International Journal of Scientific Research and Engineering Development-– Volume 9 Issue 6, Nov – Dec 2026
Available at www.ijsred.com

RESEARCH ARTICLE OPEN ACCESS

Permeability Prediction Models for Porous Media: A Critical Synthesis of Analytical, Empirical, Digital-Rock, and Machine Learning Paradigms

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Abstract:

Permeability governs fluid transport through porous media and remains a foundational parameter in petroleum reservoir engineering, groundwater hydrology, geothermal energy extraction, and geological carbon sequestration. Accurate estimation is inherently challenging because pore geometry, connectivity, grain size distribution, and multiscale heterogeneity vary significantly across lithofacies. This critical review synthesizes five distinct methodological eras: classical hydraulic-radius and Kozeny-Carman formulations; empirical core, capillary-pressure, and well-log correlations; fractal geometry and percolation models; digital-rock imaging coupled with machine learning; and physics-informed neural networks. Rather than combining disparate studies into an oversimplified ranking, this evaluation contextualizes reported performance metrics against sample scale, resolution limits, and validation design. Classical models remain valuable when their underlying geometric premises are met, whereas empirical estimators demand rigorous local calibration. Although data-driven digital rock workflows reduce the computational burden of pore-scale numerical simulation, their generalizability relies heavily on representative training sets and robust validation protocols. Similarly, physics-informed architectures embed governing conservation laws but require careful treatment of boundary constraints and extrapolation limits. The analysis concludes by identifying key priorities for transparent benchmarking, uncertainty quantification, and scale-aware upscaling in petrophysical modeling.

Keywords — Absolute permeability, Porous media, Kozeny-Carman equation, Digital rock physics, Machine learning, Fractal geometry, Physics-informed neural networks, Uncertainty quantification

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I. INTRODUCTION

Modern subsurface engineering initiatives spanning deep saline aquifer carbon dioxide storage, underground hydrogen containment, and enhanced geothermal resource recovery demand highly rigorous mathematical characterization of fluid flow through geological formations. Absolute permeability (k), which dictates the volumetric flow rate of single-phase fluids under a macroscopic pressure gradient, directly influences the economic viability and long-term containment security of subsurface projects. However, natural rock fabrics exhibit complex spatial heterogeneity across vast length scales: sub-micron, slit-shaped pore apertures in organic-rich shales control micro-darcy flow dynamics, whereas macroscopic dissolution vugs and interconnected fracture networks dominate fluid movement in highly productive carbonate reservoirs. Reconciling transport physics across these disparate spatial dimensions represents one of the most persistent challenges in petrophysics and subsurface computational mechanics.

While laboratory core flooding remains the primary empirical benchmark for permeability

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determination, core recovery is expensive, physical samples are frequently sparse, and measurements are subject to directional anisotropy and core damage artifacts. Wireline well logs offer continuous vertical coverage, yet their predictive reliability depends heavily on formation-specific calibration and tool resolution limits. To bridge these gaps, digital rock physics (DRP) has emerged as a powerful auxiliary approach. High-resolution micro-computed tomography (micro-CT) and focused-ion-beam scanning electron microscopy (FIB-SEM) image volumes are segmented into pore and solid matrix domains, enabling direct numerical simulation of pore-scale flow or the training of data-driven surrogate models [9], [10], [23]. Because direct pore-scale solvers demand substantial computational resources, machine learning surrogates have gained traction for mapping image topologies or tabular petrophysical logs directly to macroscopic permeability [12].

The evolution of permeability prediction reflects a steady methodological expansion: starting from idealized capillary tube representations [16], [4], [5] and statistical core regressions [22], [3], [21], [19], [2], [18], the field has progressed toward fractal scaling [26], [24], [25], percolation network theory [15], [11], convolutional neural networks [23], [9], transformer architectures [10], graph-based learning [14], and physics-informed neural networks [20], [7]. These paradigms are not interchangeable; they embody distinct physical assumptions, input dependencies, computational costs, and sensitivity to scale.

Although previous literature has reviewed digital imaging, pore-scale simulation, or well-log prediction in isolation, synthesizing these developments alongside classical hydraulic formulations is essential. This review critically evaluates the underlying physics, mathematical structures, data requirements, validation practices, and practical limitations across these modeling traditions.

