag{7}$$

where W_T is the terminal value, and IA is the nominal amount invested. To calculate this, we divide the number of instances that a particular strategy shows the shortfall by the total number of observations.

In addition to the shortfall probability, we calculate the average conditional shortfall to quantify the shortfall. The conditional expected shortfall is the expected value of the shortfall dependent on a shortfall occurring. It measures the average amount by which investment falls short of IA[13]. The conditional expected shortfall is:

$$CES = \left(\mathbb{E}\left[\frac{IA - W_T}{IA}\right] \;\middle|\; W_T < IA\right) \tag{8}$$

Having shortfall probability and mean conditional shortfall allows us to examine the chance of a nominal capital loss and the average amount of nominal loss in the event of a loss. The two measures give the expected value of any shortfall for a particular strategy and allow investors to choose plans subject to their loss aversion.

$$TWR_t = \frac{W_t - (W_{t-1} + CashFlow_t)}{(W_{t-1} + CashFlow_t)}$$

[8] month or a quarter, as the case may be.

[9] With the data period of 21 years, we choose to limit our time horizon to make effective use of the data. At T = 7, we can calculate the metrics up to December 2014, which we argue is at the outside margin of data for the results to remain relevant to investors today.

[10] For LSI strategy, there are only two cash flows: IA at t = 1 and W_T at $t = T$.

[11] Where the time-weighted return,.

[12] The funds are not invested into a risk-free asset.

[13] We ignore the time-value inherent in the cash-flows of a SIP given our assumption that the SIP funding is from savings.

4.1.2 ***Sampling Methodology***

The use of the rolling methodology, to an extent, simulates various price paths for the strategies from the historical prices. The approach generates a distribution of realised returns and standard deviations and, in a limited manner, deals with the path dependency of our results. A fairly obvious next step is generalising the simulation methodology to a bootstrap or Monte-Carlo approach. One could use either historical return sequences or expectations of the distribution of returns to run simulations of the price formation paths to arrive at the generalised expectations of risk and returns of these strategies (see Abeysekera and Rosenbloom (2000); Zenti (2013)). As security prices exhibit auto-correlation and their underlying distribution is challenging to be force-fitted into standard distributions, we use the bootstrap method (see Efron and Tibshirani (1994)).

The most common bootstraps for time-series data are a. the moving block bootstrap (MBB); b. the nonoverlapping block bootstrap (NBB); c. the circular block bootstrap (CBB); and d. the stationary bootstrap (SB). The first three use fixed-length blocks, while the last uses a random block length. The simulated return series is generated by sampling with replacement using blocks of returns from the pool of past returns in all cases. The bootstrap process attempts to replicate the inherent auto-correlation by re-sampling data blocks and not data points with replacement. This procedure attempts to retain any inter-temporal link in the original price series intact. Lahiri (1999) shows "both the MBB and the CBB outperform the NBB and the NBB outperforms the SB". We use the CBB for this paper which wraps returns circularly, allowing all data points have an equal chance of being picked. Specifically, we use the bootstrapping library ARCH for our computation (Sheppard, 2021)[14]. The block length serves as a smoothing parameter for the bootstrap. The block length determines the bias and the variance of a block bootstrap estimator. Following Patton et al. (2009), the ARCH library has an implementation to identify the optimal block size. We calculate the block size, b, using rolling monthly returns as 76 [15].

The relevant variables for a SIP strategy are its start date, its frequency, t, and its term, T. For our simulation, we randomise the start date for every iteration. From the randomised start date, we generate the SIP schedule such that each SIP investment occurs on a business/trading day. This procedure accounts for any seasonal day-of-month and other calendar effects. We run 50,000 simulations for monthly and quarterly SIPs and the equivalent LSI strategy for T = 1, 3, 5 and 7 years. Using these iterations, we compute the metrics used in the empirical analysis. To compare the returns and standard deviations of the SIP strategies against the LSI strategies, we define the bootstrapped p-value as the probability that the realised return or standard deviation of the SIP strategy is higher than the realised return of the LSI. A p-value of 5% (95%) would indicate that the SIP strategy under-performs (outperforms) the LSI strategy on the level of returns or has lower (higher) risk as measured by standard deviation at the 5% significance level.

[14] In this implementation, the MBB does not wrap around, and so observations near the start or end of the series will be systematically under-sampled. Consequently, the author does not recommend it, and we do not use it.

[15] We varied the block size, including using various fixed block sizes (ranging from b = 12..180), and while the returns, standard deviations and bootstrap p-values change based on the value of b, the block size, our overall conclusions remain. In this paper, we do not report summary outcomes for alternative bs

To estimate the statistical significance of the difference in the shortfall probabilities and mean conditional shortfall for strategies, we divide the 50,000 simulated return series into sub-samples each of 500 items. We calculate and compare the shortfall probabilities and average conditional shortfall for the strategies in each sub-sample. Under the null hypothesis that the strategies have the same shortfall probability/mean conditional shortfall, one strategy should not consistently provide a lower chance than the other strategy. Again, we define the bootstrapped p-value as the probability that the shortfall probability or mean conditional shortfall for a SIP plan is higher than the shortfall probability for the equivalent LSI. Therefore, a p-value of 5% (95%) indicates that the SIP has a lower (higher) shortfall probability or mean conditional shortfall than its equivalent LSI at the 5% significance level.

4.2 ***Data***

All daily NAV of funds (from 1 December 1999) and the daily index value of the S&P BSE 100 Total Return Index (from 1 August 2006) are from Refinitiv Datastream. We select sub-categories from the Debt, Hybrid and Equity AMFI fund categories with at least two mutual fund schemes. The period of observation is till 31 December 2021. The complete listing of the mutual funds is in Table A1 of the Appendix.

