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Hexagonal Neutrosophic Fuzzy Modeling for Uncertainty-Based Transportation Problem Optimization

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Abstract:

In real-world applications, classical transportation problems are often inadequate for addressing uncertainties arising from traffic congestion, unpredictable weather conditions, fluctuating transportation costs, and other unforeseen factors. To address these challenges, neutrosophic theory has been incorporated into transportation problem modelling to represent uncertainty more effectively. This paper aims to develop a systematic approach for determining the Basic Feasible Solution (BFS) of a Neutrosophic Transportation Problem using Hexagonal Neutrosophic Fuzzy Numbers (HNFNs). The relevant parameters of the transportation table are represented by HNFNs, which enable the effective representation of imprecision, indeterminacy, and vagueness associated with real-world transportation scenarios. Subsequently, a score function is employed to defuzzify these neutrosophic fuzzy numbers into crisp values, thereby facilitating computational implementation. A structured algorithm is developed to determine the BFS of the transportation problem using the proposed approach. Furthermore, a comparative analysis with existing methods is conducted to evaluate the effectiveness and computational performance of the proposed method, demonstrating its potential to support decision-making under uncertain conditions.

Keywords —**Hexagonal neutrosophic fuzzy number, Harmonic Mean, Transportation Problem, Optimization, Score function.**

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I. INTRODUCTION

In this modern era, the role of fuzzy optimization techniques in healthcare, engineering and industrial sectors has garnered massive attention due to their high accuracy. In 1965, Zadeh [7] introduced the concept of fuzzy sets with a membership function which is a best mathematical manner for representing vagueness. Sometimes, the membership function in Fuzzy Set was not appropriate one to describe the uncertainty of a problem. After development of FS theory, in 1986[1] Atanassov developed the concept of Intuitionistic fuzzy sets (IFSs) which is very useful to deal with the vagueness. It is an extension of the fuzzy sets, which included the degree of membership and degree of non-membership function. Further it is observed that a membership degree and non-membership degree does not state properly to the uncertainty situation in the real -life problems. However, real-world transportation environments are often characterized by imprecise, vague, and uncertain information due to fluctuating costs, variable traffic conditions, unforeseen delays, and incomplete knowledge about supply or demand.

To address such uncertainties, researchers have extended fuzzy set theory and intuitionistic

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fuzzy sets to neutrosophic sets which is introduced by Smarandache in 2005. Among these, Neutrosophic Sets (NS) offer a more generalized and flexible mathematical framework to handle indeterminacy and inconsistency explicitly, by incorporating degrees of truth, falsity, and indeterminacy independently. This becomes particularly powerful when dealing with uncertain parameters in transportation systems. Building upon this, Hexagonal Fuzzy Numbers (HFNs)—an extension of traditional fuzzy numbers—are capable of modeling uncertainty in a more detailed manner by capturing multiple levels of information through six defining points. When integrated with neutrosophic theory, Hexagonal Neutrosophic Fuzzy Numbers (HNFNs) allow for a more nuanced representation of uncertain data, offering improved flexibility and expressiveness over triangular or trapezoidal fuzzy models. For instance, Kanchana and Kavitha (2024) explored single-valued neutrosophic transportation problems and conducted a sensitivity analysis to assess the robustness of solutions. Similarly, Kumar and Karthikeyan (2022) proposed an algorithmic solution for transportation problems using trapezoidal neutrosophic numbers.

Hexagonal fuzzy numbers (HFNs), as implemented by Rashmanlou et al. (2014), provide a more detailed and flexible structure for modeling uncertain data with six defining points. When integrated into the neutrosophic framework, Hexagonal Neutrosophic Fuzzy Numbers (HNFNs) allow a richer and more accurate representation of imprecise parameters in transportation problems. This hybridization facilitates better decision-making in scenarios with multiple layers of uncertainty.