II. REVIEW APPROACH AND SCOPE

This paper is structured as a critical narrative review and methodological synthesis rather than a registered systematic review or meta-analysis. The literature evaluation encompasses foundational and contemporary contributions published between 1927 and 2025 across major academic repositories, including Scopus, Web of Science, OnePetro, and IEEE Xplore. Literature identification was conducted through December 2025 using targeted Boolean combinations of core petrophysical search terms: ("absolute permeability" OR "intrinsic permeability") AND ("porous media" OR "petrophysics") AND ("Kozeny-Carman" OR "digital rock physics" OR "machine learning" OR "physics-informed neural networks" OR "fractal geometry"). Candidate studies were selected purposively based on theoretical significance, mathematical completeness, empirical validation rigor, and representation of major methodological paradigms across classical, fractal, digital-rock, and physics-informed modeling, yielding the 26 primary works synthesized in this review.

To establish a rigorous comparative basis, the review evaluates models across six critical diagnostic axes:

1. Underlying physical and geometric assumptions governing the pore network;
2. Mathematical formulations and precise parameter definitions;
3. Input data requirements, spatial support, and measurement scale;
4. Degree of physical interpretability versus black-box empirical behavior;
5. Computational overhead during training and operational deployment; and
6. Validation rigor, out-of-distribution generalization, and uncertainty quantification.

Reported statistical performance metrics (such as coefficient of determination R^2, root mean square error RMSE, and mean absolute error MAE) are interpreted within the context of each study's specific sample lithology, image resolution, numerical solver, and cross-validation design. Direct quantitative ranking across disparate studies is avoided due to non-standardized datasets and evaluation protocols. The primary focus centers on single-phase absolute permeability (k), with

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multiphase relative permeability and reactive transport noted as advanced extensions.

III. PHYSICAL BASIS AND CLASSICAL MODELS

A. Darcy's Law and Permeability Tensor

For steady, laminar, single-phase flow of an isothermal Newtonian fluid through a fully saturated porous medium, Darcy's law establishes the linear proportionality between macroscopic volumetric flux (Darcy velocity, q) and the fluid pressure gradient (∇p) [8]:

$$q = -(k / \mu)\ \nabla p \quad (1)$$

where q represents the superficial velocity vector (m/s), k is the intrinsic absolute permeability (m²), μ is the dynamic fluid viscosity (Pa·s), and p is the fluid pressure (Pa). In anisotropic formations, fluid transport is directional, requiring permeability to be expressed as a symmetric second-order tensor K:

$$q = -(K / \mu)\ \nabla p \quad (2)$$

Enforcing mass conservation for an incompressible fluid under steady-state conditions without internal sources or sinks yields the governing flow equation:

$$\nabla \cdot q = 0 \ \Rightarrow\ \nabla \cdot (-(K / \mu)\ \nabla p) = 0 \quad (3)$$

Intrinsic permeability k is strictly a property of the porous rock matrix and is independent of fluid characteristics. It is quantified in square meters or darcies (1 Darcy ≈ 0.986923 × 10^{-12} m²). Intrinsic permeability must be differentiated from hydraulic conductivity (Kh = kρg / μ), which incorporates fluid density (ρ) and gravitational acceleration (g).

B. Kozeny-Carman Formulation and Surface Area Conventions

The Kozeny-Carman model represents a landmark analytical effort to connect macroscopic permeability to internal pore geometry by modeling the porous medium as an equivalent bundle of tortuous capillary channels [16], [4], [5]. The standard expression is written as:

$$k = \varphi^3 / (Ck\ Sv^2\ \tau^2) \quad (4)$$

where φ is fractional interconnected porosity, Sv is specific surface area per unit bulk volume (m^{-1}), Ck is the dimensionless Kozeny shape factor accounting for capillary cross-sectional geometry (typically ranging from 2.0 for circular tubes to 2.5 for non-circular channels), and τ is hydraulic tortuosity (defined as Le/L ≥ 1.0, where Le represents actual effective streamline length and L is straight-line sample length).