Table 3: Count of funds in each AMFI category

AMFI Category	Number
Debt Scheme - Liquid Fund	4
Debt Scheme - Low Duration Fund	2
Debt Scheme - Medium to Long Duration Fund	3
Debt Scheme - Corporate Bond Fund	3
Debt Scheme - Gilt Fund	4
Hybrid Scheme - Aggressive Hybrid Fund	7
Equity Scheme - ELSS	4
Equity Scheme - Large Cap Fund	2
Equity Scheme - Large & Mid Cap Fund	4
Equity Scheme - Mid Cap Fund	2
Equity Scheme - Flexi Cap Fund	4

5 ***Results and Discussion***

5.1 ***SIPs using Debt Mutual Funds***

Table 4 shows the summary results for debt and hybrid mutual funds AMFI subcategories in our sample. For liquid funds, the LSI strategy generally outperforms SIP strategies. As the price path of liquid funds generally increases monotonically, buy-and-hold returns are better under such a price formation. The very low realised standard deviation is similar across the strategies and T. All three plans have zero realised shortfall percentage across T. A hypothetical investor with perfect foresight might prefer LSI, but if budget-constrained, would have accepted either SIP strategy as well.

Low Duration, Medium to Long Duration, Corporate Bond and Gilt schemes show similar trends: returns increase along with risk across these categories, with Gilt funds showing the highest realised returns and volatility. LSI generally delivers the highest returns as T increases, while at T = 1, the monthly SIP strategy occasionally has the highest return by a very narrow margin. The underlying prices are largely monotonic, so the empirical results are intuitive. As expected, across all the categories, realised risk increases across T. Generally, LSI has the highest realised variance for all the schemes across T with a few exceptions.

The differences are minor on a risk-adjusted basis, with raw Sharpes staying within a narrow band at T=7. At T=1, the monthly SIP strategy generally outperforms the other two strategies on a risk-adjusted basis, and the spread in raw Sharpe is wider than at higher Ts, reflecting the variation in realised returns and realised risk due to the impact of entry prices in the short tenor. As T increases, LSI offers a marginally higher risk-adjusted return. Shortfall probabilities are non-zero for all the scheme categories across the three strategies at T = 1. These range from 1.3%-7.1%, 1.2%- 6.8%, and 0-5.8% for the monthly SIP, quarterly SIP and LSI strategies, respectively. LSI strategies have the lowest probability of shortfall, followed by quarterly SIP strategies, with the monthly SIP strategies having the highest percentages. This result follows intuition - for generally upward sloping price trajectories, an early entry point will have the lowest probability and duration of showing a loss. Entering multiple times increases the probability and duration of showing shortfalls as the harmonic average price is likely to be higher than the immediately following price point. For T > 1, no strategy shows a shortfall. As realised standard deviation increases, the probability of shortfall increases as well. The mean conditional shortfall is small (around 1% or less) across the schemes for the three strategies.

Our hypothetical investor with perfect foresight choosing an investment strategy for any of these funds is likely to choose between a monthly SIP and the LSI for short Ts. As the investment horizon gets longer, like for the liquid schemes, the budget-constrained investor would be increasingly indifferent across strategies.

Our final scheme type in this Section is the 'Aggressive Hybrid Fund'. These funds hold "65% to 80% of their investment in equity & equity related instruments and 20% to 35% in Debt instruments". With the addition of equity, this category's return standard deviations are significantly higher than the pure debt scheme categories.

Table 4: Summary performance of 3 strategies using Debt Schemes

AMFI Category	Strategy	Period (Years), T metric	1	3	5	7
Debt Scheme - Liquid Fund	SIP Monthly	Mean Realised Return	6.81	6.81	6.91	7.09
		Mean Realised Std	0.29	0.37	0.42	0.46
		Raw Sharpe	23.25	18.52	16.47	15.40
		Shortfall Prob	0.00	0.00	0.00	0.00
		Mean Cond. Shortfall				
	SIP Quarterly	Mean Realised Return	6.82	6.81	6.91	7.09
		Mean Realised Std Raw	0.40	0.41	0.44	0.47
		Sharpe	17.03	16.69	15.56	14.96
		Shortfall Prob	0.00	0.00	0.00	0.00
		Mean Cond. Shortfall				
	LSI	Mean Realised Return	6.86	6.91	6.97	7.08
		Mean Realised Std Raw	0.23	0.37	0.43	0.46
		Sharpe	29.57	18.86	16.27	15.24
		Shortfall Prob	0.00	0.00	0.00	0.00
		Mean Cond. Shortfall				
Debt Scheme - Low Duration Fund	SIP Monthly	Mean Realised Return	7.27	7.00	6.94	7.13
		Mean Realised Std	1.00	1.25	1.20	1.18
		Raw Sharpe	7.27	5.61	5.76	6.02
		Shortfall Prob	1.28	0.00	0.00	0.00
		Mean Cond. Shortfall	0.82			
	SIP Quarterly	Mean Realised Return	7.27	7.01	6.95	7.13
		Mean Realised Std Raw	1.15	1.29	1.23	1.20
		Sharpe	6.35	5.41	5.63	5.92
		Shortfall Prob	1.15	0.00	0.00	0.00
		Mean Cond. Shortfall	0.72			
	LSI	Mean Realised Return	7.27	7.14	7.13	7.22
		Mean Realised Std Raw	1.12	1.32	1.26	1.22
		Sharpe	6.48	5.40	5.68	5.91