This paper analyses the initial basic feasible solution of the neutrosophic transportation problems with hexagonal fuzzy neutrosophic numbers and propose an effective methodology to resolve a neutrosophic transportation problem. This paper discusses a score function, and present a sequential method of the proposed algorithm to solve the neutrosophic transportation problem. Subsequently, the other sections of the paper are addressed in the following manner: Section 2 emphasizes the basic definitions. The Hexagonal neutrosophic transportation problem and its formulation in section 3. In section 4, the algorithm and in Section 5 explains the numerical example as an illustrative demonstration. The important outcomes of the work done are discussed in section 6 and concludes the article.

II. PRELIMINARIES

2.1 Neutrosophic Fuzzy Set

A Neutrosophic Fuzzy set $\breve{\aleph}$ can be characterized by

$\breve{\aleph} = \{< x, \rho_{\breve{\aleph}}(x), \tau_{\breve{\aleph}}(x), \vartheta_{\breve{\aleph}}(x) > / x \in X\}$ where

$\rho_{\breve{\aleph}}(x) is\ represented\ as\ truth$ membership function, $\tau_{\breve{\aleph}}(x)$ as indeterminancy membership function $and\ \vartheta_{\breve{\aleph}}(x) is\ falsity\ membership\ function$, $\rho_{\breve{\aleph}}, \tau_{\breve{\aleph}}, \vartheta_{\breve{\aleph}} : X \to -]0,1[^{+}$ and $-0 \leq \rho_{\breve{\aleph}} + \tau_{\breve{\aleph}} + \vartheta_{\breve{\aleph}} \leq 3^{+}$.

2.2 Neutrosophic Hexagonal Fuzzy Number

A Neutrosophic Hexagonal Fuzzy Number is defined in a Neutrosophic set

$\breve{\aleph} = \{< x, \rho_{\breve{\aleph}}(x), \tau_{\breve{\aleph}}(x), \vartheta_{\breve{\aleph}}(x) > : x \in X\}$ are

$\rho_{\breve{\aleph}}(x) = \left(\rho_{\breve{\aleph}}{}^{1}(x), \rho_{\breve{\aleph}}{}^{2}(x), \rho_{\breve{\aleph}}{}^{3}(x), \rho_{\breve{\aleph}}{}^{4}(x), \rho_{\breve{\aleph}}{}^{5}(x), \rho_{\breve{\aleph}}{}^{6}(x)\right)$,

$\tau_{\breve{\aleph}}(x) = \left(\tau_{\breve{\aleph}}{}^{1}(x), \tau_{\breve{\aleph}}{}^{2}(x), \tau_{\breve{\aleph}}{}^{3}(x), \tau_{\breve{\aleph}}{}^{4}(x), \tau_{\breve{\aleph}}{}^{5}(x), \tau_{\breve{\aleph}}{}^{6}(x)\right) and$

$\vartheta_{\breve{\aleph}}(x) = \left(\vartheta_{\breve{\aleph}}{}^{1}(x), \vartheta_{\breve{\aleph}}{}^{2}(x), \vartheta_{\breve{\aleph}}{}^{3}(x), \vartheta_{\breve{\aleph}}{}^{4}(x), \vartheta_{\breve{\aleph}}{}^{5}(x), \vartheta_{\breve{\aleph}}{}^{6}(x)\right)$

respectively.