Crucial Convention Distinction: When specific surface area is defined per unit solid grain volume (Sgv, m^{-1}), bulk surface area Sv relates to grain surface area via Sv = Sgv(1 - φ). Substituting this into Equation (4) yields the standard modified Kozeny-Carman equation [4], [5]:

$$k = \varphi^3 / [Ck\ Sgv^2\ (1 - \varphi)^2\ \tau^2] = \varphi^3 / [cm\ Sgv^2\ (1 - \varphi)^2] \quad (5)$$

where cm = Ck τ² is the dimensionless composite Kozeny constant, historically approximated as cm ≈ 5.0 for random packings of uniform spheres [4], [5]. Equations (4) and (5) demonstrate why Sv and Sgv cannot be used interchangeably without accounting for the (1 - φ)² term. Furthermore, tortuosity must be treated consistently: if an empirical composite constant already embeds tortuosity, a separate tortuosity multiplier must not be applied. While the Kozeny-Carman approach provides robust physical insight for clean granular packs, its assumptions break down in vuggy, fractured, poorly sorted, or clay-rich media where pore-throat constrictions and complex topological connectivity dominate flow.

C. Empirical Correlations and Hydraulic Flow Units

Empirical models estimate permeability by correlating core plug measurements or wireline log responses with petrophysical properties. Timur [22] formulated a widely used empirical regression for clean sandstone reservoirs relating permeability k (in millidarcies, mD) to porosity and irreducible water saturation (Swi):

$$k = 0.136\ (\varphi^{4.4} / Swi^2) \quad (6)$$

The numerical coefficient in Equation (6) is tied directly to the original unit convention [22], where porosity φ and irreducible water saturation Swi are entered as percentages (e.g., 20 rather than 0.20) and permeability is output in millidarcies.

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Substituting fractional inputs without adjusting the coefficient yields erroneous predictions.

Other empirical formulations incorporate grain size statistics and mercury injection capillary pressure (MICP) characteristics. Berg [3] correlated permeability k (in millidarcies, mD) with fractional porosity φ (dimensionless), median grain diameter d50 (in millimeters, mm), and graphic phi-scale sorting dispersion σ_φ (in dimensionless phi units):

$$k = 5.1 \times 10^{-3}\, \varphi^{5}\, d50^{2}\, \exp(-1.38\, \sigma_{\varphi}) \quad (7)$$

Panda and Lake [18] developed a statistical permeability model for unconsolidated granular packs incorporating particle-size distribution moments, where k is permeability (in millidarcies, mD), C is a dimensionless empirical proportionality constant, φ is fractional porosity (dimensionless), dg is mean grain diameter (in micrometers, μm), sg is the dimensionless particle-size skewness, and σ_{g}^{2} denotes dimensionless normalized sorting variance:

$$k = C\, [\varphi^{3} / (1 - \varphi)^{2}]\, dg^{2}\, \exp(-4sg + 4\sigma_{g}^{2}) \quad (8)$$

Swanson [21] correlated air permeability (ka, in millidarcies) with mercury capillary pressure data by identifying the apex of the ratio of mercury saturation (Sb, bulk volume percentage) to capillary pressure (Pc, in psi):

$$ka = 399\, [(Sb / Pc)_{max}]^{1.691} \quad (9)$$

Equation (9) leverages the apex of the capillary pressure curve where the ratio Sb/Pc reaches its maximum value, capturing the size of the dominant connected pore throats governing flow [21].

Pittman [19] established logarithmic regressions relating air permeability (K_air, in millidarcies) to percentage porosity ($\varphi_{pct} = 100\varphi$) and the pore-throat radius corresponding to 25% mercury saturation (r25, in micrometers), utilizing base-10 logarithms ($\log_{10}$):

$$\log_{10}(K_{air}) = -1.221 + 1.512\, \log_{10}(r25) + 1.415\, \log_{10}(\varphi_{pct}) \quad (10)$$

Using r25 rather than r35 isolates the specific mercury saturation threshold that correlates most reliably with interconnected pore networks in sandstones, while percentage porosity aligns with Pittman's regression coefficients [19].