		Shortfall Prob	0.05	0.00	0.00	0.00
		Mean Cond. Shortfall	0.09			
		Mean Realised Return	8.38	7.84	7.51	7.66
		Mean Realised Std	3.19	3.98	4.35	4.58
	SIP Monthly	Raw Sharpe	2.63	1.97	1.73	1.67
		Shortfall Prob	4.78	0.00	0.00	0.00
		Mean Cond. Shortfall	0.58			
		Mean Realised Return	8.37	7.84	7.52	7.66
		Mean Realised Std Raw	3.50	4.11	4.43	4.64
Debt Scheme - Medium to Long Duration Fund	SIP Quarterly	Sharpe	2.39	1.91	1.70	1.65
		Shortfall Prob	3.85	0.00	0.00	0.00
		Mean Cond. Shortfall	0.47			
		Mean Realised Return	8.35	7.93	7.68	7.72
		Mean Realised Std Raw	3.83	4.33	4.61	4.78
	LSI	Sharpe	2.18	1.83	1.67	1.62
		Shortfall Prob	2.77	0.00	0.00	0.00
		Mean Cond. Shortfall	0.58			
		Mean Realised Return	8.03	7.51	7.15	7.26
		Mean Realised Std	2.54	3.16	3.45	3.66
	SIP Monthly	Raw Sharpe	3.16	2.38	2.07	1.98
		Shortfall Prob	3.08	0.00	0.00	0.00
		Mean Cond. Shortfall	0.80			
		Mean Realised Return	8.03	7.51	7.15	7.26
		Mean Realised Std Raw	2.80	3.27	3.52	3.71
Debt Scheme - Corporate Bond Fund	SIP Quarterly	Sharpe	2.87	2.30	2.03	1.95
		Shortfall Prob	2.88	0.00	0.00	0.00
		Mean Cond. Shortfall	0.67			
		Mean Realised Return	8.02	7.61	7.29	7.28
		Mean Realised Std Raw	3.05	3.44	3.66	3.83
	LSI	Sharpe	2.63	2.21	1.99	1.90
		Shortfall Prob	1.94	0.00	0.00	0.00
		Mean Cond. Shortfall	0.52			
		Mean Realised Return	9.67	8.77	8.15	8.21
		Mean Realised Std	4.38	5.39	5.81	6.05
	SIP Monthly	Raw Sharpe	2.21	1.63	1.40	1.36
		Shortfall Prob	7.11	0.00	0.00	0.00
		Mean Cond. Shortfall	0.98			
		Mean Realised Return	9.65	8.78	8.16	8.22
		Mean Realised Std Raw	4.81	5.58	5.94	6.14
Debt Scheme - Gilt Fund	SIP Quarterly	Sharpe	2.01	1.57	1.38	1.34
		Shortfall Prob	6.82	0.00	0.00	0.00
		Mean Cond. Shortfall	0.80			
		Mean Realised Return	9.59	8.95	8.46	8.40
		Mean Realised Std Raw	5.28	5.88	6.17	6.32
	LSI	Sharpe	1.82	1.52	1.37	1.33
		Shortfall Prob	5.81	0.00	0.00	0.00
		Mean Cond. Shortfall	1.20			
		Mean Realised Return	17.69	16.87	16.34	15.28
		Mean Realised Std	13.36	15.23	15.86	16.42
	SIP Monthly	Raw Sharpe	1.32	1.11	1.03	0.93
		Shortfall Prob	21.40	5.92	1.11	0.00
		Mean Cond. Shortfall	7.72	11.07	3.87	
		Mean Realised Return	17.39	16.80	16.33	15.30
		Mean Realised Std Raw	14.76	15.78	16.23	16.71
Hybrid Scheme - Aggressive Hybrid Fund	SIP Quarterly	Sharpe	1.18	1.06	1.01	0.92
		Shortfall Prob	20.25	6.12	1.13	0.00
		Mean Cond. Shortfall	7.77	10.32	3.53	
		Mean Realised Return	16.36	15.69	15.98	15.58
		Mean Realised Std Raw	16.24	16.64	16.87	17.25
	LSI	Sharpe	1.01	0.94	0.95	0.90
		Shortfall Prob	20.89	4.68	0.00	0.00

		Mean Cond. Shortfall	13.75	13.60		

This table shows the metrics of adopting 3 strategies SIP Monthly, SIP Quarterly, and LSI using debt and hybrid schemes for T = 1, 3, 5, and 7 years. The realised return and variances are annualised and are the means of the metrics for the underlying mutual fund for each subcategory. Raw sharpe uses the means of the realised returns and realised std dev. The shortfall probability is the realised probability of shortfall ($W_T < IA$) and the mean conditional shortfall is the average of realised shortfalls when they occur.

The realised standard deviation for LSI strategies increased to 16.25-17.25% on average. Here the evidence of the effects of path dependence is more evident. At T=1, the returns of the two SIP strategies are 100bps more than the LSI, with the monthly SIP performing the best. The realised risk for the monthly SIP is also the lowest. At T = 7, the returns and risk are similar to what we expect from the mathematical model and the empirical evidence of other debt funds: LSI has the highest realised return and volatility with the monthly SIP the lowest realised return and risk. Raw Sharpes converge over T with all three strategies being very close at T=7.

At T = 1, across the three strategies in our observation period, the realised shortfall probability is about 20%. The realised mean conditional shortfall at T=1 is highest for LSI (13.75%) against the approximately 7.7% for the equivalent two SIP strategies. As T increases, the shortfall probability falls faster for the LSI strategy. At T=5, LSI has no realised shortfall probabilities, while the SIP strategies show a shortfall probability close to 1% with an over 3.5% realised conditional shortfall. At T=3, the SIP strategies have shortfall probabilities around 6% compared to under 5% for LSI. The realised conditional shortfall is between 10-11% and over 13.5% for the two strategies. The shortfall probability and the amount of shortfall in the event of a shortfall for a 3-year SIP are relatively significant.

Over long periods (T=5, 7), the LSI has better metrics on returns and shortfall percentage and worse metrics on risk and raw Sharpe. SIP strategies perform better when T = 1, 3 in terms of returns, risk and raw Sharpe, but the probability of a capital loss is non-trivial. These results are contrary to those reported by Israelsen (1999) that DCA was a superior strategy for low-volatile funds in a study of 35 large-cap equity funds in the US over the ten years 1988 to 1998. Our hypothetical investor faces an increasingly difficult decision based on this evidence.

The empirical evidence confirms intuition for debt funds with low volatilities: LSI tend to outperform, although as T increases, the realised returns of the SIP and LSI strategies converge. As the price paths are less variable and generally show a monotonic increase over time, the results also reflect the insight of our mathematical formulation of the problem. Realised volatility, too, follows intuition and the theoretical model. As realised returns and risks converge, a budget and liquidity constrained SIP investor will experience returns and volatility not very different from the LSI investor.