2.3 score Function

For a neutrosophical Hexagonal fuzzy number

$$\breve{\aleph} = \begin{pmatrix} \left(\rho_{\breve{\aleph}}{}^{1}(x), \rho_{\breve{\aleph}}{}^{2}(x), \rho_{\breve{\aleph}}{}^{3}(x), \rho_{\breve{\aleph}}{}^{4}(x), \rho_{\breve{\aleph}}{}^{5}(x), \rho_{\breve{\aleph}}{}^{6}(x)\right) \\ \left(\tau_{\breve{\aleph}}{}^{1}(x), \tau_{\breve{\aleph}}{}^{2}(x), \tau_{\breve{\aleph}}{}^{3}(x), \tau_{\breve{\aleph}}{}^{4}(x), \tau_{\breve{\aleph}}{}^{5}(x), \tau_{\breve{\aleph}}{}^{6}(x)\right) \\ \left(\vartheta_{\breve{\aleph}}{}^{1}(x), \vartheta_{\breve{\aleph}}{}^{2}(x), \vartheta_{\breve{\aleph}}{}^{3}(x), \vartheta_{\breve{\aleph}}{}^{4}(x), \vartheta_{\breve{\aleph}}{}^{5}(x), \vartheta_{\breve{\aleph}}{}^{6}(x)\right) \end{pmatrix}$$

and its score function is defined as

$\emptyset\left(\breve{\aleph}\right) = \left| \{\left(\frac{1}{6}\sum_{k=1}^{6} \rho_{\breve{\aleph}}{}^{k}\right) - \left(\frac{1}{6}\sum_{k=1}^{6} \tau_{\breve{\aleph}}{}^{k}\right) - \left(\frac{1}{6}\sum_{k=1}^{6} \vartheta_{\breve{\aleph}}{}^{k}\right)\} \right|$ is used to convert hexagonal fuzzy number to crisp number.

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2.4 Harmonic Mean

The harmonic mean (HM) of a set of n positive numbers $x_1, x_2, x_3, \ldots\ldots, x_n$ is defined as:

$$HM = \frac{n}{\sum_{i=1}^{n} x_i}$$

III. MATHEMATICAL FORMULATION OF NEUTROSOPHIC TRANSPORTATION PROBLEM:

The components $c_{ij}, a_i \text{ and } b_j$ are all in neutrosophic cost parameters,

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{\aleph} x_{ij}$ Subject to

$$\sum_{i=1}^{m} x_{ij} = a_i^{\aleph}, i = 1,2,\cdots,m$$

$$\sum_{j=1}^{n} x_{ij} = b_j^{\aleph}, j = 1,2,\cdots,n$$

$$x_{ij} \geq 0$$

Where $a_i^{\aleph}$ is the supply at the point i, $b_j^{\aleph}$, is the demand required in j[th] destination, x_{ij} denotes the number of units transported from the source i to the destination j and $c_{ij}^{\aleph}$ indicates the cost of transportation from the source i to the destination j.

IV. PROPOSED METHOD FOR SOLVING NTP

1. Convert all hexagonal fuzzy neutrosophic data into their corresponding crisp data using the score function (2.3) mentioned above.
2. Verify the given NT table is balanced or not. If it is balanced then go to step 3, Otherwise, it should be balanced by adding a dummy row or a dummy column
3. Find the harmonic mean (H-M) of each row/column.
4. Assign the H-M in decreasing order.
5. First select the first ordered row\column in which take the lowest cell. After all assignment values are complete, remove the corresponding column or row.
6. Repeat the above process until all the supply and demand values are satisfied.
7. Finally, compute initial basic feasible solution and total minimal transportation cost.

V. ILLUSTRATION

A health authority needs to distribute critical medical supplies from regional warehouses to district hospitals during a pandemic. Here, the cost, supply and demand of transportation in each cell is in a neutrosophic hexagonal manner (table 1).

TABLE I

The neutrosophic transportation table

	D1	D2	D3	Supply
S1	$c_{11}^{\aleph}$	$c_{12}^{\aleph}$	$c_{13}^{\aleph}$	$S_1^{\aleph}$
S2	$c_{21}^{\aleph}$	$c_{22}^{\aleph}$	$c_{23}^{\aleph}$	$S_2^{\aleph}$
S3	$c_{31}^{\aleph}$	$c_{32}^{\aleph}$	$c_{33}^{\aleph}$	$S_3^{\aleph}$
Demand	$D_1^{\aleph}$	$D_2^{\aleph}$	$D_3^{\aleph}$	