To handle reservoir heterogeneity independently, Amaefule et al. [2] introduced the Hydraulic Flow Unit (HFU) framework based on the Flow Zone Indicator (FZI). Using fractional effective porosity (φe, dimensionless) and permeability k (in millidarcies, mD), the Reservoir Quality Index (RQI, in micrometers) and normalized porosity index (phiz, dimensionless) are defined as:

$$RQI = 0.0314\, \sqrt{(k / \varphi e)} \quad (11)$$

$$phiz = \varphi e / (1 - \varphi e) \quad (12)$$

This yields the operational Flow Zone Indicator [2], incorporating shape factor Fs (dimensionless), hydraulic tortuosity τ (dimensionless), and specific grain surface area Sgv (in μm^{-1}):

$$FZI = RQI / phiz = [0.0314\, \sqrt{(k / \varphi e)}] / [\varphi e / (1 - \varphi e)] = 1 / (\sqrt{Fs}\, \tau\, Sgv) \quad (13)$$

In Equations (11) and (13), the conversion factor 0.0314 arises explicitly from units where permeability k is expressed in millidarcies (mD) and effective porosity φe is fractional, yielding RQI in micrometers. Because RQI is expressed in micrometers and the normalized porosity index phiz is dimensionless, FZI retains units of micrometers (μm), consistent with $1 / (\sqrt{Fs}\, \tau\, Sgv)$ where Sgv is in μm^{-1}.

Samples exhibiting comparable FZI values share similar pore-throat attributes and can be grouped into distinct hydraulic flow units [2]. Overall, classical and empirical models provide rapid estimations but require careful calibration against laboratory measurements to capture local fabric complexities across heterogeneous intervals.

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TABLE I. HISTORICAL DEVELOPMENT OF SELECTED ANALYTICAL AND EMPIRICAL PERMEABILITY MODELS

Model / Source	Formulation	Assumptions	Limitations
Kozeny [16]	$k = \varphi^3 / (C_k S_v^2 \tau^2)$	Tortuous capillary bundle	Neglects pore shape variations
Carman [4], [5]	$k = \varphi^3 / [c_m S_{gv}^2 (1 - \varphi)^2]$	Tortuous capillary bundle	Composite Kozeny constant assumed constant
Timur [22]	$k = 0.136 (\varphi^{4.4} / S_{wi}^2)$	Empirical core plug regression	Restricted to clean sandstone formations
Berg [3]	$k = 5.1 \times 10^{-3} \varphi^5 d_{50}^2 \exp(-1.38 \sigma_\varphi)$	Log-normal grain size distribution	Sensitive to sorting variance accuracy
Panda & Lake [18]	$k = C [\varphi^3 / (1 - \varphi)^2] d_g^2 \exp(-4s_g + 4\sigma_g^2)$	Particle-size statistics and sorting	Scatters in tight or heterogeneous lithologies
Swanson [21]	$k_a = 399 [(S_b / P_c)_{max}]^{1.691}$	Mercury capillary pressure apex ratio	Requires full MICP curve measurement
Pittman [19]	$\log_{10}(K_{air}) = -1.221 + 1.512 \log_{10}(r25) + 1.415 \log_{10}(\varphi_{pct})$	r25 pore-throat radius & percentage porosity	Requires core calibration per sandstone facies
Amaefule et al. [2]	$FZI = RQI / \phi_z = [0.0314 \sqrt{(k / \varphi_e)}] / [\varphi_e / (1 - \varphi_e)]$	Hydraulic Flow Units & effective porosity φ_e	Requires extensive core calibration across intervals

IV. FRACTAL GEOMETRY AND PERCOLATION APPROACHES

To address the limitations of idealized capillary models in complex pore networks, fractal geometry and percolation theory introduce scale-invariant descriptions of rock fabric. Yu and Cheng [26] derived an analytical fractal permeability model for bi-dispersed porous media by modeling tortuous flow paths across fractal pore size distributions. For a representative cross-sectional area A (m^2) and straight-line sample length L_0 (m), intrinsic permeability k (m^2) is governed by the pore-area fractal dimension Df ($1 < D_f < 2$ in two dimensions, $2 < D_f < 3$ in three dimensions), tortuosity fractal dimension DT ($1 < D_T < 2$), and maximum pore-throat diameter λ_{max} (m) [26]:

$$k = (\pi / 128) (L_0^{(1 - D_T)} / A) [D_f / (3 + D_T - D_f)] \lambda_{max}^{(3 + D_T)} \quad (14)$$

Equation (14) is derived under the self-similar scaling condition $\lambda_{min} / \lambda_{max} \ll 1$ without introducing empirical fitting constants, demonstrating how permeability scales directly with pore-size heterogeneity and tortuosity exponent [26]. Yu [25] further showed that the effective Kozeny constant varies systematically with fractal dimensions and porosity rather than remaining invariant. Xu and Yu [24] extended fractal capillary-bundle theory to incorporate pore-wall surface roughness.