5.2 *SIPs using Equity Mutual Funds*

Table 5 summarises the metrics for selected AMFI equity schemes. Generally, the same conclusions from SIPs using debt funds apply: at T = 7, LSI tends to have higher realised returns; at T = 1, the realised returns of the monthly SIP strategy tend to be higher; realised volatility for LSI strategy is higher across T; raw Sharpes are higher for SIP strategies, and LSI strategies have much lower realised shortfall probabilities than SIP strategies at T = 7. Mean conditional shortfalls are highest for LSI strategies at T=1, and as T increases, SIP strategies show a higher conditional shortfall. At T=7, LSI strategies have not shown any conditional shortfalls for our sample period. However, both SIP strategies show a small shortfall probability and mean conditional shortfalls at T = 7.

Large Cap schemes are required to allocate at least 80% of their investments to firms that are in the top 100 by market capitalisation[16]. At T = 7, the LSI strategy has the highest realised returns and volatility and has had no realised shortfall during the period. The SIP strategies have higher realised returns at T = 1,3 and 5. Risk-adjusted returns converge over T, with the SIP strategies showing superior raw Sharpes at T=1, but by T = 7, the raw Sharpes are virtually indistinguishable. When T=1, the realised shortfalls probabilities are almost 1 in 4 across all three strategies. The monthly SIP has the highest realised shortfall probability. This result contradicts the "SIPs reduce risk" messaging popular in the media/wealth advisers. Regarding the average conditional shortfall, at T= 1, LSI shows the largest mean realised conditional shortfall of almost 15%. At T=3, both the SIP strategies show the largest shortfall probability, with all three strategies having average realised conditional shortfalls over 10%. An investor hoping to preserve capital via a SIP is likely disappointed with the empirical; evidence that contradicts the "safer strategy" myth.

The pattern is similar for ELSS, Large & Mid Cap, Mid Cap and Flexi Cap fund categories[17]. However, the shortfall probabilities and the conditional shortfall for SIPs across all schemes categories are significant. Possibly, this is a reflection of the generally rising but volatile equity returns experienced during the observation period. The realised conditional shortfall remains highest for LSI at T=1, reflecting the significant volatility inherent in the Indian equity markets. At T=3 years, the SIP and the LSI strategies show shortfall probabilities above 5% (and occasionally close to 10%) with average conditional shortfalls over 10%. Clearly, on these two measures of risk, SIPs are equally affected. Our hypothetical investor with perfect foresight is likely to face a difficult choice between the three strategies. Like for debt funds, the evidence for equity funds indicates that a SIP investor with longer time horizons will likely experience a slightly lower return than an equivalent LSI investor. The difference in the average realised returns over long periods (T = 7) is small: mean spread of 41 bps and max spread of 96 bps.

The empirical evidence is consistent across debt and equity asset classes. Subject to the price paths followed by each category during our observation period, the results are broadly aligned to the mathematical model laid out in section 3. First, LSI strategies tend to have higher realised returns and volatilities than SIP strategies. The empirical data is consistent with the theoretical framework for long time horizons (T = 7) for all the mutual fund categories. At lower T, the volatility of prices affects the empirical outcomes, and the results are noisy. The observed trend reflects the intuition where the prices are less volatile, e.g. liquid funds. Our results are also

- Though the Securities and Exchange Board of India introduced this classification in 2017/2018; there is some evidence that many large-cap funds have maintained such a strategy in previous years. See Raju (2021).

[17] These are SEBI classifications from 2017/2018. Some funds may have altered their strategies since 1999. Our methodology handles such changes via transformations in the fund's price. We caution against using the realised means and standard deviations as expected averages for the fund categories. There is a range of other technical tools available for more robust estimation.

Table 5: Summary performance of 3 strategies using Equity Schemes

AMFI Category	Strategy	Period (Years), T metric	1	3	5	7
Equity Scheme - ELSS	SIP Monthly	Mean Realised Return	24.35	21.63	20.15	17.66
		Mean Realised Std	18.32	20.99	21.88	22.62
		Raw Sharpe	1.33	1.03	0.92	0.78
		Shortfall Prob	25.97	8.21	3.67	0.33
		Mean Cond. Shortfall	9.36	16.72	7.80	4.31
	SIP Quarterly	Mean Realised Return	23.82	21.52	20.14	17.70
		Mean Realised Std Raw	20.21	21.72	22.38	23.00
		Sharpe	1.18	0.99	0.90	0.77
		Shortfall Prob	24.89	7.80	3.33	0.28
		Mean Cond. Shortfall	9.28	17.01	7.36	3.88
	LSI	Mean Realised Return	22.09	19.93	19.67	18.29
		Mean Realised Std Raw	22.23	22.90	23.25	23.73
		Sharpe	0.99	0.87	0.85	0.77
		Shortfall Prob	24.72	8.87	0.39	0.00
		Mean Cond. Shortfall	16.69	15.87	4.83	
Equity Scheme - Large Cap Fund	SIP Monthly	Mean Realised Return	22.63	20.49	19.10	17.13
		Mean Realised Std	17.45	19.74	20.34	20.92
		Raw Sharpe	1.30	1.04	0.94	0.82
		Shortfall Prob	25.01	8.25	1.65	0.44
		Mean Cond. Shortfall	8.64	11.69	8.89	5.41
	SIP Quarterly	Mean Realised Return	22.14	20.39	19.09	17.16
		Mean Realised Std Raw	19.23	20.42	20.80	21.29
		Sharpe	1.15	1.00	0.92	0.81
		Shortfall Prob	24.56	8.18	1.52	0.41
		Mean Cond. Shortfall	8.30	11.27	9.36	4.68
	LSI	Mean Realised Return	20.59	18.88	18.76	17.79
		Mean Realised Std Raw	21.15	21.52	21.62	21.96
		Sharpe	0.97	0.88	0.87	0.81
		Shortfall Prob	23.31	6.91	0.50	0.00
		Mean Cond. Shortfall	15.59	12.63	4.20	
Equity Scheme - Large & Mid Cap Fund	SIP Monthly	Mean Realised Return	21.68	18.95	17.44	15.46
		Mean Realised Std	18.45	21.10	21.87	22.56
		Raw Sharpe	1.17	0.90	0.80	0.69
		Shortfall Prob	26.96	9.72	3.47	0.57
		Mean Cond. Shortfall	10.16	13.34	7.80	5.18
	SIP Quarterly	Mean Realised Return	21.11	18.83	17.43	15.49
		Mean Realised Std Raw	20.34	21.83	22.35	22.95
		Sharpe	1.04	0.86	0.78	0.68
		Shortfall Prob	27.23	9.54	3.13	0.44
		Mean Cond. Shortfall	9.64	13.08	7.77	5.22
	LSI	Mean Realised Return	19.28	17.12	16.96	15.88
		Mean Realised Std Raw	22.38	23.02	23.24	23.68
		Sharpe	0.86	0.74	0.73	0.67
		Shortfall Prob	24.56	11.00	1.88	0.00
		Mean Cond. Shortfall	19.32	13.66	4.07	
Equity Scheme - Mid Cap Fund	SIP Monthly	Mean Realised Return	30.71	26.56	23.91	20.91
		Mean Realised Std	19.68	22.51	23.40	24.20
		Raw Sharpe	1.56	1.18	1.02	0.86
		Shortfall Prob	24.37	7.09	2.83	0.00
		Mean Cond. Shortfall	10.49	17.92	6.92	
	SIP Quarterly	Mean Realised Return	30.02	26.44	23.92	20.96
		Mean Realised Std Raw	21.68	23.28	23.92	24.61
		Sharpe	1.38	1.14	1.00	0.85
		Shortfall Prob	24.65	6.85	2.63	0.00
		Mean Cond. Shortfall	9.97	17.71	6.77	