Where each time of transportation $c_{ij}^{\aleph}$ as neutrosophic numbers are follows:

$c_{11}^{\aleph}$= ((1,6,7,1,2,3), (2,3,5,9,8,7), (2.5,6,8.3,1.2,2.3,3.4))

$c_{12}^{\aleph}$ = ((1.5,3.5,5.5,2.5,2.6,2.6), (2,4,6,8,9,10), (1.5,3.5,5.5,2.3,2.4,3.5))

$c_{13}^{\aleph}$ = ((3,5,7,1,9,7), (1.5,2.5,3.5,7.8,8.9,5.6), (2,4,6,1,2,3))

$c_{21}^{\aleph}$ = ((2,3,4,4,5,6), (1.5,2.5,3.5,5.6,6.7,7.8), (1.5,2.5,3.5,8.9,9.8,7.8))

$c_{22}^{\aleph}$= ((1,3.5,4,7,5,4), (1.5,1,2.5,5.6,3.4,6.7), (1.2,5,3,4.2,5,8.9))

$c_{23}^{\aleph}$ = ((1.5,2.5,3.5,3.4,4.5,5.6), (1,1.5,3,6.7,4.5,2.3), (2,3,4,3,5,8))

$c_{31}^{\aleph}$ = ((3,4,6,3,6,8), (1.5,2.5,4.5,8.7,7.5,4.5), (3,5,7,3,5,8))

$c_{32}^{\aleph}$= ((1,5,8,1,2,3), (1.5,4.5,7.5,4,6,7), (4,6.5,9,1,7,3))

$c_{33}^{\aleph}$ = ((3,5,8,9,1,2), (1.5,3,6.5,4.5,3.4,2.7), (4,7,9,2,3,4))

$S_1^{\aleph}$ = ((10,15,20,25,35,21), (14,16,22,45,23,34), (12,15,19,12,13,45))

$S_2^{\aleph}$ = (20,25,30,35,45,46), (24,26,32,25,35,38), (22,25,29,45,46,34))

$S_3^{\aleph}$ = ((15,20,25,56,65,34), (19,21,27,12,23,34) (17,20,24,50,56,68))

$D_1^{\aleph}$ = ((13,18,23,21,32,54), (17,19,25,12,13,14),

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$(15,18,22,16,17,21))$

$D_2{}^{\breve{\aleph}}$

$= ((15,20,25,19,20,21), (19,21,27,22,23,24), (17,20,24,25,26,28))$

$D_3{}^{\breve{\aleph}}$

$= ((17,22,27,28,29,30), (21,23,29,76,32,55), (19,22,66,34,25,27))$

Solution

The process of finding IBFS is represented in Table 2-5.

TABLE 2
The finally obtained crisp NBT table

	D₁	**D₂**	**D₃**	**Supply**
S₁	6.29	6.59	2.63	24
S₂	6.27	3.92	3.83	30
S₃	5.03	6.83	3.1	26
Demand	8	26	46	

$$\left[\sum_{i=1}^{m} x_{ij} = a_i{}^{\breve{N}}\right] = \left[\sum_{j=1}^{n} x_{ij} = b_j{}^{\breve{\aleph}}\right] = 80$$

The table is balanced

TABLE 3
The process continues...