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Percolation theory examines connectivity in disordered media near critical thresholds. Katz and Thompson [15] formulated a percolation model linking intrinsic permeability k (m^2) to a characteristic critical pore-throat diameter lc (m) associated with the continuous spanning cluster:

$$k = (1 / 226)\, lc^2\, (\sigma / \sigma_0) \quad (15)$$

Here, lc is the critical pore-throat diameter controlling electrical and hydraulic percolation (m), σ is the bulk electrical conductivity of the brine-saturated rock (S/m), and σ_0 is the electrical conductivity of the saturating brine (S/m), making the conductivity formation-factor ratio σ / σ_0 dimensionless [15]. Mavko and Nur [17] analyzed percolation thresholds within the Kozeny-Carman framework, showing that as porosity approaches a critical percolation limit, connected pathways disappear and permeability vanishes despite non-zero total porosity. Ghanbarian et al. [11] reviewed tortuosity definitions, emphasizing that geometric, hydraulic, electrical, and diffusive tortuosity coefficients are fundamentally distinct and not interchangeable.

V. DIGITAL ROCK PHYSICS AND MACHINE LEARNING

A. Image-Based Permeability Prediction (2D/3D CNNs, Transformers, GNNs)

Digital rock physics (DRP) workflows extract 3D pore architecture from micro-CT or FIB-SEM scans, segmenting voxels into pore and mineral phases. Permeability is then computed either via numerical fluid flow solvers (such as the Lattice Boltzmann Method or Navier-Stokes finite volume solvers) or through data-driven surrogate models [23], [9]. Modern machine learning architectures map spatial image features directly to permeability values:

- 2D Convolutional Neural Networks: Extract local fabric textures and planar porosity from thin sections or SEM slices, benchmarked against computational fluid dynamics (CFD) simulations on synthetic 2D media [23].
- Three-dimensional CNNs: Process volumetric 3D image subvolumes to capture spatial pore geometry and connectivity, evaluated against Lattice Boltzmann Method (LBM) flow simulations [9].
- Swin Transformers: Leverage shifted-window self-attention mechanisms to capture long-range spatial dependencies across image patches [10].
- Graph Neural Networks (GNNs): Represent pore space topology as mathematical graphs (nodes for pore bodies, edges for connecting throats), trained against LBM simulation datasets [14].

B. Tabular Petrophysical Machine Learning

For well-log and core data integration, tabular models such as Random Forests, Support Vector Regression, and neural networks estimate permeability from wireline logs [12]. Rigorous cross-validation design is vital here: because depth-adjacent measurements in a single well exhibit strong spatial autocorrelation, conventional random row-wise splitting places correlated data in both training and test subsets, resulting in overly optimistic performance estimates [12]. Validating models properly requires holding out entire wells or independent core sets [12]. Furthermore, feature scaling and hyperparameter optimization play critical roles in mitigating overfitting when dealing with highly skewed petrophysical distributions across heterogeneous formations [12].

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TABLE II. COMPARATIVE OVERVIEW OF MACHINE-LEARNING APPROACHES AND VALIDATION CONTEXTS

Architecture	Input Data	Primary Advantages	Limitations & Validation Context
2D Convolutional Neural Networks	2D thin-section images / SEM slices	Low computational cost, fast inference	Lacks 3D pore connectivity representation [23]
3D Convolutional Neural Networks	Volumetric 3D micro-CT subvolumes	Directly captures spatial pore geometry	High computational and memory overhead [9]
Multi-scale 3D CNNs	Multi-resolution 3D image pyramids	Captures hierarchical pore structures	Sensitive to image resolution degradation [9]
Swin Transformers	2D image patches	Long-range self-attention mechanisms	High memory footprint during training [10]
Graph Neural Networks (GNNs)	Pore network extraction graphs	Direct topological representation of paths	Relies heavily on accurate pore network extraction [14]
Random Forest / SVR	Tabular wireline logs and core data	Handles non-linear petrophysical trends	Prone to overfitting without spatial splitting [12]