Equity Scheme - Flexi Cap Fund	LSI	Mean Realised Return	27.78	24.71	23.72	21.86
		Mean Realised Std Raw	23.80	24.52	24.85	25.38
		Sharpe	1.17	1.01	0.95	0.86
		Shortfall Prob	23.14	8.43	0.07	0.00
		Mean Cond. Shortfall	18.56	13.77	1.12	
	SIP Monthly	Mean Realised Return	22.88	20.39	19.02	17.15
		Mean Realised Std	18.58	21.18	22.06	22.86
		Raw Sharpe	1.23	0.96	0.86	0.75
		Shortfall Prob	25.97	8.06	1.95	0.26
		Mean Cond. Shortfall	9.58	13.40	7.49	3.06
	SIP Quarterly	Mean Realised Return	22.35	20.28	19.01	17.17
		Mean Realised Std Raw	20.48	21.92	22.55	23.25
		Sharpe	1.09	0.93	0.84	0.74
		Shortfall Prob	25.31	7.84	1.77	0.21
		Mean Cond. Shortfall	9.31	13.17	7.70	2.54
	LSI	Mean Realised Return	20.63	18.70	18.59	17.63
		Mean Realised Std Raw	22.53	23.11	23.44	23.98
		Sharpe	0.92	0.81	0.79	0.73
		Shortfall Prob	24.63	8.35	0.29	0.00
		Mean Cond. Shortfall	17.10	12.90	4.37	

This table shows the metrics of adopting 3 strategies SIP Monthly, SIP Quarterly, and LSI using equity mutual fund schemes in our sample for T = 1, 3, 5, and 7 years. The realised return and variances are annualised and are the means of the metrics for the underlying funds for each subcategory. Raw sharpe uses the means of the realised returns and realised std dev. The shortfall probability is the realised probability of shortfall (W_T<IA) and the mean conditional shortfall is the average of realised shortfalls when they occur.

similar to findings reported by other studies (Constantinides, 1979; Knight and Mandell, 1993; Leggio and Lien, 2003; Kirkby et al., 2020). The difference in the average realised returns over long periods is small (a mean spread of 7 bps and max spread of 30 bps for debt and mean spread of 41 bps and max spread of 96 bps for equity).

To check if the return distributions are from the same underlying distribution of returns, we run the Anderson Darling k-sample test to compare the return distributions of the three strategies for T = 1, 3, 5, 7 pairwise. The null hypothesis is that the two samples are from the same distribution. Tables 6 and 7 summarise the results. They show that the ks test statistic is significant between the return distributions of SIP and LSI strategies, while the return distributions are similar for the two SIP strategies. But the differences as seen in the spread of mean realised returns and standard deviations at T=7 are small. Therefore, the statistical difference is unlikely to change the decision on the choice of strategy between a SIP and LSI in the long run. Importantly the results suggest that the inflexible investment schedule of SIP strategies do not hurt the returns experienced by an investor materially over the long run. Additionally, an investor with liquidity or budget constraints requiring a quarterly SIP will also experience similar outcomes as an investor with a monthly SIP when tenors are long. The longer the SIP term, T, the smaller the influence of the frequency, t. With short tenors SIPs, an investor would require to understand the security's return distribution to decide on which frequency to choose

Table 6: Summary Anderson Darling k-sample test: Debt mutual funds

		SIP Quarterly				LSI			
Cat	period metric	1	3	5	7	1	3	5	7
Debt Scheme - Liquid Fund	SIP Monthly	-0.87	-0.36	0.21	3.63**	1.58*	38.63***	76.57***	93.34***
	SIP Quarterly					0.99	33.55***	69.90***	88.90***
Debt Scheme - Low Duration Fund	SIP Monthly	-0.60	-0.74	2.16**	0.86	5.75***	32.07***	148.80***	189.77***

	SIP Quarterly					5.16***	28.74***	137.53***	185.49***
Debt Scheme - Medium to Long Duration Fund	SIP Monthly	0.52	-0.61	-0.31	-0.93	22.63***	68.31***	46.80***	110.92***
	SIP Quarterly					15.88***	61.26***	42.73***	107.03***
Debt Scheme - Corporate Bond Fund	SIP Monthly	0.01	-0.90	-0.79	-0.89	12.50***	17.65***	15.41***	55.74***
	SIP Quarterly					7.94***	16.38***	14.90***	51.25***
Debt Scheme - Gilt Fund	SIP Monthly	0.08	-0.09	-0.70	-0.75	12.35***	69.80***	37.23***	89.30***
	SIP Quarterly					8.07***	62.40***	34.59***	90.22***
Hybrid Scheme - Aggressive Hybrid Fund	SIP Monthly	0.21	-0.54	-0.97	-0.85	10.28***	40.39***	19.94***	110.33***
	SIP Quarterly					6.92***	35.84***	17.96***	102.08***