	(1)D₁	(2)D₂	(6)D₃	Supply	H-M
(5)S₁	6.29	6.59	2.63	24	4.34
(4)S₂	6.27	3.92	3.83	30	4.43
(3)S₃	5.03	6.83	3.1	26	4.49
Demand	8	26	46		
H-M	5.8	5.422	3.11		

TABLE 4
The finally allocated table

	D₁	**D₂**	**D₃**
S₁	6.29	6.59	2.63***(24)***
S₂	6.27	3.92***(26)***	3.83***(4)***
S₃	5.03***(8)***	6.83	3.1***(18)***
Demand	8	26	46

The initial basic feasible solution of the HNTP is

$$Z = 2.63 * 24 + 3.92 * 26 + 3.83 * 4 + 5.03 * 8 + 3.1 * 18 = \mathbf{276.4}$$

VI RESULTS AND DISCUSSION

The results presented in the comparison of the solutions to the neutrosophic transportation problem via hexagonal numbers obtained using the proposed method with those derived from existing methods. Table 5 presents a comparative analysis of three different methods used to solve the neutrosophic transportation problem—North West Corner Method (NWCM), Vogel's Approximation Method (VAM), and the proposed method. The objective is to minimize transportation costs under conditions of uncertainty, which are effectively modeled using neutrosophic logic. As evident from the results, this method yields the highest transportation cost, Rs. 441.9975. This is due to its nature of allocating based purely on positional logic without integrating any cost components or degrees of uncertainty. VAM is a more refined heuristic that attempts to improve cost efficiency by considering penalties associated with not choosing the cheapest options. The solution obtained using VAM, Rs. 352.16, shows a noticeable improvement over NWCM, reflecting its more strategic approach. However, it still lacks the capacity to fully exploit the uncertainty and indeterminacy embedded in neutrosophic environments. The proposed method achieves higher performance levels than both traditional approaches by producing the least transportation cost, Rs. 276.4. This method is fine-tuned for neutrosophic conditions and incorporates degrees of truth, indeterminacy, and falsity in the cost matrix. It demonstrates a higher level of adaptability and accuracy in decision-making under uncertainty, making it a more effective solution for real-world ambiguous scenarios. The decreasing trend in total cost—from NWCM to VAM to the proposed optimized method—clearly highlights the superiority of advanced, uncertainty-aware optimization techniques. The proposed method not only minimizes transportation costs effectively but also provides a robust framework to handle the vagueness and indeterminacy inherent in complex logistics problems.

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TABLE 5
Analysing the Different Approaches

Methods	Transportation Cost
North West Corner Method	Rs.352.16
Vogel's Approximation Method	Rs.278.44
Proposed optimized method	Rs.276.4

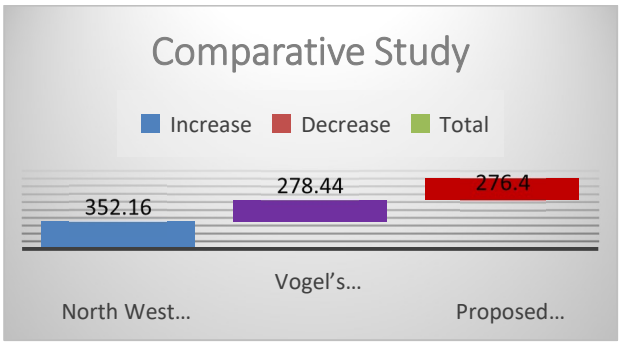


Fig. 1 Comparative study of different methods to find IBFS

7. CONCLUSIONS

In recent years, neutrosophy has emerged as a powerful framework for addressing uncertainty and indeterminacy in real-world scenarios. This study focuses on solving balanced transportation problems using hexagonal neutrosophic numbers through a newly developed method for finding initial basic feasible solutions. By representing all relevant parameters with hexagonal neutrosophic values, the approach captures uncertainty more accurately and realistically. The proposed method offers a structured and practical means of making decisions under uncertain conditions and minimize the cost effectively. Comparative analysis with existing methods highlights the enhanced flexibility and improved quality of solutions achieved by the proposed technique (Figure 1). The comparative analysis clearly illustrates the limitations of classical methods when applied to environments influenced by uncertainty and vagueness. While NWCM and VAM offer baseline solutions, they lack the sophistication to handle indeterminate data effectively. The proposed optimized method, built upon the foundation of neutrosophic logic, provides a substantial improvement in cost efficiency.

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