VI. PHYSICS-INFORMED AND MULTISCALE APPROACHES

Physics-informed neural networks (PINNs) embed governing partial differential equations directly into the network training loss function [20]. For steady-state, single-phase, incompressible Darcy flow with constant viscosity and no internal sinks, the pressure field p(x) and permeability field K(x) must satisfy mass conservation and Darcy's law [20]:

$$\nabla \cdot q = \nabla \cdot (-(K(x) / \mu) \nabla p(x)) = 0 \quad (16)$$

The composite training objective minimizes a weighted sum of observational mismatch, interior partial differential equation (PDE) residuals, and boundary constraints [20]:

$$L_{total} = w_{data} L_{data} + w_{physics} L_{physics} + w_{boundary} L_{boundary} \quad (17)$$

where L_data measures the mean squared discrepancy against sparse core or pressure observations, L_physics penalizes the squared residual of Equation (16) evaluated at interior collocation points via automatic differentiation, and L_boundary enforces prescribed Dirichlet pressure and Neumann no-flux conditions along domain boundaries; w_data, w_physics, and w_boundary are scalar weighting hyperparameters balancing gradient magnitudes during optimization [20]. Chen et al. [7] applied transfer-learning physics-informed convolutional neural networks to simulate multiphase flow with time-varying controls. Concurrently, Aldakheel et al. [1], Chaaban et al. [6], and Heider et al. [13] demonstrated advanced machine-learning-supported multiscale frameworks for capturing anisotropic and deformable poromechanical behavior in heterogeneous porous media.

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TABLE III. COMPREHENSIVE METHODOLOGICAL TRADE-OFF MATRIX

Paradigm	Interpretability	Data Requirements	Computational Expense	Lithology Generalization
Classical Kozeny-Carman	High (Explicit variables: φ, Sv)	Low (Porosity, specific surface area)	Minimal (Instantaneous analytical)	Limited to granular media matching assumptions
Empirical Correlations	Moderate (Regression coefficients)	Moderate (Core plugs, MICP, logs)	Minimal (Algebraic evaluations)	Low outside calibrated formations
Fractal Geometry Models	High (Dimensions Df, DT)	Moderate (Image fractal metrics, MICP)	Low to Moderate (Analytical multi-scale)	Conditional on self-similarity range
Deep Learning (CNN/Transformers)	Moderate (Learned feature maps)	High (Large labeled 3D image sets)	High (Intensive GPU training)	Requires testing on unseen lithologies
Physics-Informed ML (PINNs)	Moderate (PDE constraints embedded)	Moderate (Sparse data + PDE residuals)	High (PDE residual optimization)	Conditional on PDE validity and boundary setup
Hybrid Multi-Scale Systems	Moderate to High (Explicit physics)	High (Multi-resolution CT + core)	High (Coupled micro-macro solvers)	Problem-specific cross-scale transfer

VII. COMPARATIVE SYNTHESIS

A key takeaway from this review is that dataset-independent R^2 values should never be used in isolation to rank permeability prediction models [23], [9], [10]. The coefficient of determination is sensitive to the variance and dynamic range of the target dataset. Predictive performance must instead be evaluated alongside absolute error measures, logarithmic error evaluation ($\log_{10}$ k), and rigorous out-of-distribution testing across diverse lithologies.

VIII. RESEARCH GAPS AND STRATEGIC RECOMMENDATIONS

1. Open-Source Benchmarking: Advancing reproducible petrophysical modeling requires standardized, publicly accessible benchmark datasets pairing 3D micro-CT volumes with validated multi-scale core measurements.

2. Rigorous Uncertainty Quantification: Data-driven and physics-informed models must incorporate Bayesian frameworks or deep ensembles to distinguish aleatoric noise from epistemic prediction uncertainty [20].

3. Scale-Aware Upscaling: Future research must prioritize scale-bridging operators that account for tensor anisotropy and Representative Elementary Volume (REV) stability across hierarchical domains [13], [6].

IX. CONCLUSIONS

Permeability prediction has evolved through distinct analytical, empirical, fractal, digital-rock, and physics-informed paradigms, each operating across specific spatial scales and physical assumptions. No single modeling approach is universally optimal; prediction success depends on data quality, image resolution, and validation

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design. Future advancements rely on open benchmarking, physical constraint integration, and robust scaling methodologies.

ACKNOWLEDGMENT

The authors wish to acknowledge the International Journal of Scientific Research and Engineering Development (IJSRED) and Causal Productions for providing the author formatting templates utilized in the preparation of this publication.

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