This table shows the Anderson Darling k-sample test statistics using the returns of strategies as pairs. We compare the monthly-quarterly, monthly-LSI and quarterly-LSI pairs across T = 1, 3, 5, and 7 years and scheme sub-category. Significance is shown next to the statistics. (p < 0.01: ***, < 0.05: ** and < 0.01 < 0.1: *)

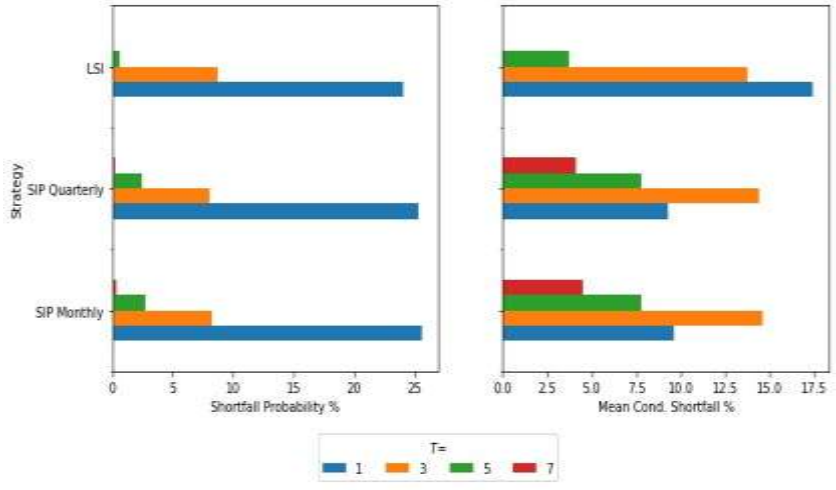
Table 7: Summary Anderson Darling k-sample test: Equity mutual funds

		SIP Quarterly				LSI			
Cat	period metric	1	3	5	7	1	3	5	7
Equity Scheme - ELSS	SIP Monthly	-0.04	-0.62	-0.89	-0.79	9.93***	40.17***	29.66***	77.57***
	SIP Quarterly					6.06***	38.49***	26.04***	73.44***
Equity Scheme - Large Cap Fund	SIP Monthly	-0.20	-0.48	-0.93	-0.75	9.61***	38.66***	16.38***	48.12***
	SIP Quarterly					6.82***	34.15***	14.08***	43.72***
Equity Scheme - Large & Mid Cap Fund	SIP Monthly	0.21	-0.50	-1.04	-0.75	11.13***	37.46***	14.12***	83.03***
	SIP Quarterly					7.69***	33.36***	12.65***	77.08***
Equity Scheme - Mid Cap Fund	SIP Monthly	-0.20	-0.49	-0.81	-0.65	11.38***	44.09***	23.09***	80.38***
	SIP Quarterly					7.69***	39.78***	20.10***	75.91***
Equity Scheme - Flexi Cap Fund	SIP Monthly	-0.01	-0.49	-0.99	-0.84	11.45***	34.77***	17.47***	78.74***
	SIP Quarterly					7.67***	31.13***	14.99***	71.70***

This table shows the Anderson Darling k-sample test statistics using the returns of strategies as pairs. We compare the monthly-quarterly, monthly-LSI and quarterly-LSI pairs across T = 1, 3, 5, and 7 years and scheme sub-category. Significance is shown next to the statistics.
(p < 0.01: ***, < 0.05: ** and < 0.01 < 0.1: *)

The mathematical model and the empirical data indicate that the LSI strategy has higher volatility than equivalent SIP strategies. Like we saw for returns, the standard deviations also converge in the long run. Here, too, our results are consistent with other studies (Smith and Artigue, 2018). When volatility of the underlying security is high, SIPs might demonstrate less volatility in short time horizons. However, if the volatility of the underlying security is low, SIPs are likely to show higher volatility in the short run compared to an equivalent LSI. If SIPs are saving strategies for the long run, then their volatility of returns is likely to be similar to LSI volatility. The "safer strategy" myth is valid only under specific conditions.

Figure 2: Shortfall probabilities and conditional shortfall for SIP and LSI strategies using equity schemes



Source: Author's calculations

Our results on the shortfall probabilities and conditional shortfalls show that Equity SIP strategies experienced higher shortfall probabilities than LSI after T= 3 years. Figure 2 shows the mean shortfall probabilities and the conditional shortfalls for all equity schemes in our sample. At T = 1, the shortfall probabilities are uniformly high (almost 25%), with the LSI showing the most significant conditional shortfall. At T = 3, the shortfall

probabilities and conditional shortfall are similar. SIP strategies no longer protect the investor better than an LSI strategy. At T = 5 and 7, the SIP strategies show a higher shortfall probability, although this is now small (2-3%). However, when the shortfalls occur, on average, the conditional shortfall is almost 8% - not small for an investor who has been patient for five years. This empirical result goes counter to the marketing messages extolling the virtues of a SIP as a "safer" investment method. Indeed, sequence risk inherent in all investment is very much in evidence for SIP strategies.

Finally, the empirical evidence shows differences between the monthly and quarterly SIP when T is low, but as T increases, the differences between the two diminish. The frequency of SIP, t, is directly related to the harmonic average of the entry prices (equation 3). As the number of investments increases (a function of T, the SIP term), the harmonic means will converge to the mean of the prices for T. An investor wanting to minimise standard deviation in W_T for a SIP with a short T should look to increase frequency, t. If T is long, terminal value outcomes between monthly or quarterly SIPs will converge. As said earlier, as T increases, the effect of t is more negligible.

5.3 Simulation Results

We run a simulation using S&P BSE 100 TR index prices for the three strategies as a robustness check. We change the underlying security and the data period (1 August 2006 through 31 December 2021) to examine the conclusions from the empirical analysis. Table 8 summarises the metrics and the bootstrap p-values.

Table 8: Summary metrics of 3 strategies with S&P BSE 100 TR index using Circular Bootstrap with 50,000 Iterations

Period	Metric	SIP Monthly	SIP Quarterly	LSI
	Mean Realised Return	16.75	16.43	15.34
	Mean Realised Std	16.72	18.37	20.26
	Raw Sharpe	1.00	0.89	0.76
	Shortfall Prob	24.75	24.75	22.00
1	Mean Cond. Shortfall	9.42	9.42	16.07
	Bootstrap p-val: Return	0.49	0.46	
	Bootstrap p-val: Std Dev	0.00	0.03	
	Bootstrap p-val: Shortfall Prob	0.97	0.76	
	Bootstrap p-val: Cond. Shortfall	0.00	0.00	
	Mean Realised Return	13.40	13.36	12.63
	Mean Realised Std	19.66	20.33	21.48
	Raw Sharpe	0.68	0.66	0.59
	Shortfall Prob	7.55	7.55	5.50
3	Mean Cond. Shortfall	17.23	17.23	7.39
	Bootstrap p-val: Return	0.57	0.58	
	Bootstrap p-val: Std Dev	0.00	0.00	
	Bootstrap p-val: Shortfall Prob	1.00	0.93	
	Bootstrap p-val: Cond. Shortfall	1.00	1.00	
	Mean Realised Return	12.74	12.73	12.36
	Mean Realised Std	20.55	20.96	21.80
	Raw Sharpe	0.62	0.61	0.57
	Shortfall Prob	3.79	3.79	1.10
5	Mean Cond. Shortfall	9.85	9.85	0.64
	Bootstrap p-val: Return	0.55	0.53	
	Bootstrap p-val: Std Dev	0.00	0.00	

	Bootstrap p-val: Shortfall Prob	1.00	1.00	
	Bootstrap p-val: Cond. Shortfall	1.00	1.00	
7	Mean Realised Return	12.59	12.59	12.37
	Mean Realised Std	21.08	21.39	22.05
	Raw Sharpe	0.60	0.59	0.56
	Shortfall Prob	4.91	4.91	1.29
	Mean Cond. Shortfall	15.41	15.41	13.53
	Bootstrap p-val: Return	0.56	0.56	
	Bootstrap p-val: Std Dev	0.00	0.00	
	Bootstrap p-val: Shortfall Prob	1.00	1.00	
	Bootstrap p-val: Cond. Shortfall	1.00	1.00	

This table shows the metrics of adopting 3 strategies SIP Monthly, SIP Quarterly, and LSI using the S&P BSE100 TR index for T = 1, 2, 3, and 4 years. The realised return and variances are annualised and are the means of the metrics for the number of simulations for each period. Raw sharpe uses the means of the realised returns and realised std dev. The shortfall probability is the realised probability of shortfall ($W_T < IA$) and the mean conditional shortfall is the average of realised shortfalls when they occur. The bootstrapped p-value for the annualised return (standard deviation) is the probability that the realised return (standard deviation) of the respective monthly or quarterly SIP plan is higher than the realised return (standard deviation) of the LSI. The bootstrapped p-value for the shortfall probability (mean conditional shortfall) is the probability that the shortfall probability (mean conditional shortfall) under the respective monthly or quarterly SIP is higher than the respective metric for the equivalent LSI. S&P BSE 100 TR index data from 1 August 2006 through 31 December 2021.

The results from the simulation are similar to the empirical evidence reported for the three strategies using debt, hybrid and equity mutual funds: returns and volatilities converge as T increases, LSI has the highest volatility, and SIP strategies have higher shortfall probabilities and conditional shortfalls at $T > 3$ years. The one big difference is that the simulation's LSI strategy does not have the highest return at $T = 7$. The autocorrelations and inherent distributions of price returns are maintained in the simulation and hence results. We posit that the results arise from the sample dataset[18]. As we would expect, when T increases, the spread between SIP returns and LSI returns narrows. Though the mean realised returns are higher for the SIP strategies than their LSI equivalent across T, the bootstrap p-values range between 45-55%, indicating that the differences are not statistically significant. SIP strategies do not outperform equivalent LSI strategies. Importantly, they are not worse. Our results are aligned with Knight and Mandell (1993); Lei and Li (2007); Zenti (2013).

Like returns, volatilities converge over time. However, the volatilities for LSI strategies remain consistently above those of SIP strategies, as evidenced by the bootstrap p-values. Further, between the monthly and quarterly SIPs, the monthly SIP shows a lower realised standard deviation as implied by the theoretical model in section 3. This difference in realised standard deviation reduces as T increases.

Consequently, SIP strategies do better in our simulation on a risk-adjusted basis, but as T increases, the risk-adjusted returns converge. The convergence of risk and returns of both SIP strategies over time implies T, the tenor, and to a minor degree, t, the frequency of a SIP affects its outcomes. This conclusion is aligned with the findings of Kirkby et al. (2020) and our empirical analysis above.

[18] When we run the simulations using different start points for the S&P BSE 100 Index, at $T = 7$, LSI returns are indeed higher than SIP returns. As expected, the simulated returns change depending on the data series. The broader set of results, however, continues to show the same pattern.

The simulation results for shortfall probabilities and mean conditional shortfall echo the results from the empirical analysis: SIP strategies have higher shortfall probabilities and average conditional shortfall for T > one year. This result is similar to that by Lei and Li (2007) but contrary to what Zenti (2013) reports.

The conclusions from the empirical analysis and the bootstrap simulation are very similar. The findings are what the theoretical framework implies. In addition, the results are identical to several other international studies on the subject.

6 ***Conclusion***

We started by outlining a theoretical framework for a comparative analysis of SIP and LSI. This framework lays out some essential theoretical insights into the nature of the SIP. Specifically, the framework shows why the average entry price of a SIP will be lower than or equal to the average prices on the entry dates, why expected returns for LSI strategies tend to be higher than those for SIP, why the expected volatility is likely to be lower, and the path dependence of SIP returns and risk. It is essential to recognise that a SIP strategy will potentially significantly underperform a buy-and-hold approach in a scenario where prices are monotonically decreasing. The deterministic nature of a SIP and the inherent path dependence of its returns poses a sequence risk that investors should consider. Our empirical analysis and simulation results assume that our data's generally rising price trend holds over the long term.

Our empirical analysis using mutual fund data and the bootstrap analysis using the S&P BSE 100 TR Index indicate that SIPs and LSI returns are not very different over long time horizons. This finding, similar to many academic studies in other markets, highlights the critical point that the deterministic nature of a SIP does not lead to significantly lower returns than an equivalent buy-and-hold strategy. Thus, SIPs are a practical method for budget- and liquidity-constrained investors to accumulate investments over time. Within the Indian context, this is an essential validation of the benefits of a SIP.

By design, under some specific conditions, the volatility of SIP returns are lower than buy-and-hold strategy returns. Over short periods, SIP could show lower volatilities and superior risk-adjusted returns than their LSI equivalents when the price volatility is high. The generalisation of this specific condition could be the basis of the popular myth that SIPs are "safer" investments. As the tenor of the investment increases, the volatilities, like returns, converge. Using return volatility as a metric, SIPs are no safer than LSI over the long term. The convergence of returns and volatilities, in the long run, is what makes SIP an attractive accumulation strategy, not necessarily a "safer" one.

The results indicate that while the SIP frequency can play a role in determining the outcomes of a SIP in the short term, it becomes less relevant as the SIP tenor increases. The path-dependent nature of SIPs requires a study of the return distribution to "optimise" the SIP frequency for short tenors. However, when SIP is used as long term wealth accumulation strategies, possibly over decades, the convergence of risk-adjusted returns implies that the SIP frequency does not materially change outcomes. This result highlights that an investor with a long tenor SIP can choose the frequency based on their unique liquidity and budget constraints without compromising long-term effects. In a sense, a long term SIP is a great equaliser.

Our analysis shows that SIP strategies have higher shortfall probabilities and higher mean conditional shortfalls than equivalent LSI strategies. The shortfall probabilities and mean conditional shortfalls remain elevated even with tenors of 5 years. This result contradicts the popular refrain that SIPs are "safer" investment strategies. SIPs have a lot going for them and, we argue, are essential as a disciplined and regular saving and investment plan with administrative ease. However, investors should consider the higher shortfall probability and the mean conditional shortfall in their investment plans.

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Appendix

Table A1: Funds in our sample

AMFI Category	SNo	Fund Name	Data From
Debt Scheme - Liquid Fund	1	Nippon India Liquid-Retail-Growth	1999-12-31
	2	JM Liquid-Growth	1999-12-31
	3	Aditya BSL Liquid Fund-Retail-Growth	1999-12-31
	4	Franklin India Liquid-Growth	1999-12-31
Debt Scheme - Low Duration Fund	5	UTI Treasury Advantage-Discont-Growth	1999-12-31
	6	Aditya BSL Low Duration Fund-Growth	1999-12-31
Debt Scheme - Medium to Long Duration Fund	7	SBI Magnum Income-Regular-Growth	1999-12-31
	8	Kotak Bond Plan A-Growth	1999-12-31
	9	Aditya BSL Income-Retail-Growth	1999-12-31
Debt Scheme - Corporate Bond Fund	10	Aditya BSL Corporate Bond-Growth	1999-12-31
	11	L&T Triple Ace Bond Fund-Growth	1999-12-31
	12	Franklin India Corporate Debt-Growth	1999-12-31
Debt Scheme - Gilt Fund	13	ICICI Prudential Gilt Fund-Growth	1999-12-31
	14	DSP Government Securities Fd-Growth	1999-12-31
	15	Aditya BSL Government Securities-Growth	1999-12-31
	16	Kotak Gilt Investment-Growth	1999-12-31
Hybrid Scheme - Aggressive Hybrid Fund	17	DSP Equity & Bond-Regular Plan-Growth	1999-12-31
	18	SBI Equity Hybrid - Rgr - Gth	1999-12-31
	19	ICICI Prudential Equity & Debt-Growth	1999-12-31
	20	Franklin India Equity Hybrid Fund-Growth	1999-12-31
	21	Aditya BSL Equity Hybrid '95 Fund-Growth	1999-12-31
	22	UTI Hybrid Equity Fund-Growth	1999-12-31
	23	Tata Hybrid Equity-Regular-Growth	1999-12-31
Equity Scheme - ELSS	24	Principal Tax Savings Fund	1999-12-31
	25	Franklin India Taxshield-Growth	1999-12-31
	26	HDFC TaxSaver-Growth	1999-12-31
	27	ICICI Prudential Long Term Equity-Growth	1999-12-31
Equity Scheme - Large Cap Fund	28	Franklin India Bluechip Fund-Growth	1999-12-31
	29	Tata Large Cap Fund-Growth	1999-12-31
Equity Scheme - Large & Mid Cap Fund	30	Aditya BSL Equity Advantage-Rgr-Growth	1999-12-31
	31	HDFC Large and Mid Cap Fund-Growth	1999-12-31
	32	ICICI Prudential Large & Mid Cap-Growth	1999-12-31
	33	Nippon India Vision-Growth	1999-12-31
Equity Scheme - Mid Cap Fund	34	Franklin India Prima Fund-Growth	1999-12-31
	35	Nippon India Growth-Growth	1999-12-31
Equity Scheme - Flexi Cap Fund	36	Franklin India Flexi Cap Fund-Growth	1999-12-31
	37	LIC MF Flexi Cap-Growth	1999-12-31
	38	Aditya BSL Flexi Cap Fund - Regular - Growth	1999-12-31
	39	HDFC Flexi Cap Fund-Growth	1999-12-31

This table shows the funds in our sample sorted by their AMFI category. The first date of price history available in our data sample for each mutual fund is also shown.